गणित

माध्यमिक कक्षाओं के लिए

भाग 1

(कक्षा VI की पाठ्यपुस्तक)

संपादक मण्डल

गणित

माध्यमिक कक्षाओं के लिए

भाग 1

(कक्षा VI की पाठ्यपुस्तक)

मनमोहन सिंह अरोरा आर० एम० भागवत एस० डी० चोपड़ा

संपादक

मनमोहन सिंह अरोरा



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

इस पुस्तक का प्रथम संस्करण राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् की अनुमति से जूलाई 1977 में मैसर्स दी मैकमिलन कम्पनी ऑफ इन्डिया लिमिटेड द्वारा प्रकाशित हुआ था तथा राष्ट्रीय परिषद् की अनुमति से जूलाई 1978 में उन्हीं के द्वारा पुनर्मुद्रित हुआ था। इसके बाद के पुनर्मुद्रण राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् द्वारा प्रकाशित किये गये है।

प्रथम संस्करण

जूलाई 1977
आषाढ़ 1899

पुनर्मुद्रण

जूलाई 1978
आषाढ़ 1900
जून 1979
ज्येष्ठ 1901
मार्च 1980
चैत्र 1902

P.D. 10T

मूल्य : रु० 3.10

मुख पृष्ठ चित्र : सी० पी० टंडन

प्रकाशन विभाग में, श्री विनोद कुमार पंडित राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, श्री अरविन्द मार्ग, नई दिल्ली 110016 द्वारा प्रकाशित तथा पर्ल आफसेट प्रेस, 5/33, कीर्तिनगर इन्डस्ट्रीयल एरिया, नई दिल्ली 110015 में मुद्रित।

प्राक्कथन

यह पुस्तक भारतीय रंगभूमि के अनुरूप गणित प्रस्तुत करती है। यह गणित न तो 'आधुनिक' है और न ही 'परम्परागत'। यहाँ हमारे सामने जो भी है वह है अनावश्यक पृथकता का त्याग करते हुए और विद्यार्थी को दैनिक जीवन में गणित की अनुरूपता और अनुप्रयोगों की जानकारी देते हुए तथा इसके साथ ही आगे की खोज और अध्ययन के लिए उसकी कल्पना को 'जागृत' करते हुए, उदाहरणों की सहायता से विकसित किया हुआ एक सरल, सुन्दर और सुरूप गणित। इस पाठ्यपुस्तक में कक्षा में करने हेतु अनेकों गतिविधियाँ सुझाई गई हैं और विषय सामग्री को विद्यार्थी के लिए 'परिपूर्ण' बनाने हेतु इनको उदारता से रैखिक आकृतियों एवं चित्रों द्वारा समझाया गया है। परिषद्, संपादक मंडल के अध्यक्ष डा० एम० पी० सिंह और उसके सदस्यों की आभारी है जिनके मार्ग दर्शन में यह काम पूरा किया गया।

पहले इस सामग्री का मंडल के सदस्यों के बीच विवेचन किया गया और इसके बाद 1 नवम्बर से 3 नवम्बर 1976 तक राष्ट्रीय शिक्षा संस्थान में आयोजित एक कार्यशाला में, जिसमें इस सामग्री की ध्यानपूर्वक जाँच और इसमें सुधार हेतु सुझाव प्राप्त करने के लिए संपूर्ण भारत से अनुभवी अध्यापकों और विषय-विशेषज्ञों को आमंत्रित किया गया था, इसका समीक्षात्मक विवेचन किया गया।

पुस्तक का अंतिम लेखन और विषय-संपादन डा० मनमोहन सिंह अरोरा ने किया। डा० बी० देवकीनन्दन, श्री जी० डी० दल, डा० एस०, के० सिंह गौतम एवं श्री महेन्द्र शंकर ने आवश्यक सहायता प्रदान की। डा० आर० पी० गुप्ता, डा० बी० देवकीनन्दन एवं श्री ईश्वर चन्द्र ने उत्तर तैयार किए। हिन्दी संस्करण का विषय-संपादन डा० बी० देवकीनन्दन एवं श्री महेन्द्र शंकर द्वारा किया गया। मैं इनमें से प्रत्येक का विशेष रूप से प्रो० मनमोहन सिंह अरोरा का आभारी हूँ जिन्होंने अपने व्यस्त कार्यक्रम के होते हुए भी संपादन कार्य को स्वीकार किया और बहुत कम समय के अंदर ही इसे पूर्ण किया।

इस पुस्तक को अध्यापकों और विद्यार्थियों के हाथों में सौंपते हुए मुझे प्रसन्नता हो रही है। पुस्तक के अगले संस्करण में सुधार हेतु, परिषद् पाठकों के अनुभवों का स्वागत करेगी क्योंकि हमने सदैव विद्यार्थियों के लिए अच्छे से अच्छे रूप में उचित सामग्री प्रदान करने का प्रयत्न किया है।

एस० के० मित्रा
संयुक्त निदेशक
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

नई दिल्ली
जून, 1977

प्रस्तावना

शिक्षा की 10+2 पद्धति के अंतर्गत राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् ने कक्षाओं VI से VIII तक की गणित की पाठ्यपुस्तकों और इनसे संबंधित सामग्री तैयार करने में सहायता हेतु एक संपादक मंडल का निर्माण किया। संपादक मंडल ने वर्तमान पुस्तकें, जोकि राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् ने तैयार की थीं, को देखने के अतिरिक्त व्यापक रूप से संपूर्ण स्कूल ढाँचे के लिए और विशेष रूप से कक्षा VI के लिए प्रस्तावित गणित के पाठ्यक्रम की जाँच की। ऐसा अनुभव किया गया कि शिक्षा की 10+2 पद्धति के तत्वज्ञान और उद्देश्यों तथा वर्तमान पुस्तकों के प्रयोग करने वालों से जो समालोचनाएँ प्राप्त हुई हैं उनको देखते हुए, इन वर्तमान पुस्तकों में एक दीर्घ संशोधन की आवश्यकता है। मंडल का प्रत्येक सदस्य इस तथ्य से भी अभिज्ञ था कि विभिन्न विषयों को पढ़ाने में जो विधि उपयोग में लाई जाए उसमें हमारी जनता और समाज की आवश्यकताएँ और आकांक्षाएँ प्रतिबिम्बित हों।

अत: इस पुस्तक में सरलता और सुन्दरता के साथ संकल्पनाओं का प्रस्तुतीकरण किया गया है। प्रत्येक संकल्पना को उपयुक्त उदाहरणों द्वारा प्रोत्साहित किया गया है। जहाँ तक संभव हो सका है अनावश्यक दृढ़ता का त्याग किया गया है। संकल्पना का उपयोग करने में विद्यार्थी के समर्थ होने के लिए अभ्यास हेतु हल किए उदाहरण प्रचुर मात्रा में दिए गए हैं। जहाँ संभव हो सका है, यह दृष्टिगत रखते हुए कि विद्यार्थी शिक्षण-अध्ययन प्रक्रिया में पूर्ण रूप से संबद्ध रहे, अध्यापक के लिए कक्षा में करने योग्य गतिविधियाँ सुझाई गई हैं। पुस्तक में रैखिक आकृतियाँ और चित्र उदारता से दिए गए हैं जिससे कि विषय सामग्री 'परिपूर्ण' हो जाए।

इस पुस्तक की कुछ और मुख्य विशेषताएँ निम्न पेराग्राफों में वर्णित हैं :

(1) संख्याओं की चर्चा केवल धनपूर्णांकों और पूर्ण संख्याओं तक ही सीमित रखी गई है। इसलिए भिन्नों, परिमेय संख्याओं, और दशमलव भिन्नों को कक्षा VII के लिए छोड़ दिया गया है।

(2) संख्याओं के कुछ रोचक गुणों, जैसे कि विभाज्यता की जाँच इत्यादि, को शामिल किया गया है, यद्यपि इनकी उपपत्ति नहीं दी गई है।

(3) विद्यार्थी को आधुनिक पारिभाषिक शब्द का केवल वहाँ ही ज्ञान कराया गया है जहाँ उसकी आवश्यकता प्रतीत होती है तथा जहाँ इससे उसे कुछ लाभ होता हो। उदाहरणार्थ, यह कहा गया है कि पूर्ण संख्याओं का योग क्रमविनिमेय है। परन्तु इसके साथ ही, विद्यार्थी को सुझाव दिया गया है कि वह इस नाम की चिंता न करे। उसे तो केवल यह तथ्य सीखना और देखना चाहिए कि दो पूर्ण संख्याओं के योग में क्रम का कोई प्रभाव नहीं पड़ता। जहाँ तक संभव हो सका है, अनावश्यक आधुनिक पारिभाषिक शब्दों और संकेतों का परित्याग किया गया है। उदाहरणार्थ, विद्यार्थी को केवल एक बार संख्या और संख्यांक के बीच का अंतर बता दिया गया है। इसके आगे दोनों के बीच में कोई अंतर नहीं रखा गया है और दोनों शब्दों को एक दूसरे के स्थान पर प्रयोग किया गया है।

(4) प्रत्येक अनुच्छेद के अंत में दिए गए प्रश्नों की संख्या इस प्रकार है कि विद्यार्थी उनको सरलतापूर्वक कुछ घंटों के कार्य के रूप में कर सकता है। अनुच्छेद छोटे हैं और इसी प्रकार एकक भी। इससे विद्यार्थी को, जो कुछ उसने सीखा है उसकी जानकारी प्राप्त करने में सहायता मिलेगी।

(5) उपयुक्त स्थानों पर विविध प्रश्नावलियों को शामिल किया गया है ताकि विद्यार्थी पढ़ी हुई सामग्री की पुनरावृत्ति कर सके।

(6) प्रत्येक एकक के प्रारम्भ में उसका सारांश दिया गया है ताकि विद्यार्थी यह जान सके कि वह इस विशेष एकक में क्या पढ़ने जा रहा है।

(7) अंकगणित, बीजगणित और ज्यामिति को मिलाकर एक ही पाठ्यपुस्तक बनाई गई है।

(8) लाभ और हानि, साधारण ब्याज इत्यादि के अनुप्रयोगों का अध्ययन करते समय वास्तविक जीवन से संबंधित एवं सरल स्थितियों को शामिल किया गया है।

(9) इसके लिए कि विषय सामग्री विद्यार्थी के लिए 'परिपूर्ण' हो सके, ज्यामिति के अधिकांश परिणामों का सुझाई गई गतिविधियों द्वारा सत्यापन किया गया है।

(10) पुस्तक की भाषा सरल और रोचक है। $11^{+}-12^{+}$ आयु के बच्चे के शब्दकोश को दृष्टिगत रखने का निरन्तर ध्यान रखा गया है। जहाँ तक हो सका है, सरल और छोटे वाक्यों का प्रयोग किया गया है।

मुझे यह लिखने में गर्व है कि हमें जो कार्य सौंपा गया था वह हमें इसके लिए दिए गए अल्प समय में ही पूर्ण हो गया। निस्संदेह यह मेरे साथियों में से प्रत्येक के स्वेच्छ और अपरिमित सहयोग से हुआ और मैं उनका आभारी हूँ। मंडल के प्रत्येक सदस्य ने कठोर परिश्रम और निष्ठा के साथ, अपनी शक्ति के अनुसार इसमें योगदान दिया। वास्तव में, गणित के लिए अपने को समर्पित करते हुए मंडल के प्रत्येक सदस्य ने जिस दलीय भावना से कार्य किया केवल उस के फलस्वरूप ही, विद्यार्थी के लिए, यह एक अच्छी प्रकार लिखी हुई एवं रोचक पुस्तक तैयार की जा सकी।

विशेषरूप से मैं प्रो० मनमोहन सिंह अरोरा का कृतज्ञ हूँ कि उन्होंने अपने अन्य भारी व्यावसायिक कार्यों के होते हुए भी कम से कम समय में इस पुस्तक का संपादन किया। इस कार्य में उनको डा० बी० देवकीनन्दन, श्री जी० डी० दल, डा० एस० के० सिंह गौतम एवं श्री महेन्द्र शंकर ने आवश्यक सहायता प्रदान की। पुस्तक के निर्माण कार्य का निरीक्षण डा० आर० पी० गुप्ता, डा० बी० देवकीनन्दन, डा० एस० के० सिंह गौतम, डा० राम औतार एवं श्री महेन्द्र शंकर द्वारा किया गया। डा० आर० पी० गुप्ता, डा० बी० देवकीनन्दन एवं श्री ईश्वर चन्द्र ने उत्तर प्रदान किए। हिन्दी संस्करण का विषय-संपादन डा० बी० देवकीनन्दन एवं श्री महेन्द्र शंकर ने किया। मैं सच्चे हृदय से इन में से प्रत्येक का बहुत कृतज्ञ हूँ।

जिन प्रतिबन्धों के अंतर्गत हमें कार्य करना पड़ा उनके कारण यह बहुत कुछ संभव है कि पूर्ण सावधानियों के लेते हुए भी कुछ छपाई संबंधी या अन्य अशुद्धियाँ हमारे ध्यान से बच गई हों।

इस पुस्तक में सुधार हेतु किन्ही भी सुझावों का मंडल स्वागत करेगा और अत्यन्त कृतज्ञता के साथ उन्हें स्वीकार करेगा।

एम० पी० सिंह
अध्यक्ष
संपादक मंडल

नई दिल्ली
जून, 1977

कृतज्ञताज्ञापन

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद् निम्न व्यक्तियों की आभारी है जिन्होंने 1 से 3 नवम्बर 1976 तक एन० आई० ई० कैम्पस, नई दिल्ली में आयोजित एक कार्यशाला में इस पाठ्यपुस्तक की प्रथम सामग्री का समीक्षात्मक विवेचन किया :

1. श्री ए० आर० आहूजा
   धनपतमल विरमानी स्कूल, रूप नगर, दिल्ली
2. डा० मनमोहन सिंह अरोरा
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
3. डा० राम औतार
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
4. श्रीमती जी० टी० एस० चड्डा
   केन्द्रीय विद्यालय, आर० के० पुरम, नई दिल्ली
5. श्रीमती सुशील चावला
   केन्द्रीय विद्यालय, गोल मार्केट, नई दिल्ली
6. डा० एस० डी० चोपड़ा
   कुरुक्षेत्र विश्वविद्यालय, कुरुक्षेत्र
7. श्री एम० एस० दहिया
   राजकीय उच्चतम माध्यमिक बालक विद्यालय, रूप नगर, दिल्ली
8. डा० बी० देवकीनन्दन
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
9. श्री जी० डी० दत्त
   राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
10. डा० एस० के० सिंह गौतम
    राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
11. डा० आर० पी० गुप्ता
    राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
12. श्री एस० आर० जयपाल
    केन्द्रीय विद्यालय, आर० के० पुरम, नई दिल्ली
13. श्री आर० कानन
    बेसेंट थियोसोफिकल हाई स्कूल, थिरुवानमियूर, मद्रास
14. श्री एन० कृष्णमाचारी
    आर० के० एम० रेसीडेन्शल हाई स्कूल माईलापोर, मद्रास

15. श्रीमती एस० मल्होत्रा
    लेडी इरविन हायर सैकेन्डरी स्कूल, नई दिल्ली
16. कु० जे० मंसुखानी
    लेडी इरविन हायर सैकेन्डरी स्कूल, नई दिल्ली
17. कु० ऊषा मेहता
    दिल्ली पब्लिक स्कूल, नई दिल्ली
18. कु० विमला गीतवानी
    राजकीय कन्या उच्चतर माध्यमिक विद्यालय
    आ० ए० आर० आई० पूसा, नई दिल्ली
19. श्री बबीआह नायडू
    राजकीय जूनियर कालेज, हैदराबाद
20. श्री आर० जी० नागिया
    केन्द्रीय विद्यालय, दिल्ली छावनी, दिल्ली
21. श्री मोहिन्दर राज
    केन्द्रीय विद्यालय, जनकपुरी, नई दिल्ली
22. श्री थलकान सिंह राठौर
    राजकीय जूनियर बेसिक स्कूल, जबलपुर
23. श्री रमेश चन्द्र सनाठ्या
    राजकीय सैकेन्डरी स्कूल, कोठारिया, राजस्थान
24. डा० बी० एम० शाह
    एम० एस० यूनिवर्सिटी आफ बड़ौदा, बड़ौदा
25. श्री महेन्द्र शंकर
    राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्, नई दिल्ली
26. श्रीमती एस० शर्मा
    स्प्रिंगडेल्स सीनियर स्कूल, नई दिल्ली
27. श्री इन्द्रजीत सिंह
    एस० एस० खालसा हायर सैकेन्डरी स्कूल, नई दिल्ली
28. डा० एम० पी० सिंह
    इन्डियन इन्सटीट्यूट आफ टैक्नोलोजी, नई दिल्ली
29. श्रीमती एस० सूद
    केन्द्रीय विद्यालय, एन्डरूजगंज, नई दिल्ली
30. श्री किशनलाल श्रीवास्तव
    राजकीय मिडिल स्कूल, जबलपुर

अध्यापक के लिए निर्देश

अध्यापक का एक अत्यधिक महत्वपूर्ण कार्य है कि विषय सामग्री को विद्यार्थी के सम्मुख इस प्रकार प्रस्तुत करना कि वह उसकी कल्पना को 'जागृत' कर सके। एक 'अच्छे' अध्यापक से केवल अपने विद्यार्थियों को लाभ पहुँचाने की ही आशा नहीं की जाती बल्कि उससे यह भी आशा की जाती है कि वह ऐसे कदम भी उठाए ताकि उनके लाभ को 'परिपूर्ण' रखा जा सके। शिक्षण-अध्ययन प्रक्रिया में, पुस्तक बहुत से साधनों में से केवल एक साधन है। वास्तव में जो कार्य करने वाला है वह अध्यापक ही है। अतः इस पुस्तक को पढ़ाते समय निम्न बातों को ध्यान में रखना चाहिए :

(1) जहाँ तक संभव हो सका है विद्यार्थी को प्रारम्भ में पहले यह बताया गया है कि किसी विशेष एकक में वह क्या पढ़ने जा रहा है। तब प्रत्येक संकल्पना का एक सरल उदाहरण या एक सरल स्थिति के द्वारा परिचय कराया गया है। इसके बाद विद्यार्थी को तर्क संगत और क्रमबद्ध चरणों की एक शृंखला की सहायता से कठिनता के वांछित अक्षांश तक ले जाया जाता है।

(2) कोई 'आधुनिक' गणित नहीं है और कोई 'परम्परागत' गणित नहीं है। दोनों ही नाम भ्रमात्मक हैं। इस पुस्तक में हमारे सामने जो भी है वह है अनावश्यक दृढ़ता से रहित, वास्तविक जीवन की स्थितियों के आशाजनक अनुरूप तथा इसके साथ ही आगे के अध्ययन और खोज के लिए विद्यार्थी की कल्पना को 'जागृत' करने में समर्थ एक सरल, सुन्दर और सुरूप गणित। जो आवश्यक है वह है, जहाँ संभव हो गतिविधियों का प्रयोग करते हुए, विद्यार्थी को गणित का अनुभव कराते हुए और उसे उसके अपने हाथ और मस्तिष्क से गणित 'करवाते' हुए आधुनिकतम शिक्षण।

(3) लिखने का ढंग कुछ शाब्दिक है। अनौपचारिक और अनुसंधानिक विधि के पक्ष में दृढ़ता का त्याग किया गया है।

(4) कठिन प्रश्नों को तारांकित किया गया है।

(5) पुस्तक में विभिन्न स्थानों पर 'क्यों?' लगा दिया गया है। यहाँ अध्यापक को विद्यार्थी द्वारा उत्तर देने पर जोर देना चाहिए। केवल अंत में ही, यदि आवश्यक हो तो, अध्यापक को स्वयं उत्तर देने पर विचार करना चाहिए।

(6) प्रश्नों को कठिनता के आरोहीक्रम में रखा गया है। विद्यार्थी के योग्यता स्तर को देखते हुए गृह कार्य हेतु इन प्रश्नों की संख्या में वृद्धि की जा सकती है या इनमें से केवल कुछ ही प्रश्न गृह कार्य के लिए दिए जा सकते हैं।

विद्यार्थी के लिए निर्देश

आप इस गणित की पुस्तक का अध्ययन आरम्भ कर रहे हैं। यदि आप प्रारम्भ में ही सीखने की कुछ 'अच्छी' प्रवृत्तियाँ बना लेंगे, तो आप पाएँगे कि गणित का अध्ययन बहुत ही सार्थक और रुचिकर है। कुछ 'अच्छी' सीखने की प्रवृत्तियाँ सुझाव के तौर पर नीचे दी जा रही हैं :

1. **गणित केवल कुछ कार्य करके ही सीखी जाती है।** केवल अपनी पाठ्यपुस्तक को ही न पढ़ें। सदैव एक पेंसिल और कागज लीजिए और पाठ्यसामग्री की सहायता से 'कार्य' प्रारम्भ कीजिए।

2. पाठ्यसामग्री में जहाँ जहाँ आपको शब्द 'क्यों?' मिले, वहाँ आप उसके कारण देने का प्रयास करें।

3. **एक ही प्रश्न पर अधिक समय व्यतीत न करें।** यह सदैव अच्छा है कि आप नए प्रश्न को हल करें और कुछ समय बाद शुद्ध मस्तिष्क से उस प्रश्न को हल करें जिसमें आप कठिनाई अनुभव करते हैं।

4. मनुष्य के मस्तिष्क में केवल सीमित सूचनाएँ ही इकट्ठी रह सकती हैं। जिस वस्तु का अधिकतर उपयोग नहीं होता, उसे प्रायः भंडार से हटा दिया जाता है। **इसलिए यह अच्छा होगा कि आप प्रत्येक एकक के मूलभूत परिणामों की संक्षिप्त सूची बना लें और समय-समय पर उसका पुनरावलोकन करें।**

संकेत-सूची

$+$	:	योग
$-$	:	घटाना/व्यवकलन
$\div$	:	विभाजन
$\times$	:	गुणन
$<$	:	कम है/छोटा है
$=$	:	बराबर है/समान है
$>$	:	अधिक है/बड़ा है
$\|\ \|$	:	निरपेक्ष मान
$\%$	:	प्रतिशत
$\angle$	:	कोण
$\triangle$	:	त्रिभुज
$\parallel$	:	समांतर है
$\perp$	:	लम्ब है

SYLLABUS FOR CLASS VI

UNIT I: NATURAL NUMBERS AND WHOLE NUMBERS

Natural numbers; numbers and numerals; place-value and the digit zero; the number zero; representation of whole numbers on a number line.

UNIT II: OPERATIONS ON WHOLE NUMBERS

(All properties to be formulated through examples.) Addition and its properties; subtraction and its relationship to addition; multiplication and its properties; multiplication as repeated addition; the distributive property for whole numbers; division and its relationship to multiplication; division as repeated subtraction; the algorithm for division.

Properties of 0 (zero) and 1 (unity); impossibility of division by zero (motivated through repeated subtraction.)

UNIT III: INTEGERS

The need for integers; order on the number line; absolute value.

UNIT IV: OPERATIONS ON INTEGERS

Addition, subtraction, multiplication and division of integers and their properties; rules of sign through patterns.

UNIT V: POWERS OF INTEGERS

Exponential notation; computation of squares, cubes, etc. of the given integers; square roots of perfect squares (positive integers) by factor method.

UNIT VI: PROPERTIES OF NUMBERS

Factors and multiples; prime and composite numbers; prime factorization property; divisibility tests for 2, 3, 4, 5, 6, 8, 9, 10, 11.

UNIT VII: **ALGEBRAIC EXPRESSIONS WITH INTEGRAL COEFFICIENTS**

Use of letters to denote numbers; terms of an expression; algebraic expressions up to and including trinomials; addition and subtraction of algebraic expressions; use of brackets and other grouping symbols; multiplication of binomials. (18)

UNIT VIII: **INTRODUCTION TO EQUATIONS**

Use of letters to denote unknown quantities; solution of equations by
(a) trial and error,
(b) using properties of operations.
Applications in solution of word problems. (12)

UNIT IX: **RATIOS, PERCENTAGES AND THEIR APPLICATIONS**

Ratio, proportion; direct and inverse variation; percentages. Applications to real life situations: problems in profit and loss; simple interest and problems of national importance. (25)

UNIT X: **INCIDENCE PROPERTIES IN THE PLANE**

The following incidence properties are to be observed and subsequently assumed:
(a) Through any two different points in a plane, there is exactly one line. This line lies wholly in the plane.
(b) Two different lines in a plane may be: (i) intersecting, or (ii) parallel.
If they intersect, they intersect in exactly one point; collinearity; concurrent lines. (6)

UNIT XI: **MEASUREMENT OF LINE SEGMENTS**

Comparison of line segments; equal segments; choice of standard unit of measurement; ruler and its uses; construction of line segment of a given length; construction of a segment whose length is equal to sum or difference of lengths of two given line segments. (8)

UNIT XII: **ANGLES**

Rays and angles; vertex and arms of an angle; comparison of angles; straight, right, complete and zero

angles; (degree) measure of an angle; use of protractor to measure angles and construct angles of given magnitudes; acute, obtuse and reflex angles; adjacent angles; linear pair; complementary and supplementary angles; set squares; the interior and exterior of an angle.
Construction of angles of 30°, 45°, 60° and 90°, using set squares.

UNIT XIII: PARALLEL LINES

Parallel lines; transversals; corresponding angles; alternate angles; interior and exterior angles.
Using set squares; (i) to construct a line parallel to a given line; (ii) to draw a perpendicular to a given line from a point outside it.

UNIT XIV: TRIANGLES

A triangle; its interior and exterior; vertices, sides and angles of a triangle; sum of the angles of a triangle; isosceles, equilateral, scalene, acute, obtuse and right triangles; sum of the two sides of a triangle.

UNIT XV: CIRCLES

Arc, circumference, radius, chord, diameter, segments and sectors of a circle; interior and exterior of a circle.

UNIT XVI: COMPASSES AND RULER CONSTRUCTIONS

The student should be familiar with the following constructions, using compasses and ruler:

(i) Constructing an angle equal to a given angle;
(ii) Bisecting a given angle;
(iii) Dividing a circular region into six equal sectors;
(iv) Construction of angles of 15°, $22\frac{1}{2}$°, 30°, 45°, 60°, 75°, 90°, 120°, 135°, 150° and 240°;
(v) Drawing a line parallel to a given line from a point outside it;
(vi) Constructing a triangle when lengths of its three sides are given.

UNIT XVII: RECOGNITION OF POLYHEDRA

Congruence of plane figures; regular polygons; vertical and horizontal directions. Recognition of faces, edges and vertices of cubes, cuboids, right prisms, pyramids; net of a cube; counting of vertices, edges and faces of polyhedra; Euler's formula.

UNIT XVIII: LINEAR SYMMETRY

Symmetric and non-symmetric figures; line(s) of symmetry; properties of pairs of symmetric figures; construction of symmetric figures through paper cutting and paper folding, etc.

(i) To construct a point symmetric to a given point with respect to a given line of symmetry.

(ii) Given two points, to construct their line of symmetry.

(iii) To construct a line segment symmetric to a given line segment with respect to a given line of symmetry.

विषय सूची

विषय सूची

पृष्ठ

एकक I

धनपूर्णांक और पूर्ण संख्याएँ

आप पिछली कक्षाओं में धनपूर्णांक (*natural numbers*) और पूर्ण संख्याओं (*whole numbers*) के कुछ तथ्यों और गुणों के विषय में पहले ही पढ़ चुके हैं। इनमें से कुछ तथ्यों और गुणों का हम इस एकक में पुनरावलोकन करेंगे।

1.1 धनपूर्णांक

आपको अपनी छठी कक्षा के लिए कितनी पुस्तकें खरीदनी पड़ीं? आपके परिवार में कितने सदस्य हैं? एक सप्ताह में कितने दिन होते हैं? अंग्रेजी वर्णमाला में कितने अक्षर हैं? उपर्युक्त प्रश्नों के उत्तर में संख्याओं का प्रयोग करना पड़ेगा। आप कह सकते हैं कि आपको छठी कक्षा के लिए आठ पुस्तकें खरीदनी पड़ीं या कि आपके परिवार में **पाँच** सदस्य हैं। आप जानते हैं कि एक सप्ताह में **सात दिन** होते हैं तथा अंग्रेजी वर्णमाला में **छब्बीस** अक्षर होते हैं।

उपर्युक्त उदाहरणों में संख्याएं आठ, पाँच, सात, छब्बीस **धनपूर्णांक** हैं। इनके अतिरिक्त और भी अनेक धनपूर्णांक हैं जैसे कि एक, दो, तीन इत्यादि। इन संख्याओं का किसी संग्रह (*collection*) की वस्तुएं गिनने या गणन (*count*) करने में प्रयोग किया जाता है। इसलिए इन संख्याओं को **गणन संख्याएं** (*counting numbers*) भी कहते हैं।

यदि आपकी कक्षा में आंध्रप्रदेश का कोई विद्यार्थी है तथा यदि उससे पूछा जाता है कि उसे छठी कक्षा के लिए कितनी पुस्तकें खरीदनी पड़ीं तो वह कह सकता है कि **एनिमिडी** (*Enimidi*)। **एनिमिडी** तेलगु भाषा का शब्द है जिसका अर्थ है **आठ**। संख्या आठ के अलग अलग भाषाओं में अलग अलग नाम हैं जैसे कि तामिल में **एट्टू** (*Ettu*), अंग्रेजी में **एट** (*Eight*), संस्कृत में **अष्ट** (*Ashta*), जर्मन में **आच्ट** (*Acht*)। इसी प्रकार प्रत्येक संख्या के विभिन्न भाषा में विभिन्न नाम* होते हैं।

***अध्यापक के लिए निर्देश**

[कक्षा में करने योग्य गतिविधि (*activity*) के लिए सुझाव]

प्रत्येक विद्यार्थी को सुझाव दीजिए कि वह जितनी भी भाषाओं में हो सके अपने साथियों, पड़ोसियों और मित्रों से संख्याओं एक से दस तक के विभिन्न नामों का पता लगाए। तब इस सूचना को कक्षा में इकट्ठा करके एक सारणी के रूप में प्रस्तुत करें। इसके बाद एक चार्ट बनाया जा सकता है और उसे कक्षा में प्रदर्शित किया जा सकता है।

1.2 संख्या और संख्यांक

नामों के अतिरिक्त लेखन कार्य में, हम संख्याएँ निरूपित करने के लिए संकेतों का प्रयोग करते हैं। ये संकेत भी अलग अलग भाषाओं में अलग अलग प्रकार के हैं। उदाहरणार्थ संख्या **तीन** को हिन्दी में ३, अंग्रेजी में 3, उर्दू में ٣ लिखते हैं।' यदि आज प्राचीनकालीन मिस्र का कोई व्यक्ति जीवित आ जाए तो वह तीन के लिए लिखे गए उपर्युक्त संकेतों में किसी को भी नहीं पहचानेगा और वह तीन को निरूपित करने के लिए संकेत ||| का प्रयोग करेगा।

संख्याओं के संकेतों* को **संख्यांक** (*numerals*) कहते हैं। [इस प्रकार, संख्याओं और उनके संख्या-संकेतों अर्थात् संख्यांकों में अंतर है। परन्तु इस स्तर पर हम इन दोनों संकल्पनाओं (*concepts*) के इस अंतर पर कोई महत्व नहीं देंगे ;]

1.3 स्थानीय मान तथा अंक 0

जब 'छोटे' संग्रहों तक ही गिनना सीमित था तो विभिन्न गणन संख्याओं के लिए विभिन्न संख्याओं का प्रयोग करना संभव था। परन्तु जब बड़े बड़े संग्रहों के गिनने की आवश्यकता हुई तो प्रत्येक गणन संख्या के लिए एक भिन्न संकेत रखना असुविधाजनक प्रतीत हुआ। इसके लिए क्रमशः वस्तुओं के निश्चित साइज़ के छोटे संग्रह या समुच्चय बनाने के सिद्धान्त पर आधारित एक विधि का विकास हुआ।

उदाहरणार्थ, मिस्रवासियों ने दस दस वस्तुओं के संग्रहों का प्रयोग किया। उन्होंने संख्या **दस** को ∩ से निरूपित किया। तब **ग्यारह** को ∩| (अर्थात् दस और एक) लिखा गया। **बारह** को ∩|| से निरूपित किया गया, इत्यादि। **उन्नीस** को ∩||||| |||| तथा **बीस** को ∩∩ लिखा गया। **सौ** के लिए संकेत ? का प्रयोग किया गया। यद्यपि यह विधि पहली विधियों से अच्छी थी परन्तु फिर भी बड़ी संख्याओं को निरूपित करने के लिए यह विधि बहुत ही असुविधाजनक थी।

संख्याओं को संख्या-संकेतों से निरूपित करने की एक बहुत ही सुविधाजनक विधि जो कि आजकल बहुधा प्रयोग की जाती है **स्थानीय मान** (*place value*) की संकल्पना पर आधारित है। इस विधि का आविष्कार प्राचीन हिन्दू गणितज्ञों ने किया और अरबवासियों ने इसे पश्चिमी देशों तक पहुँचाया। इस विधि को **हिन्दू-अरेबिक अंकन पद्धति** (*Hindu-Arabic System of Numeration*) कहते हैं। आजकल, संसार के अधिकतर देश इसी विधि का प्रयोग करते हैं। अतः इस पद्धति को **अंतर्राष्ट्रीय अंकन पद्धति** (*International System of Numeration*) कहना उपयुक्त होगा।

इस पद्धति में मूल संग्रह का साइज़ 'दस' चुना गया है। यह शायद इसलिए कि जब मानव को पहले पहल गिनने की आवश्यकता प्रतीत हुई तो उसने अपने दोनों हाथों की उंगलियों का प्रयोग किया था। अतः दस 'एकक' (*unit*) से एक 'दस' बनता है, दस 'दस' से एक 'सौ' बनता है, दस

---

*अध्यापक के लिए निर्देश

(कक्षा में करने योग्य गतिविधि के लिए सुझाव)

प्रत्येक विद्यार्थी को सुझाव दीजिए कि वह जितनी भी भाषाओं में हो सके अपने साथियों, पड़ोसियों और मित्रों से संख्याओं एक से दस तक के लिए विभिन्न संख्या-संकेतों का पता लगाए। तब इस सूचना को कक्षा में इकट्ठा करके एक सारणी के रूप में प्रस्तुत करें। इसके बाद एक चार्ट बनाया जा सकता है और उसे कक्षा में प्रदर्शित किया जा सकता है।

'सौ' से एक 'हज़ार' बनता है, इत्यादि। **एकक स्थान** (*unit's place*) से प्रारम्भ कर और उसके बाईं ओर को चलते हुए उपर्युक्त संग्रहों में से प्रत्येक को क्रम से एक एक स्थान (*place*) निर्दिष्ट किया गया है। इस प्रकार, एकक स्थान से ठीक बाईं ओर पहले **दस का स्थान** (*ten's place*) है, दस के स्थान से ठीक बाईं ओर पहले **सौ का स्थान** (*hundred's place*) है, इत्यादि।

सुविधा की दृष्टि से हम नीचे इन स्थानों को एक चार्ट* के रूप में निरूपित कर रहे हैं :

दस करोड़ 10,00,00,000	करोड़ 1,00,00,000	दस लाख 10,00,000	लाख 1,00,000	दस हज़ार 10,000	हज़ार 1,000	सौ 100	दस 10	एकक 1
10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

संख्याओं को अब संकेतों 1, 2, 3, 4, 5, 6, 7, 8, 9 का प्रयोग करके निरूपित किया जा सकता है। इनमें प्रत्येक संकेत का मान उसके द्वारा धारित स्थान पर निर्भर करता है। उदाहरणार्थ संख्या **एक सौ तेइस** एक सौ, दो दस, और तीन एकक के संग्रह को निरूपित करती है।

आपको स्मरण होगा कि इसे सौ के स्थान पर 1, दस के स्थान पर 2 तथा एकक के स्थान पर 3 रखकर लिखा जाता है और इस प्रकार हमें संकेत 123 प्राप्त होता है। एक सौ तीन वस्तुओं के संग्रह के विषय में आप क्या कहेंगे? इस संग्रह में एक सौ, कोई दस नहीं तथा तीन एकक हैं। निश्चय ही इस संग्रह को संकेत 13 से निरूपित करना सही नहीं होगा। (क्यों?) हमें तीनों स्थानीय मानों अर्थात् 'सौ', 'दस' और 'एकक' का प्रयोग करना चाहिए। साथ ही हमारे पास एक संकेत यह निरूपित करने के लिए होना चाहिए कि इस संग्रह में एक भी दस नहीं है। यह हमें एक अतिरिक्त संकेत '0' (शून्य) शामिल करने से प्राप्त होता है। जिस स्थान के संगत कोई संग्रह न हो उस स्थान को दर्शाने के लिए 0 का एक स्थान धारक (*place holder*) के रूप में प्रयोग किया जाता है। संस्कृत में भी इसे शून्य कहते हैं जिसका अर्थ है **रिक्त** (*empty*)। इससे यह संकेत मिलता है कि शून्य द्वारा धारित स्थान रिक्त है। इस प्रकार **एक सौ तीन** को संकेत 103 से निरूपित किया जाता है।

यद्यपि स्थानीय मान की संकल्पना का प्रयोग दूसरे देशों जैसे कि बेबीलोनिया के निवासियों द्वारा प्रारम्भ किया गया परन्तु इस बात का श्रेय हिन्दुओं द्वारा आविष्कारित अंक 0 को है जिसकी सहायता से एक बहुत ही उपयोगी अंकन पद्धति का निर्माण हुआ।

1, 2, 3, 4, 5, 6, 7, 8, 9 और 0 में से प्रत्येक संकेत को **अंक** (*digit*) कहते हैं। 45, दो अंकों की संख्या (या संख्यांक) है, 127, तीन अंकों की संख्या है, 8, एक अंक की संख्या है।

जब किसी संख्या में एक से अधिक अंक होते हैं तो प्रत्येक अंक का मान उसके स्थान के अनुसार होता है। इस प्रकार, प्रत्येक अंक का **अंकित मान** (*face value*) के साथ साथ **स्थानीय मान** (*place value*) भी होता है। किसी अंक विशेष का अंकित मान सदैव समान रहता है चाहे वह

*हमने चार्ट में संकेतनों 10^1, 10^2, 10^3, इत्यादि का प्रयोग किया है। आपको याद होगा कि $10^1=10$, $10^2=10\times10$, $10^3=10\times10\times10$, इत्यादि।

किसी भी स्थान पर आए। उदाहरणार्थ, संख्या 127 में 2 का अंकित मान 'दो' है जबकि उसका स्थानीय मान 2×10 अर्थात् 20 है।

अब, स्थानीय मान के सिद्धान्त का उपयोग करते हुए किसी भी संख्या को उसके विभिन्न अंकों के स्थानीय मानों के **सूचित योग** (*indicated sum*) के रूप में लिखा जा सकता है। उदाहरणार्थ,

$$\begin{aligned}3127 &= 3\times 10^3 + 1\times 10^2 + 2\times 10^1 + 7\times 1\\ &= 3\times 1000 + 1\times 100 + 2\times 10 + 7\times 1\\ &= 3000 + 100 + 20 + 7\\ 2024769 &= 2\times 10^6 + 0\times 10^5 + 2\times 10^4 + 4\times 10^3 + 7\times 10^2 + 6\times 10 + 9\times 1\\ &= 2\times 1000000 + 0\times 100000 + 2\times 10000 + 4\times 1000 +\\ &\quad 7\times 100 + 6\times 10 + 9\times 1\\ &= 2000000 + 20000 + 4000 + 700 + 60 + 9\end{aligned}$$

जब कोई संख्या सूचित योग के रूप में लिख दी जाती है, जैसा कि ऊपर दिखाया गया है, तो हम कहते हैं कि हमने संख्या को **प्रसारित संकेतन** (*expanded notation*) में व्यक्त कर दिया है।

किसी संख्या को लिखते समय यह परम्परागत है कि सबसे बड़े स्थान पर 0 को छोड़ कर कोई और अंक लिखा जाता है। उदाहरणार्थ संख्या **छःसौ पच्चीस** को 0625 **न** लिखकर 625 लिखा जाएगा।

प्रश्नावली 1.1

1. निम्न में से प्रत्येक में 'सात' का स्थानीय मान क्या है ?
   (i) 1070 (ii) 560897 (iii) 7000902 (iv) 70000000
2. संख्या 80475 में 4 और 7 के स्थानीय मानों का गुणनफल ज्ञात कीजिए।
3. संख्या 6213 में 2 के स्थानीय मान और अंकित मान का अंतर ज्ञात कीजिए।
4. वह अंक निर्धारित कीजिए जिसका 7210 में स्थानीय मान 10 है।
5. निम्न में से प्रत्येक के तदनुरूपी संख्या लिखिए :
   (i) $5\times 1000 + 3\times 100 + 2\times 10 + 6\times 1$
   (ii) $7\times 1000000 + 8\times 10000 + 9\times 100 + 5\times 10 + 2\times 1$
   (iii) $2\times 10^6 + 3\times 10^3 + 8\times 10^2 + 1\times 1$

6. निम्न में से प्रत्येक को प्रसारित संकेतन में लिखिए :

(i) 2406 (ii) 15968 (iii) 400000 (iv) 11010101

7. अंकों 3, 5, 7 का प्रयोग करते हुए 3 अंकों की सभी संभव संख्याएँ लिखिए जबकि एक अंक एक संख्या में केवल एक ही बार प्रयोग किया जाता है। इन संख्याओं का योग ज्ञात कीजिए।

8. अंकों 4, 2, 8, 6 का प्रयोग करते हुए 4 अंकों की कोई भी पाँच संख्याएँ लिखिए जबकि एक अंक संख्या में केवल एक ही बार प्रयोग किया जाता है।

9. अंकों 9, 2 और 0 का प्रयोग करते हुए 3 अंकों की कितनी विभिन्न संख्याएँ बनाई जा सकती हैं जबकि कोई भी अंक दोहराया न जाए ?

10. 5 अंकों की एक ऐसी संख्या लिखिए जिसके अंकों को उलटे क्रम में लिखने पर संख्या में कोई परिवर्तन न हो। संख्या को पढ़िए।

11. 4 से लेकर 98 तक के सभी धनपूर्णांकों में एकक स्थान पर अंक 5 कितनी बार आता है ?

1.4 संख्या-शून्य

हम यह देख चुके हैं कि किस प्रकार एक स्थान धारक के रूप में संकेत '0' को शामिल करके, स्थानीय मान के सिद्धान्त पर हम किसी भी संख्या को, चाहे वो कितनी भी बड़ी हो, लिखने में समर्थ हो सके। अंक '0' से भिन्न परन्तु उतनी ही आवश्यक **संख्या शून्य** (*number zero*) की संकल्पना है।

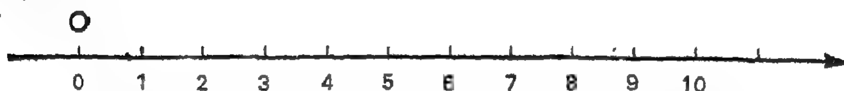
यदि आपको उपहार में एक टॉफियों का डिब्बा मिले और आप सभी टॉफियों को अपने मित्रों में बाँट दें तो डिब्बे में कितनी टॉफियाँ शेष रहेंगी ? कोई नहीं ! या यदि आपकी कक्षा की समाप्ति पर सभी विद्यार्थी कक्षा से बाहर चले जाएँ तो कक्षा में कितने विद्यार्थी शेष रहेंगे ? कोई नहीं ! स्पष्ट है कि डिब्बे में शेष बची हुई टॉफियों या कक्षा में शेष बचे हुए विद्यार्थियों की संख्या व्यक्त करने के लिए किसी भी गणन संख्या का प्रयोग नहीं किया जा सकता। अतः ऐसे संग्रह के अवयवों की संख्या, जिसकी सभी वस्तुओं को बाहर निकाल लिया गया है, को भी व्यक्त करने की आवश्यकता है। ऐसे संग्रहों के अवयवों की संख्या को व्यक्त करने के लिए **संख्या शून्य** (जिसे संकेत '0' द्वारा व्यक्त किया जाता है) का प्रयोग किया जाता है।

संख्या '0' और गणन संख्याएँ (या धनपूर्णांक) 1, 2, 3, ... मिलकर एक समुच्चय बनाते हैं जिसे **पूर्ण संख्याओं का समुच्चय** (*set of whole numbers*) कहते हैं। इस प्रकार पूर्ण संख्याएँ हैं : **0, 1, 2, 3, 4, 5, ...** ।

1.5 पूर्ण संख्याओं का संख्या रेखा पर निरूपण

पूर्ण संख्याओं को **संख्या रेखा** (*number line*) पर निरूपित करना बहुत उपयोगी है। इस निरूपण से संख्याओं के कुछ गुण खोजने में सहायता मिलती है। आप यह जानते ही हैं कि संख्या रेखा पर पूर्ण संख्याएँ किस प्रकार निरूपित करते हैं। हम यहाँ संक्षेप में इस विधि का पुनरावलोकन करेंगे।

हम एक रेखा खींचते हैं और उस पर कहीं भी एक बिंदु O अंकित कर लेते हैं। अब O से प्रारम्भ कर और उसके दाईं ओर बराबर दूरियों पर हम एक के बाद एक निशान लगाते हैं। (देखिए आकृति 1.1)



आकृति 1.1 : पूर्ण संख्याओं का संख्या रेखा पर निरूपण

बिंदु O पर 0 लिख दिया जाता है। अन्य बिंदुओं पर क्रम से 1, 2, 3,...लिख देते हैं जैसा कि आकृति 1.1 में दिखाया गया है। हम देखते हैं कि इस प्रकार से हम **किसी भी** पूर्ण संख्या को **संख्या रेखा** पर निरूपित कर सकते हैं।

हम यह भी देखते हैं कि **दो, एक से** 'एक अधिक' है, **तीन, दो से** 'एक अधिक' है, **चार, तीन से** 'एक अधिक' है, इत्यादि।

दो, एक का **परवर्ती** (*successor*) कहलाता है, तीन, दो का **परवर्ती** कहलाता है, इत्यादि। प्रत्येक गणन संख्या का **एक और केवल एक परवर्ती** अर्थात् गणितज्ञ की भाषा में **एक अद्वितीय परवर्ती** (*a unique successor*) होता है।

क्या कोई सबसे बड़ी गणन संख्या है? क्या कोई सबसे बड़ी पूर्ण संख्या है? इन दोनों प्रश्नों का उत्तर है: **नहीं**! क्यों? यह इसलिए कि 'एक अधिक' की प्रक्रिया बिना किसी अंत के चल सकती है।

एक बार संख्याओं को संख्या रेखा पर निरूपित करने के बाद उनकी तुलना सरलता से की जा सकती है। उदाहरणार्थ,

$2 > 1$, चूँकि 2, 1 के दाईं ओर स्थित है;

$3 > 2$, चूँकि 3, 2 के दाईं ओर स्थित है;

$4 > 1$, चूँकि 4, 1 के दाईं ओर स्थित है, इत्यादि।

साथ ही, $2 < 3$, चूँकि 2, 3 के बाईं ओर स्थित है;

$3 < 4$, चूँकि 3, 4 के बाईं ओर स्थित है;

$2 < 5$, चूँकि 2, 5 के बाईं ओर स्थित है;

$0 < 6$, चूँकि 0, 6 के बाईं ओर स्थित है, इत्यादि।

परन्तु यदि हमसे यह पूछा जाए कि 112 और 326 में कौन सी संख्या छोटी है तो हम क्या करेंगे ? निश्चय ही, ऐसी बड़ी संख्याओं को पहले संख्या रेखा पर निरूपित करना और फिर उनकी पारस्परिक स्थितियाँ देखना अव्यावहारिक होगा। परन्तु फिर भी संख्या रेखा से हमें इस समस्या के हल के लिए कुछ संकेत अवश्य मिल जाता है। आइए देखें।

जब $2<3$, तो हमें किसी ऐसे धनपूर्णांक की आवश्यकता है जिसे 2 में जोड़ने से योग 3 हो जाए (इस स्थिति में यह धनपूर्णांक 1 है : $2+1=3$);

जब $3<4$, तो हमें किसी ऐसे धनपूर्णांक की आवश्यकता है जिसे 3 में जोड़ने से योग 4 हो जाए (इस स्थिति में यह धनपूर्णांक 1 है : $3+1=4$);

जब $2<5$, तो हमें धनपूर्णांक 3 की आवश्यकता है जिससे कि $2+3=5$;
जब $0<6$, तो हमें धनपूर्णांक 6 की आवश्यकता है जिससे कि $0+6=6$।

दूसरे शब्दों में, यदि दो (असमान) पूर्ण संख्याएँ दी हुई हों तो इनमें से वह पूर्ण संख्या **छोटी** होती है जिसमें कोई धनपूर्णांक जोड़ने से योग दूसरी पूर्ण संख्या के बराबर हो जाए।

इस प्रकार, 112 और 326 में 112 छोटी है चूँकि $112+214=326$। संख्या रेखा से **मध्य-स्थिति** (*betweenness*) का गुण भी प्रदर्शित होता है। उदाहरणार्थ, 3, संख्याओं 2 और 4 के **मध्य** (*between*) स्थित है, 8, 5 और 9 के **मध्य** स्थित हैं। हम लिखते हैं: $2<3<4$, $5<8<9$।

उदाहरण 1: 4 अंकों की छोटी से छोटी और बड़ी से बड़ी पूर्ण संख्याएँ लिखिए।

हल : 4 अंकों की छोटी से छोटी संख्या के लिए हजार के स्थान पर सबसे छोटा शून्येतर (*non-zero*) अंक होना चाहिए। निस्संदेह यह 1 है। अतः सौ, दस और एकक के स्थानों में से प्रत्यक पर सबसे छोटे अंक अर्थात् 0 की आवश्यकता है। इस प्रकार, 4 अंकों की सबसे छोटी संख्या 1000 है। 4 अंकों की सबसे बड़ी संख्या के लिए यह स्पष्ट है कि प्रत्येक स्थान पर सबसे बड़ा अंक होना चाहिए। निस्संदेह, सबसे बड़ा अंक 9 है। अतः 4 अंकों की बड़ी से बड़ी संख्या 9999 है।

उदाहरण 2: अंकों 0, 1 और 4 का प्रयोग करते हुए 3 अंकों की छोटी से छोटी एवं बड़ी से बड़ी संख्याएँ बनाइए जबकि कोई अंक दोहराया न जाए।

हल: छोटी से छोटी संख्या के लिए हम सौ के स्थान पर अंक 0 नहीं ले सकते। (क्यों ?) इस स्थान पर 1 होना चाहिए। (क्यों ?) दस और एकक के स्थान पर क्रमशः 0 और 4 होना चाहिए। इस प्रकार छोटी से छोटी संख्या 104 है। बड़ी से बड़ी संख्या 410 है।

प्रश्नावली 1.2

1. संख्याओं 7, 14, 189 तथा 8645 के परवर्ती लिखिए।
2. पूर्ण संख्याओं के समुच्चय में 0 का परवर्ती क्या है ?
3. 8 और 9 क मध्य कितने धनपूर्णांक हैं ?

4. 1 और 8 के मध्य कितनी पूर्ण संख्याएँ हैं?

5. सबसे छोटा धनपूर्णांक बताइए।

6. निम्न को लिखने के लिए संकेतों '>' या '<' का प्रयोग कीजिए :

(i) 73, 61 से अधिक है,

(ii) 18, 29 से कम है,

(iii) 59, 57 और 63 के मध्य स्थित है,

(iv) 0 किसी भी धनपूर्णांक से कम है।

7. निम्न संख्या युग्मों की तुलना कीजिए। प्रत्येक युग्म में बताइए कि कौन सी संख्या छोटी है।

(i) 68708, 69006 (ii) 80008, 78888

(iii) 300000, 3000 (iv) 5555, 0

8. 6 अंकों की 5 पर अंत होने वाली छोटी से छोटी संख्या तथा 2 पर अंत होने वाली बड़ी से बड़ी संख्या लिखिए।

9. अंकों 6, 0, 3, 5 का प्रयोग करते हुए 4 अंकों की बड़ी से बड़ी तथा छोटी से छोटी संख्याएँ लिखिए जबकि प्रत्येक अंक संख्या में केवल एक ही बार प्रयोग किया जाता है।

10. सभी अंकों 4, 1, 9, 7, 5, 2 का प्रयोग करते हुए बड़ी से बड़ी और छोटी से छोटी पूर्ण संख्याएँ बनाइए जबकि प्रत्येक अंक एक संख्या में एक ही बार प्रयोग किया जाता है।

एकक II

पूर्ण संख्याओं पर संक्रियाएँ

हम पहले से ही जानते है कि पूर्ण संख्याओं को किस प्रकार जोड़ा, घटाया, गुणा तथा भाग किया जाता है। आप अनौपचारिक रूप से इन संक्रियाओं के कुछ गुणों से भी परिचित हैं। इस एकक में हम इनमें से कुछ गुणों का पुनरावलोकन करेंगे।

2.1 योग पर विचार

आप यह भलीभांति जानते हैं कि दो **पूर्ण संख्याओं को किस प्रकार जोड़ा** जाता है। यदि संख्याएँ 'छोटी' हों तो हम अपने मन ही में इनको जोड़कर योग ज्ञात कर सकते हैं। उदाहरणार्थ $4+5=9$ अन्यथा हम स्तम्भ विधि से योग (*column addition*) भी ज्ञात कर सकते है। उदाहरणार्थ,

$$\begin{array}{r} 409836 \\ +\ \ 83729 \\ \hline 493565 \\ \hline \end{array}$$

क्या दो संख्याओं के योग पर इसका कुछ अंतर पड़ता है कि हम इन संख्याओं को किस क्रम में जोड़ते हैं? दूसरे शब्दों में यदि हमें 21 और 36 का योग ज्ञात करना हो तो क्या इससे कुछ अंतर पड़ेगा कि हम 21 में 36 जोड़ते हैं या 36 में 21 जोड़ते हैं? नहीं!
इस प्रकार,

$$21+36=36+21$$

वास्तव में यह किन्हीं भी दो पूर्ण संख्याओं के लिए सत्य है। हम कहते है कि (पूर्ण संख्याओं का) **योग क्रमविनिमेय** (*commutative*) है। इतना इस गुण का नाम आवश्यक नही है जितना यह स्मरण रखना कि

किन्हीं दो संख्याओं का योग ज्ञात करने में इसका कोई महत्व नहीं है कि हम संख्याओं को किस क्रम में जोड़ते हैं।

अब यदि हम गणित की भाषा और उस भाषा को लें जिसका हम बोलने या लिखने में प्रयोग करते हैं तो हम इनमें एक महत्वपूर्ण अंतर यह देखेंगे कि गणित की भाषा में विशेष चिन्हों (signs) और संकेतों का प्रयोग किया जाता है। इनसे हमें अपने विचार संक्षेप और अच्छे से अच्छे रूप में व्यक्त करने में सहायता मिलती है। उदाहरणार्थ, मान लीजिए कि हम किन्हीं दो पूर्ण संख्याओं को

और b से व्यक्त करते हैं। अब इन चिन्हों और संकेतों का प्रयोग करने से **योग का क्रमविनिमेय ण** निम्न प्रकार व्यक्त किया जा सकता है :

यदि a और b कोई दो पूर्ण संख्याएँ हों तो

$$a+b=b+a$$

इस प्रकार हमने किन्हीं दो पूर्ण संख्याओं के लिए (योग के) एक गुण को संक्षेप में और एक सुन्दर रीके से व्यक्त कर दिया।

यदि हम **योग के लिए एक संक्रिया सारणी** (*operation table for addition*) बना लें तो यह गुण र भी अच्छी प्रकार से समझा जा सकता है। आइए केवल पहली पांच पूर्ण संख्याएँ ही लें। निम्न सारणी प्राप्त होगी :

दूसरी संख्या

+	**0**	**1**	**2**	**3**	**4**
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

पहली संख्या

मुख्य विकर्ण

हम देखते है कि संक्रिया सारणी, मुख्य विकर्ण (*main diagonal*) के सापेक्ष सममित *metrical*) है। सारणी से यह सरलता से देखा जा सकता है कि $2+1=1+2$, $2=2+3$, $4+3=3+4$, इत्यादि।

हम सारणी से यह भी देखते हैं कि

$0+0=0$ |

$0+1=1+0=1$

$0+2=2+0=2$

$0+3=3+0=3$

$0+4=4+0=4$

दूसरे शब्दों में **शून्य और किसी पूर्ण संख्या का योग स्वयं वह पूर्ण संख्या होती है।** इस गुण **र का योज्य गुण** (*addition property of zero*) कहते हैं तथा **0 योग के लिए तत्समक अवयव**

(*identity element for addition*) कहलाता है। पुनः नाम का यहाँ कोई महत्व नहीं है। इस स्तर पर आवश्यकता इस बात की है कि इस गुण का ज्ञान हो और इस ज्ञान को व्यावहारिक स्थितियों में प्रयोग में लाने की योग्यता हो।

संकेतों का प्रयोग कर हम इस गुण को निम्न प्रकार व्यक्त कर सकते हैं :

यदि a कोई पूर्ण संख्या हो, तो

$$0+a=a+0=a$$

अब यदि हमें **तीन संख्याओं को जोड़ना** हो तो हम क्या करेंगे? आइए एक उदाहरण पर विचार करें। एक स्कूल एक विविध मनोरंजन कार्यक्रम आयोजित करता है। एक दिन सतीश ने 73, बीना ने 56 तथा रोहित ने 109 टिकट बेचे। मान लीजिए आपसे कहा जाता है कि बेचे गए टिकटों की कुल संख्या ज्ञात कीजिए। आप शायद पहले 73 में 56 जोड़ेंगे और बाद में इस योग में 109 जोड़ देंगे। आपकी योग ज्ञात करने की इस विधि को निम्न प्रकार दर्शाया जा सकता है :

$$(73+56)+109=129+109=238$$

कोष्ठकों से तात्पर्य है कि पहले 73 और 56 को जोड़ना है।

आइए अब देखें कि यदि हम पहले 56 और 109 को जोड़ें और फिर योग में 73 जोड़ें तो हमें क्या योग प्राप्त होता है। दूसरे शब्दों में

$73+(56+109)$ का क्या मान है? हम देखते हैं कि

$$73+(56+109)=73+165=238$$

इस प्रकार,

$$(73+56)+109=73+(56+109)$$

उपर्युक्त उदाहरण से (पूर्ण संख्याओं के) योग का एक अन्य महत्वपूर्ण गुण स्पष्ट होता है, वह यह कि **योग सहचारी** (*associative*) **होता है।** पुनः नाम का इतना महत्व नहीं है जितना यह स्मरण रखने का कि

किन्हीं तीन पूर्ण संख्याओं का योग ज्ञात करने में इस बात से कोई अंतर नहीं पड़ता कि पहले हम कौन सी दो संख्याएं लेते हैं और फिर उनके योग में अंतिम संख्या जोड़ते हैं।

चिन्हों और संकेतों का प्रयोग कर हम योग के **साहचर्य गुण** (*associative property*) को निम्न प्रकार व्यक्त कर सकते हैं :

यदि a, b और c कोई तीन पूर्ण संख्याएं हों तो

$$(a+b)+c=a+(b+c)$$

इस गुण के फल स्वरूप ही हम प्रायः इन समान योगों को $a+b+c$ लिखते हैं।

आइए एक और उदाहरण लें।

उदाहरण 1 : 433, 567 तथा 2698 का योग ज्ञात कीजिए।

हल : स्पष्ट है कि पहले 433 और 567 को जोड़कर उनके योग में 2698 जोड़ कर योग ज्ञात करना अपेक्षाकृत सरल है। हम देखते हैं कि

$$(433+567)+2698=1000+2698=3698$$

[यदि हमने निम्न क्रिया स योग ज्ञात किया होता तो हमें योग ज्ञात करने में कुछ कठिनाई प्रतीत होती।

$433+(567+2698)=433+3265=3698$]

अंत में, हम चार या उससे अधिक संख्याओं का योग किस प्रकार ज्ञात करते हैं? हम योग के क्रमविनिमेय और साहचर्य गुणों का, यदि आवश्यक हो तो कई बार, प्रयोग करते हैं। इस स्तर पर यह आवश्यक नहीं कि हम प्रत्येक पग (step) पर इन गुणों का प्रयोग बताते जाएं। इस संदर्भ में जो महत्वपूर्ण है वह है 'इस प्रयोग का परिणाम'। परिणाम है कि

यदि कई संख्याएँ दी हुई हों तो यह आवश्यक नहीं कि उनको उसी क्रम में जोड़ा जाए जिसमें वे दी हुई हैं। हम उनके सरलतम समूह बनाकर उनका योग ज्ञात कर सकते हैं। यह गुण **योग का पुनर्व्यवस्थितकरण गुण (rearrangement property of addition)** कहलाता है।

उदाहरणार्थ, 27, 423, 73, 56 और 77 का योग हम निम्न प्रकार ज्ञात कर सकते हैं:

$$27+423+73+56+77=(27+73)+(423+77)+56$$
$$=100+500+56$$
$$=656$$

या, $$27+423+73+56+77=(27+423)+73+56+77$$
$$=(450+73)+56+77$$
$$=(523+56)+77$$
$$=579+77$$
$$=656$$

स्पष्ट है कि, योग ज्ञात करने की दूसरी विधि अपेक्षाकृत कम सुविधाजनक है।

प्रश्नावली 2.1

1. (क) कोई दो विषम संख्याएँ लीजिए और उनका योग ज्ञात कीजिए। यह योग सम है या विषम?

   (ख) कोई दो सम संख्याएँ लीजिए और उनका योग ज्ञात कीजिए। यह योग सम है या विषम?

2. योग ज्ञात कीजिए: (सबसे उपयुक्त संयोग प्रयोग कीजिए।)
   (क) 709, 386, 3291
   (ख) 2062, 353, 1438, 547
   (ग) 1849, 2608, 1784, 3377, 2051

3. यदि हम दो 3 अंकों की संख्याओं को जोड़ें तो योग में अधिकतम कितने अंक संभव हैं? न्यूनतम कितने अंक संभव हैं?

*4. आइए (पूर्ण संख्याओं के लिए) एक ऐसी संक्रिया '⊕' खोजें जिसका अर्थ है, "पहली संख्या का दुगुना करके उसमें दूसरी संख्या का तिगुना जोड़ दें।" इस प्रकार, 2⊕3 = 4 + 9 = 13, 0⊕1 = 0 + 3 = 3, इत्यादि।

(क) 0 ⊕ 4 का क्या अर्थ है? 1 ⊕ 2 का क्या अर्थ है? 2 ⊕ 5 का क्या अर्थ है?

(ख) क्या 2⊕3 = 3⊕2 है? दूसरे शब्दों में क्या ⊕ एक क्रमविनिमेय संक्रिया है?

5. चित्र में दिए हुए मैजिक वर्ग* (*magic square*) पर विचार कीजिए जिसमें प्रत्येक पंक्ति, प्रत्येक स्तंभ एवं प्रत्येक विकर्ण के योग समान है।

(क) पहले स्तंभ की संख्याओं का क्या योग है?

(ख) मैजिक वर्ग पूर्ण कीजिए।

6	1	8
7	5	3
2	9	4

2.2 व्यवकलन

आप पहले ही से परिचित हैं कि एक पूर्ण संख्या में से दूसरी पूर्ण संख्या किस प्रकार घटाते या व्यवकलित (*subtract*) करते हैं। उदाहरणार्थ, क्या आपको स्मरण है कि जब आपको 9 − 5 ज्ञात करना था तो आपने क्या किया था?

आपने नौ में से पांच 'बाहर निकाल लिया' होगा। या आपने स्वयं से ही प्रश्न किया होगा, "9 प्राप्त करने के लिए मुझे 5 में क्या जोड़ना चाहिए?" प्रत्येक स्थिति में उत्तर 4 है। हाँ, दूसरी स्थिति में 9 में से 5 **व्यवकलित** करने का अर्थ वही है जो कि ऐसी संख्या ज्ञात करने का जिसे 5 में जोड़ने पर 9 प्राप्त हो जाए। इसी कारण से **व्यवकलन** (*subtraction*), **योग का प्रतिलोम** (*inverse*) कहलाता है। इस प्रकार एक व्यवकलन प्रश्न के उत्तर की सत्यता, तदनुरूपी योग की सहायता से जाँच की जा सकती है।

अतः हम देखते हैं कि कथन 4 + 5 = 9 को दूसरे प्रकार से लिखने की विधि 9 − 5 = 4 है।

अभी तक हम केवल पूर्ण संख्याओं के विषय में ही जानते हैं। क्या आप एक पूर्ण संख्या में से दूसरी पूर्ण संख्या सदैव घटा सकते हैं। नहीं! उदाहरणार्थ हम अभी तक यह नहीं जानते कि 9 में से 11 या 18 में से 23 किस प्रकार घटाएं। दूसरे शब्दों में **यदि हम अपने को केवल पूर्ण संख्याओं तक ही सीमित**

***अध्यापक के लिए निर्देश**

(कक्षा में करने योग्य गतिविधि के लिए सुझाव)

उपर्युक्त एक 3 × 3 मैजिक वर्ग का उदाहरण है। संख्याओं को बिना दोहराए हुए 1 से 9 तक की संख्याओं से जो 3 × 3 मैजिक वर्ग बनता है उसमें प्रत्येक पंक्ति, स्तंभ और विकर्ण का योग 15 होता है। विद्यार्थियों से कहिए कि वे बिना दोहराए हुए 1 से 9 तक की संख्याओं का प्रयोग करके अनेक मैजिक वर्ग बनाएँ।

विद्यार्थियों को यह खोजने का अवसर दें कि जब 2 से 10 तक, या 3 से 11 तक, या 4 से 12 तक, इत्यादि की संख्याएँ प्रयोग की जाती हैं तो क्या होता है। प्रत्येक दशा में हमें कितना योग प्राप्त होता है?

रखें तो हम एक छोटी संख्या में से बड़ी संख्या को नहीं घटा सकते। केवल छोटी संख्याएँ ही बड़ी संख्याओं में से घटाई जा सकती हैं।
क्या 9—5=5—9 है? क्या 123—98=98—123 है? नहीं।
वास्तव में, 5—9 या 98—123 को ज्ञात भी नहीं किया जा सकता है, चूंकि हम एक छोटी संख्या में से बड़ी संख्या को घटा नहीं सकते।

प्रश्नावली 2.2

1. निम्न व्यवकलन पूरे कीजिए। तदनुरूपी योग की सहायता से अपने उत्तर की जांच कीजिये।
   (क) 736—214
   (ख) 1263—989
   (ग) 6032—3295
2. निम्न व्यवकलन पूरे कीजिए :

$$\text{(क)}\quad \begin{array}{r} 101010 \\ -98765 \\ \hline \\ \hline \end{array} \qquad\qquad \text{(ख)}\quad \begin{array}{r} 250608 \\ -79368 \\ \hline \\ \hline \end{array}$$

3. निम्न में से प्रत्येक में '*' के स्थान पर उपयुक्त अंक लिखिये :

$$\text{(क)}\quad \begin{array}{r} 895 \\ -29* \\ \hline **4 \\ \hline \end{array} \qquad\qquad \text{(ख)}\quad \begin{array}{r} 5376 \\ **59 \\ \hline 35** \\ \hline \end{array}$$

4. 5 अंकों की छोटी से छोटी संख्या और 4 अंकों की बड़ी से बड़ी संख्या का अंतर ज्ञात कीजिए।
5. एक स्टोर (Store) में 2084 क्विटल आलू थे। यदि 995 क्विटल आलू बेच दिए गए हों तो स्टोर में अब कितने क्विटल आलू शेष रह गए हैं?
6. गौतम 100 रु० का नोट लेकर बाजार जाता है वह 35 रु० का राशन, 19 रु० के जूते तथा 4 रु० का दूध खरीदता है। गौतम के पास अब कितने रुपये शेष रह जाते हैं?
7. एक टेलीविजन सेट का मूल्य 3497.00 रु० था। दीवाली के उपलक्ष में उसका मूल्य घटाकर 3279.00 रु० कर दिया गया। मूल्य में कितनी कमी हुई?
8. भारत की जनसंख्या सन् 1961 में 4392 लाख थी तथा यह सन् 1971 में बढ़कर 5481 लाख हो गई। जनसंख्या में वृद्धि ज्ञात कीजिए।

2.3 गुणन पर विचार

आप जानते हैं कि दो पूर्ण संख्याओं को किस प्रकार गुणा किया जाता है। यदि संख्याएँ छोटी हों तो हम अपने मन ही में गुणा करके इनका गुणन फल ज्ञात कर सकते हैं। उदाहरणार्थ, $4 \times 5 = 20$।

अन्यथा हम अपनी गुणन सारणियों (*multiplication table*) का प्रयोग कर सकते हैं। उदाहरणार्थ,

$$\begin{array}{r} 60238 \\ \times 247 \\ \hline 421666 \\ 240952 \\ 120476 \\ \hline 14878786 \end{array}$$

क्या संख्याओं के गुणनफल (*product*) पर इसका कुछ अंतर पड़ता है कि हम इन संख्याओं को किस क्रम में गुणा करते हैं? दूसरे शब्दों में, यदि हमें 21 और 18 का गुणनफल ज्ञात करना हो तो क्या इससे कुछ अंतर पड़ेगा कि हम 21 को 18 से गुणा करते हैं या 18 को 21 से गुणा करते हैं? नहीं! इस प्रकार, $21 \times 18 = 18 \times 21$

वास्तव में यह किन्हीं भी दो पूर्ण संख्याओं के लिए सत्य है। हम कहते हैं कि **(पूर्ण संख्याओं का) गुणन क्रम विनिमेय है।** पुनः हम विद्यार्थियों को स्मरण करा दें कि नाम का यहाँ कोई महत्व नहीं है। परन्तु विद्यार्थी को यह अवश्य ज्ञात होना चाहिए कि यह गुण क्या है और इसका किस प्रकार उपयोग किया जा सकता है।

चिन्हों और संकेतों का प्रयोग करते हुए हम **गुणन के क्रम विनिमेय गुण** (*commutativity of multiplication*) को निम्न प्रकार व्यक्त कर सकते हैं:

यदि a और b कोई दो पूर्ण संख्याएँ हों, तो

$$a \times b = b \times a$$

आइए, **गुणन के लिए एक संक्रिया सारणी** (*operation table for multiplication*) बनाएँ। पहले ही की भांति हम केवल पहली पाँच पूर्ण संख्याएँ ही लेते हैं। हमें निम्न सारणी प्राप्त होगी:

	×	दूसरी संख्या 0	1	2	3	4
पहली संख्या	0	0	0	0	0	0
	1	0	1	2	3	4
	2	0	2	4	6	8
	3	0	3	6	9	12
	4	0	4	8	12	16

मुख्य विकर्ण

हम देखते हैं कि योग सारणी की भांति ही यह संक्रिया सारणी भी मुख्य विकर्ण के सापेक्ष समंमित है। सारणी से यह बहुत ही सरलता से देखा जा सकता है कि $2\times1=1\times2$, $3\times2=2\times3$, $4\times3=3\times4$ इत्यादि।

हम सारणी से यह भी देखते हैं कि

$$0\times0=0$$
$$0\times1=1\times0=0$$
$$0\times2=2\times0=0$$
$$0\times3=3\times0=0$$
$$0\times4=4\times0=0$$

दूसरे शब्दों में, **शून्य और किसी पूर्ण संख्या का गुणनफल सदैव शून्य होता है।** संकेतन में हम कहते हैं कि **यदि** a **कोई पूर्ण संख्या हो तो**

$$0\times a=a\times0=0$$

पुन: सारणी से हम यह भी देखते हैं कि

$$1\times0=0\times1=0$$
$$1\times1=1$$
$$1\times2=2\times1=2$$
$$1\times3=3\times1=3$$
$$1\times4=4\times1=4$$

दूसरे शब्दों में, **1 और किसी पूर्ण संख्या का गुणनफल सदैव स्वयं वह पूर्ण संख्या होती है।** इस गुण को **1 का गुणन गुण** (*multiplication property of 1*) कहते हैं और **1 गुणन के लिए तत्समक अवयव** (*identity element for multiplication*) कहलाता है।

चिन्हों और संकेतों का प्रयोग करते हुए हम इस गुण को निम्न प्रकार से व्यक्त कर सकते हैं:

यदि a **कोई पूर्ण संख्या हो, तो**

$$1\times a=a\times1=a$$

क्या गुणन और योग में परस्पर कोई सम्बन्ध है? आइए देखें। यदि हम 4 और 5 का गुणा करें अर्थात् 4×5 ज्ञात करें तो वह 20 आता है। यदि हम 4 को ही पांच बार जोड़ें तो क्या होगा? हमें $4+4+4+4+4$ अर्थात् 20 प्राप्त होता है। इस प्रकार हम देखते हैं कि

$$4\times5=4+4+4+4+4$$

या, $$7\times8=7+7+7+7+7+7+7+7$$

दूसरे शब्दों में, (पूर्ण संख्याओं का) **गुणन केवल बार बार योग** (*repeated addition*) **ही है।**

आइए अब तीन संख्याओं माना 49, 5 और 4 को **गुणा** करें। हम इनका गुणनफल निम्न प्रकार से ज्ञात कर सकते हैं।

$$(49\times5)\times4=245\times4=980$$

या, $$49\times(5\times4)=49\times20=980$$

हम देखते हैं कि

$$(49\times5)\times4=49\times(5\times4)$$

उपर्युक्त उदाहरण से गुणन का एक अन्य महत्वपूर्ण गुण स्पष्ट होता है, वह यह कि (पूर्ण संख्याओं का) **गुणन सहचारी होता है।** वस्तुतः नाम की अपेक्षा यह स्मरण रखना आवश्यक है कि

किन्हीं तीन संख्याओं का गुणनफल करने में इस बात से कोई अंतर नहीं पड़ता कि पहले हम कौन सी दो संख्याएँ लेते हैं और फिर उनके गुणनफल को अंतिम संख्या से गुणा करते हैं।

वास्तव में हम उपर्युक्त उदाहरण में देखते हैं कि यदि हम पहले 5 और 4 को गुणा करें तो सम्बन्ध गुणन अत्यन्त सरल है।

चिन्हों और संकेतों का प्रयोग करते हुये हम **गुणन के साहचर्य गुण** को निम्न प्रकार व्यक्त कर सकते हैं :

यदि a, b और c कोई तीन पूर्ण संख्याएँ हों, तो

$$(b \times c) \times c = a \times (b \times c)$$

इस गुण के फलस्वरूप ही हम प्रायः इन समान गुणनफलों को $a \times b \times c$ (या केवल abc लिखते हैं।

अंत में, हम चार या उसके अधिक संख्याओं को किस प्रकार गुणा करते हैं? हम गुणन के क्रमविनिमेय और साहचर्य गुणों का, यदि आवश्यक हो तो कई बार, प्रयोग करते हैं। इस स्तर पर यह आवश्यक नहीं कि हम प्रत्येक पग पर इन गुणों का प्रयोग बताते जाएँ। इस संदर्भ में जो महत्वपूर्ण है वह है 'इस प्रयोग का परिणाम'। परिणाम है कि

यदि कई संख्याएँ दी हुई हों तो यह आवश्यक नहीं कि उनको उसी क्रम में गुणा किया जाय जिसमें वो दी हुई हैं। हम उनके सरलतम समूह बनाकर उनका गुणनफल ज्ञात कर सकते हैं। यह गुण, **गुणन का पुनर्व्यवस्थितकरण गुण** (*rearrangement property of multiplication*) कहलाता है।

उदाहरणार्थ, 68, 326, 5 और 20 का गुणनफल हम निम्न प्रकार ज्ञात कर सकते हैं :

$$\begin{aligned} 68 \times 326 \times 5 \times 20 &= (68 \times 20) \times (326 \times 5) \\ &= 1360 \times 1630 \\ &= 2216800 \end{aligned}$$

$$\begin{aligned} \text{या, } 68 \times 326 \times 5 \times 20 &= (68 \times 326) \times 5 \times 20 \\ &= (22168) \times 5 \times 20 \\ &= (22168 \times 5) \times 20 \\ &= 110840 \times 20 \\ &= 2216800 \end{aligned}$$

स्पष्ट है कि, गुणनफल ज्ञात करने की दूसरी विधि अपेक्षाकृत कम सुविधाजनक है।

प्रश्नावली 2.3

1. निम्न गुणन पूरे कीजिए :
   (क) 745×816
   (ख) 2032×613

2. गुणा कीजिए :

(क) 49381×206 (ख) 23701×4389

3. निम्न गुणनफल ज्ञात कीजिए। (सबसे उपयुक्त संयोग प्रयोग कीजिए)

(क) $4 \times 203 \times 25$ (ख) $1362 \times 5 \times 18 \times 20$

4. एक पुस्तक का मूल्य 67 पैसे है। इस पुस्तक की 43 प्रतियों का क्या मूल्य होगा ?

5. 3 अंकों की छोटी से छोटी संख्या और 4 अंकों की बड़ी से बड़ी संख्या का गुणनफल ज्ञात कीजिए।

6. तमिलनाडु में धान की एक किस्म की पैदावार 8 टन प्रति. हेक्टेअर हुई। 50 लाख हेक्टेअर में कुल कितनी पैदावार हुई ?

7. एक स्कूल में कक्षा IX के एक विद्यार्थी से 72 रु० वार्षिक शुल्क लिया जाता है। यदि कक्षा IX में 436 विद्यार्थी हों तो उनसे कुल कितने रुपये प्राप्त होंगे ?

8. हमारे देश में वर्ष 1974-75 में प्राथमिक कृषि ऋण सोसाइटियों की कुल संख्या 155088 थी। यदि एक सोसाइटी की औसत सदस्यता 233 हो तो कुल सदस्यता ज्ञात कीजिए।

*9. आइए (पूर्ण संख्याओं के लिए) एक ऐसी संक्रिया 'θ' खोजें जिसका अर्थ है, 'पहली संख्या में 6 जोड़कर उसको दूसरी संख्या से गुणा कर दें'। इस प्रकार, $2\theta4 = (2+6) \times 4 = 8 \times 4 = 32$, $0\theta3 = 18$, इत्यादि।

(क) $4\theta5$ का क्या अर्थ है ? $0\theta6$ का क्या अर्थ है ? $6\theta6$ का क्या अर्थ है ?

(ख) क्या $2\theta4 = 4\theta2$ है ? दूसरे शब्दों में, क्या 'θ' एक क्रमविनिमेय संक्रिया है ?

2.4 पूर्ण संख्याओं के लिए वितरण गुण

दो मित्र मोहन और सोहन किसी काम को 80 घंटे में पूरा करते हैं। मोहन को 2.00 रु० प्रति घंटा तथा सोहन को 1.00 रु० प्रति घंटा मिलता है। इस कार्य से वे कुल कितनी धन राशि कमा लेंगे?

मोहन और सोहन द्वारा कमाई गई कुल धन राशि

$= (80 \times 2)$ रु० $+ (80 \times 1)$ रु०

$= (160 + 80)$ रु०

$= 240$ रु०

क्या इस परिणाम को ज्ञात करने की कोई दूसरी विधि भी है। स्पष्ट है, मोहन और सोहन मिलकर 3.00 रु० प्रति घंटा कमाते हैं। अतः

मोहन और सोहन द्वारा कमाई गई कुल धन राशि $= 80 \times 3$ रु०

$= 240$ रु०

हम देखते हैं कि

$$(80 \times 2) + (80 \times 1) = 80 \times (2 + 1)$$

उपर्युक्त गुणनफलों में 80 एक **सर्वनिष्ठ गुणनखंड** (*common factor*) है। ऐसे गुणन में सर्वनिष्ठ या उभयनिष्ठ गुणनखंड स्वयं **योग पर वितरित** (*distributive over addition*) होता है। हम कहते हैं कि गुणन, योग पर वितरणात्मक है (*multiplication distributes over addition*)। चिन्हों और संकेतों का प्रयोग कर हम वितरण गुण (*distributive property*) को निम्न प्रकार व्यक्त कर सकते हैं :

यदि a, b, c पूर्ण संख्याएं हों, तो
$a \times (b+c) = (a \times b) + (a \times c)$
या, $a(b+c) = ab + ac$

प्रश्नावली 2.4

वितरण गुण का प्रयोग करके निम्न को सरल कीजिए :

1. $(73 \times 64) + (27 \times 64)$
2. $(25 \times 167) + 233 \times 25$
3. 103×65
4. 45×198
5. $603 \times 7 + 3 \times 603$
6. $263 \times 24 - 163 \times 24$
7. $65 \times 813 - 613 \times 65$

निम्न में से प्रत्येक का, दो विधियों से मान ज्ञात कीजिए :

8. $(100+3) \times 305$
9. $25(264+36)$

2.5 विभाजन

आप यह जानते हैं कि एक पूर्ण संख्या को दूसरी पूर्ण संख्या से किस प्रकार विभाजित (*divide*) करते हैं। या आपको स्मरण है कि जब आपको 4 से 24 को विभाजित करना था तो आपने क्या किया था ? आपने शायद स्वयं से पूछा होगा, '24' में कितने 4 सम्मिलित हैं ? या, आपने स्वयं से यह प्रश्न पूछा होगा, मैं 4 को किससे गुणा करूं कि 24 आ जाए ? प्रत्येक स्थिति में उत्तर 6 है। हाँ, दूसरी स्थिति में 24 को 4 से **विभाजित** करने का अर्थ वही है जो कि ऐसी संख्या ज्ञात करने का जिसे 4 से **गुणा** करने पर 24 प्राप्त हो जाए। इसी कारण विभाजन (*division*) **गुणन का प्रतिलोम** (*inverse of multiplication*) कहलाता है इस प्रकार, किसी विभाजन के प्रश्न के उत्तर की तदनुरूपी गुणन की सहायता से जाँच की जा सकती है।

अतः हम देखते हैं कि कथन $4 \times 6 = 24$ **को दूसरे प्रकार** से लिखने की विधि $24 \div 4 = 6$ है।

अब $32 \div 5$ के बारे में आप क्या सोचते हैं? हम देखते हैं कि 32 में छः '5' सम्मिलित हैं और दो शेष रह जाते है। या यह कि 32 प्राप्त करने के लिए 5 में 6 का गुणा करके, गुणनफल में 2 जोड़ दिया जाए। क्या आपको स्मरण है कि 32 **भाज्य** (*dividend*) है, 5 **विभाजक** (*divisor*) है, 6 **भागफल** (*quotient*) है तथा 2 **शेष** (*remainder*) है? हम इसे निम्न प्रकार लिखते हैं:

$$32 = (5 \times 6) + 2$$

अर्थात्, भाज्य = (विभाजक × भागफल) + शेष

यदि हम a को भाज्य, b को विभाजक, q को भागफल, r को शेष मानें तो चिन्हों और संकेतों का प्रयोग करने पर हमें b $(b \neq 0)$ **द्वारा a के विभाजन के लिए निम्न एल्गोरिथ्म** (*algorithm*) अर्थात् नियम (rule) प्राप्त होता है:

$$a = bq + r$$

स्पष्ट है कि a, b, q और r में से प्रत्येक एक पूर्ण संख्या है परन्तु b शून्य के बराबर नहीं होना चाहिए। **यह ध्यान रखिए कि $r < b$।**

आपको स्मरण होगा कि गुणन केवल बार बार योग ही है। अतः **विभाजन,** गुणन का प्रतिलोम होने के कारण, **केवल बार बार व्यवकलन** (*repeated subtraction*) ही रह जाता है। आइए $32 \div 5$ में इसका निरीक्षण करें।

$$\begin{array}{rl} 32 & \\ -\ 5 & (1) \\ \hline 27 & \\ -\ 5 & (2) \\ \hline 22 & \\ -\ 5 & (3) \\ \hline 17 & \\ -\ 5 & (4) \\ \hline 12 & \\ -\ 5 & (5) \\ \hline 7 & \\ -\ 5 & (6) \\ \hline 2 & \end{array}$$

हम उस समय तक व्यवकलित करते रहते हैं जब तक कि हमें ऐसी संख्या (इस उदाहरण में 2) न प्राप्त हो जाए जो कि उस संख्या **विभाजक** से जिसका बार बार व्यवकलन किया जा रहा है छोटी हो। व्यवकलनों की संख्या को **भागफल** तथा अंतिम संभव व्यवकलन के बाद बची हुई संख्या को **शेष** कहा जाएगा।

शून्य से विभाजन के विषय में आप क्या सोचते हैं? आइए उदाहरण के लिए 8 को शून्य से विभाजित करने का प्रयत्न करें। हमें ऐसी दो संख्याएँ q और r ज्ञात करने की आवश्यकता है कि

$$8 = 0 \times q + r$$

परन्तु शून्य को किसी भी संख्या से गुणा करने पर शून्य ही प्राप्त होता है। अत: $8=r$। परन्तु शेष, भाजक से, जो कि इस उदाहरण में शून्य है, छोटा होना चाहिए। अत: शून्य से विभाजन नहीं हो सकता। हम कहते हैं कि **शून्य से विभाजन परिभाषित नहीं है।**

प्रश्नावली 2.5

1. निम्न विभाजन पूरे कीजिए और तदनुरूपी गुणन की सहायता से अपने उत्तर की जांच कीजिए :

(क) $9261 \div 21$ (ख) $4107 \div 37$ (ग) $9348 \div 246$

2. विभाजित कीजिए :

(क) 72315 को 45 से

(ख) 328032 को 804 से

(ग) 364800 को 600 से

3. क्या विभाजन क्रम विनिमेय है? दूसरे शब्दों में, यदि a और b धन पूर्णांक हों तो क्या $a \div b = b \div a$ है?

4. 2431100 को दो संख्याओं के गुणा के रूप में व्यक्त कीजिए जिनमें एक संख्या 302 है।

5. एक चिड़ियाघर (zoo) में घूमते समय, 25 बच्चे हाथी की सवारी करना चाहते हैं। यदि एक हाथी पर एक बार में केवल 5 ही बच्चे बैठ सकते हैं तो हाथी को कितनी बार सवारी करानी पड़ेगी?

6. यदि प्रत्येक संख्या 4865 और 4901 को 29 से विभाजित किया जाए, तो शेष ज्ञात कीजिए।

7. एक सिनेमा गृह की प्रत्येक पंक्ति में 32 सीटें हैं। 475 व्यक्तियों को बैठाने के लिए कम से कम कितनी पंक्तियों की आवश्यकता है?

8. एक संख्या ज्ञात कीजिए जिसे 25 से विभाजित करने पर 24 शेष रहे। क्या आप ऐसी कुछ और संख्याएँ ज्ञात कर सकते हैं?

9. किसी वर्ष विशेष का पहला दिन शुक्रवार था। यह मानते हुए कि यह वर्ष लौंद का वर्ष (leap year) नहीं था, ज्ञात कीजिए कि अगले वर्ष का पहला दिन सप्ताह के कौन से दिन पड़ेगा?

विविध प्रश्नावली I

(एकक I और II पर)

1. संकेतों '<', '>' या '=' में से किसी एक का प्रयोग करते हुए रिक्त स्थानों को भरिए।
   (i) 123...321
   (ii) 55...55
   (iii) 89...1

2. (क) निम्न संख्याओं को आरोही क्रम में लिखिए :
   7, 15, 4, 1, 85
   (ख) निम्न संख्याओं को अवरोही क्रम में लिखिए :
   1, 85, 15, 4, 7

3. (क) अंकों 2, 4 और 7 का प्रयोग करते हुए एक अंक की तीन संख्याएँ लिखिए।
   (ख) अंकों 2, 4 और 7 का प्रयोग करते हुए तीन अंकों की एक संख्या लिखिए।

4. रिक्त स्थान भरिए :
   (i) $7\times(8+9)=7\times8+\ldots$
   (ii) $\ldots\times(75-15)=25\times75\ldots25\times15$
   (iii) $35\times(\ldots+\ldots)=35\times7+35\times3$
   (iv) $a(b+c)=\ldots+a\times c$
   (v) $a(b-c)=ab-\ldots$

5. संख्या 98056 में 8 और 5 के स्थानीय मानों का योग ज्ञात कीजिए।

6. संख्या 758 में 5 के स्थानीय मान को उसके अंकित मान से भाग दीजिए।

7. क्या $35\div5$, $5\div35$ के बराबर है ?

8. 175 में से 85 और 15 के योग को घटाइए।

9. 78 प्राप्त करने के लिए 203 में से क्या घटाना चाहिए ?

10. क्या संख्यापद्धति में कोई ऐसा स्थान भी है जहाँ किसी भी शून्येतर अंक के स्थानीय मान और अंकित मान एक ही हों ? यदि हाँ, तो वह स्थान ज्ञात कीजिए।

11. वह अंक बताइए जिसका स्थान बदलने पर भी स्थानीय मान नहीं बदलता।

12. 5602 में 6 के स्थानीय मान को 2 के स्थानीय मान से भाग दीजिए।

13. (क) कोई भी दो विषम संख्याएँ लीजिए और उनका अंतर ज्ञात कीजिए। यह अंतर विषम है या सम ?

(ख) कोई भी दो सम संख्याएँ लीजिए और उनका अंतर ज्ञात कीजिए। यह अंतर विषम है या सम ?

(ग) एक विषम और एक सम संख्या लीजिए। उनका योग विषम है या सम? उनका अंतर विषम है या सम?

14. 35 और 15 के योग और अंतर का गुणनफल ज्ञात कीजिए।

15. (क) 5 पर समाप्त होने वाली 4 अंकों की छोटी से छोटी संख्या लिखिए।

(ख) 5 पर समाप्त होने वाली 4 अंकों की बड़ी से बड़ी संख्या लिखिए।

16. सभी अंकों 5, 0, 2 और 7 का प्रयोग करते हुए बड़ी से बड़ी और छोटी से छोटी पूर्ण संख्याएँ बनाइए जबकि प्रत्येक अंक एक संख्या में एक ही बार आए।

17. 4 अंकों की छोटी से छोटी तथा 3 अंकों की बड़ी से बड़ी संख्याओं के योग और अंतर का गुणनफल ज्ञात कीजिए।

18. सभी अंकों 0, 1, 2,..., 9 का प्रयोग करते हुए बड़ी से बड़ी और छोटी से छोटी पूर्ण संख्याएँ लिखिए जबकि प्रत्येक अंक एक संख्या में एक ही बार आए। दोनों संख्याओं का अंतर भी ज्ञात कीजिए।

19. निम्न का मान ज्ञात करने के लिए वितरण गुण का प्रयोग कीजिए :

(क) 99×15 (ख) 998×25

20. किसी वर्ष विशेष में जनवरी का पहला दिन सोमवार को पड़ता है। यह मानते हुए कि यह वर्ष लौंद का वर्ष नहीं है, बताइए कि इस वर्ष में मार्च का पहला दिन सप्ताह के कौन से दिन पड़ेगा?

21. किसी लौंद के वर्ष में फरवरी का पहला दिन शुक्रवार को पड़ता है। इस वर्ष में अप्रेल का पहला दिन सप्ताह के किस दिन पड़ेगा?

22. निम्न में से प्रत्येक में '*' के स्थान पर उपयुक्त अंक लिखिए :

(क)
```
 123
 ×*
 ----
 8*1
```

(ख)
```
   527
   ×**
  ----
  *6*9
  ***4
 -----
 1*2**
```

एकक III

पूर्णांक

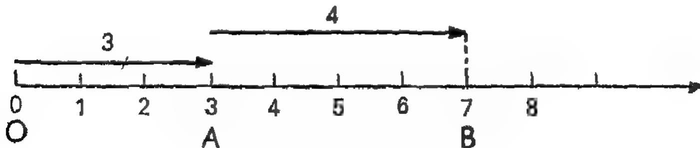
इस एकक में हम पूर्णांकों की आवश्यकता पर प्रकाश डालेंगे तथा उनको संख्या रेखा पर निरूपित करेंगे।

3.1 पूर्णांकों की आवश्यकता

अनुच्छेद 1.5 में हमने पूर्ण संख्याओं को संख्या रेखा पर निरूपित किया था। एकक II में हमने पूर्ण संख्याओं पर संक्रियाओं का पुनरावलोकन किया था। विशेष रूप से अनुच्छेद 2.2 में हमने व्यवकलन संक्रिया का अध्ययन किया था और देखा था कि यदि हम अपने को केवल पूर्ण संख्याओं तक ही सीमित रखें तो हम छोटी संख्या में से बड़ी संख्या को घटा नहीं सकते। दूसरे शब्दों में, पूर्ण संख्याओं में, हम $5-9$, $98-123$, इत्यादि ज्ञात नहीं कर सकते। अतः हमें नई संख्याएँ 'खोजने' की आवश्यकता है ताकि हम $5-9$, $98-123$, इत्यादि ज्ञात कर सकें।

आइए, पहले संख्या रेखा की सहायता से पूर्ण संख्याओं के योग और व्यवकलन ज्ञात करने की विधि का पुनरावलोकन करें।

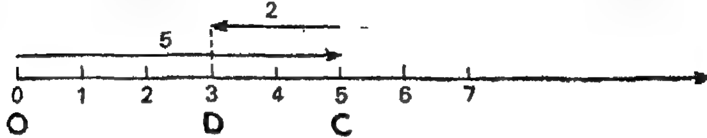
उदाहरण 1 : यदि हमें संख्या रेखा की सहायता से, उदाहरणार्थ, $3+4$ ज्ञात करना हो तो हम क्या करेंगे? हमारे पास एक संख्या रेखा है जिस पर किसी बिंदु O से प्रारम्भ करके उसके दाईं ओर को चलते हुए, एक के बाद एक, बराबर पगों (दूरियों) के निशान लगे हैं। आपको याद होगा कि प्रारम्भिक बिंदु को O तथा अन्य बिंदुओं के क्रमशः 1, 2, 3,... लिखा जाता है। अब $3+4$ ज्ञात करने के लिए हम O से प्रारम्भ करेंगे तथा पहले 3 पग दाईं ओर को चलेंगे और मान



आकृति 3.1 : $3+4=7$

लीजिए A पर आ जाते हैं। (देखिए आकृति 3.1) फिर हम A से प्रारंभ करेंगे और दाईं ओर को 4 पग और चलेंगे और इस प्रकार, मान लीजिए, B पर आ जाते हैं। अतः $3+4=7$

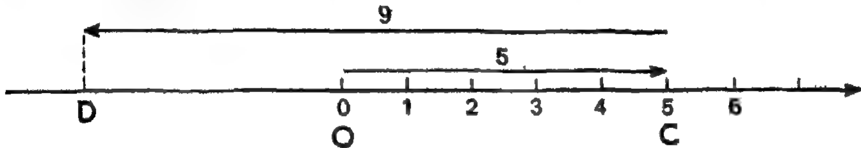
उदाहरण 2 : आइए संख्या रेखा की सहायता से, उदाहरणार्थ, 5—2 ज्ञात करें। हम यह कैसे करेंगे ? पुनः हम पहले O से प्रारम्भ करेंगे तथा दाईं ओर को 5 पग चलेंगे और, मान लीजिए,



आकृति 3.2 : 5—2=3

C पर आ जाते हैं। (देखिए आकृति 3.2) फिर हम C से प्रारम्भ करेंगे तथा बाईं ओर को 2 पग चलेंगे और इस प्रकार मान लीजिए D पर आ जाते हैं। अतः 5—2=3

उदाहरण 3 : आइए उदाहरण 2 की विधि का प्रयोग करें और उदाहरणार्थ, 5—9 ज्ञात करने का प्रयत्न करें। पहले हम O से प्रारम्भ करके दाईं ओर को 5 पग चलते है और, मान लीजिए, C पर आ जाते हैं। अब यदि हम C से प्रारम्भ करके बाईं ओर को 9 पग चलें तो हम O को पार कर जाएँगे



आकृति 3.3 : 5—9=?

और उसके बाईं ओर, मान लीजिए, किसी बिंदु D पर आ जाएँगे। (देखिए आकृति 3.3) परन्तु यह D क्या है ?

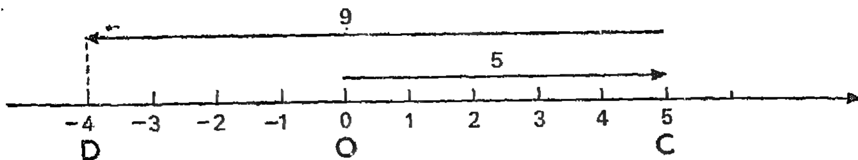
इस बिंदु को ज्ञात करने के लिए, आइए संख्या रेखा पर O से प्रारम्भ करके उसके बाईं ओर, एक के बाद एक, बराबर* पगों (दूरियों) के चिन्ह लगाएँ। साथ ही, इन बिंदुओं को क्रम से —1, —2, —3,...लिखें** (देखिए आकृति 3.4) और इन्हें क्रम से 'ऋणात्मक (*negative*) एक'*** 'ऋणात्मक दो', 'ऋणात्मक तीन', ... पढ़ें।

---

*प्रारम्भिक बिंदु के दोनों ओर दूरियाँ (पग) बराबर होनी चाहिए।

**जब (प्रारम्भिक बिंदु), O के दोनों ओर के बिंदुओं को उपर्युक्त प्रकार से लिखते हैं तो यह परम्परागत है कि O के दाईं ओर के बिंदुओं को क्रम से +1, +2, +3,...तथा बाईं ओर के बिंदुओं को क्रम से —1, —2, —3,...लिखा जाता है। इससे हमें O के दाईं ओर के पगों (steps) और बाईं ओर के पगों में भेद करने में सहायता मिलती है। इस प्रकार +3, O के बाईं ओर 3 पगों की दूरी पर एक बिंदु निरूपित करता है जबकि —3, O के बाईं ओर 3 पगों की दूरी पर एक बिंदु निरूपित करता है। परन्तु धन चिन्ह (+) को प्रायः छोड़ दिया जाता है चूँकि बिना इसके लगाए भी अर्थ सरलता से समझ में आ जाता है।

***इनको कभी कभी 'ऋण (*minus*) एक', 'ऋण दो', 'ऋण तीन', इत्यादि भी पढ़ा जाता है। परन्तु हम ऋण के स्थान पर 'ऋणात्मक' शब्द के प्रयोग को प्राथमिकता देंगे।



आकृति 3.4 : $5-9=-4$

अब हम देखते हैं कि $5-9=-4$। इस प्रकार देखिए हमने $5-9$, $98-123$, इत्यादि ज्ञात करने में समर्थ होने के लिए संख्याओं $-1, -2, -3, \ldots$ की 'खोज' की। संग्रह $0, 1, -1, 2, -2, 3, -3, \ldots$ **पूर्णांकों का समुच्चय** (*set of integers*) **कहलाता है।** जब हम इस पूरे संग्रह को लेते हैं तो $1, 2, 3, \ldots$ **धनात्मक पूर्णांक** (*positive integers*) तथा $-1, -2, -3, \ldots$ **ऋणात्मक पूर्णांक** (*negative integers*) कहलाते हैं। संख्या 0 केवल पूर्णांक है। न तो यह धनात्मक पूर्णांक है और न ही ऋणात्मक पूर्णांक।

हम देखेंगे कि जब कभी भी वास्तविक जीवन में दो परस्पर **प्रतिकूल** (*opposite*) स्थितियाँ संबद्ध होती हैं तो ऐसी स्थितियों को गणितीय रूप में व्यक्त करने के लिए पूर्णांकों के प्रयोग की आवश्यकता होती है।

उदाहरणार्थ, एक दुकानदार के विभिन्न महीनों में लाभ (*profits*) और हानियाँ (*losses*); **समुद्र सतह** (*sea level*) **से ऊपर** (*above*) **और समुद्र सतह से नीचे** (*below*) के पदों में स्थानों की **ऊंचाईयाँ;** 0°C **के ऊपर** या 0°C **के नीचे** वस्तुओं के **तापमान** (*temperatures*); इत्यादि। ऐसी स्थितियों में हम लाभ, समुद्र सतह से ऊपर ऊंचाई, 0°C से ऊपर तापमान, इत्यादि को **धनात्मक पूर्णांकों** से तथा उनके प्रतिकूलों (*opposites*) अर्थात् हानि, समुद्र सतह से नीचे ऊंचाई, 0°C से नीचे तापक्रम, इत्यादि को **ऋणात्मक पूर्णांकों** से निरूपित कर सकते हैं।

देखिए हमने ऋणात्मक पूर्णांकों को व्यक्त करने के लिए संकेत '$-$' का प्रयोग किया है। साथ ही, हम इस संकेत का व्यवकलन दर्शाने में भी प्रयोग कर चुके हैं। क्या इससे कुछ भ्रम उत्पन्न होने की संभावना है? नहीं! संदर्भ से यह सब सदैव स्पष्ट हो जाएगा। उदाहरणार्थ जब हम यह कहते हैं कि शिमले का तापमान -3°C था तो यह तुरन्त स्पष्ट हो जाता है कि यहाँ कोई व्यवकलन संबद्ध नहीं है। यहाँ केवल ऋणात्मक पूर्णांक '-3' दिखाया गया है। या जब हम कहते हैं कि $16-7$ ज्ञात कीजिए तो यह तुरन्त स्पष्ट हो जाता है कि यहाँ कोई ऋणात्मक पूर्णांक संबद्ध नहीं है। यहाँ केवल 16 में से 7 का व्यवकलन दर्शाया गया है।

प्रश्नावली 3.1

1. निम्न में से प्रत्येक का **प्रतिकूल** बताइए :
   (i) मूल्य में वृद्धि
   (ii) उत्तर को जाना
   (iii) जनसंख्या में कमी
   (iv) बैंक में रुपया जमा कराना
   (v) वज़न कम होना

2. आप निम्न तापमानों को किस प्रकार लिखेंगे ?
   (i) शून्य से 7°C ऊपर (ii) शून्य से 7°C नीचे
3. निम्न को पूर्णांकों की सहायता से दर्शाइए :
   (i) खाते (*account*) में से 25 रु० निकालना,
   (ii) खाते में 110 रु० जमा करना।

4. नीचे कुछ वस्तुओं के रुपयों में विक्रय मूल्य (*selling prices*) और क्रय मूल्य (*cost prices*) दिए हुए हैं। पूर्णांकों का प्रयोग करते हुए प्रत्येक वस्तु पर हुआ लाभ या हानि लिखिए।

वस्तु	1	2	3	4	5	6	7
विक्रय मूल्य (रुपयों में)	30	18	24	16	21	7	12
क्रय मूल्य (रुपयों में)	20	12	19	23	43	10	15
लाभ (रुपयों में)	+10			−7			

5. यदि '+200' यह प्रदर्शित करता है कि कोई विशेष शहर समुद्र सतह से 200 मीटर की ऊँचाई पर है तो निम्न से क्या प्रदर्शित होगा ?
   (i) +300 (ii) −100 (iii) +70
6. संख्या रेखा पर निम्न संख्याएँ अंकित कीजिए :
   −6, 3, −8, 10, +11, −9
7. निम्न मापनों (*measures*) को पूर्णांकों की सहायता से निरूपित कीजिए :
   (i) मृत सागर (*dead sea*) की सतह, समुद्र सतह से लगभग 390 मीटर नीचे है।
   (ii) एवरेस्ट पहाड़ (*mount everest*) समुद्र सतह से लगभग 8840 मीटर ऊपर है।

3.2 निरपेक्ष मान

दो मित्र, राम और दत्त, एक ही बिंदु O से प्रारम्भ करके परस्पर विपरीत दिशाओं में 3 मीटर चलते हैं। तब राम की स्थिति को +3 (या केवल 3) से तथा दत्त की स्थिति को −3 से निरूपित किया जा सकता है। अब मान लीजिए कि उनकी चलने की दिशाओं में हमारी कोई रुचि नहीं है तथा केवल प्रत्येक ने O से जितनी दूरी चली है उसी में हमारी रुचि है। तो राम और दत्त दोनों ने ही प्रारम्भिक बिंदु O से 3 मीटर की दूरी चली है। हम कहते है कि दोनों में से प्रत्येक द्वारा मीटरों में चली हुई दूरी 'तीन' का **निरपेक्ष मान** (*absolute value*) है।

किसी पूर्णांक का निरपेक्ष मान उसके चिन्ह (*sign*) **पर बिना कोई ध्यान दिए स्वयं वह पूर्णांक ही होता है।** हम पूर्णांक का निरपेक्ष मान दर्शाने के लिए उसे दो उर्ध्वाधर लकीरों '| |' के बीच में रखते

हैं। इस प्रकार, |3| = 3, |−3| = 3, |−8| = 8। चूंकि पूर्णांक 0 न तो धनात्मक है और न ही ऋणात्मक इसलिए हम कहते हैं कि **शून्य का निरपेक्ष मान शून्य है।** इसे हम लिखते है कि |0| = 0

3.3 संख्या रेखा पर क्रम

हम, पूर्ण संख्याओं में, पहले ही देख चुके हैं कि एक संख्या दूसरी संख्या से बड़ी तब होती है जब कि संख्या रेखा पर पहली संख्या दूसरी संख्या के दाईं ओर स्थित हो।

हम इस कल्पना को पूर्णांकों के लिए भी लागू कर सकते हैं। −2, −5 के **दाईं** ओर स्थित है, अतः हम कहते हैं कि −2, −5 से **बड़ा** है और इसे −2 > −5 लिखते हैं। इसी प्रकार −4, −1 के **बाईं** ओर स्थित है, अतः हम कहते हैं कि −4, −1 से **छोटा** है और इसे −4 < −1 लिखते हैं।

यह सरलता से देखा जा सकता है कि शून्य **प्रत्येक ऋणात्मक पूर्णांक से बड़ा है** तथा **प्रत्येक धनात्मक पूर्णांक से छोटा है।**

संक्षेप में, **एक पूर्णांक, दूसरे पूर्णांक से बड़ा होता है जबकि वह संख्या रेखा पर दूसरे पूर्णांक के दाईं ओर स्थित हो।** साथ ही, **एक पूर्णांक, दूसरे पूर्णांक से छोटा होता है जबकि वह संख्या रेखा पर दूसरे पूर्णांक के बाईं ओर स्थित हो।**

प्रश्नावली 3.2

1. खाली स्थानों में उपयुक्त संकेत '>' या '<' भरिए ताकि निम्न में से प्रत्येक सत्य हो।

   (i) 0...−1 (ii) −2...2
   (iii) −1...18 (iv) 0...7
   (v) −8...−3 (vi) 4...−1
   (vii) −8...−13 (viii) −6...−2

2. निम्न को आरोही क्रम (*increasing order*) में लिखिए :

   −3, 17, −10, 16, −4

3. निम्न को अवरोही क्रम (*decreasing order*) में लिखिए :

   −3, −10, 0, −4, −5, 21

4. निम्न में से प्रत्येक युग्म में कौनसी संख्या बड़ी है?

   (i) 140, 120 (ii) −140, 120
   (iii) 140, −120 (iv) −140, −120

5. निम्न में कौन छोटा है?

   (i) −2871 या −4948 (ii) −10785 या 126
   (iii) −28913 या −120610 (iv) 999 या 9999

6. −7 और 3 के बीच में कितने पूर्णांक हैं? कितनी पूर्ण संख्याएँ हैं?

7. निम्न में से प्रत्येक का निरपेक्ष मान लिखिए :

   −1, 0, 8, −7, +13

एकक IV

पूर्णांकों पर संक्रियाएँ

इस एकक में हम यह सीखेंगे कि पूर्णांकों को किस प्रकार जोड़ा, घटाया, गुणा और भाग किया जाता है। साथ ही इस एकक में हम इन संक्रियाओं के गुणों का भी अध्ययन करेंगे।

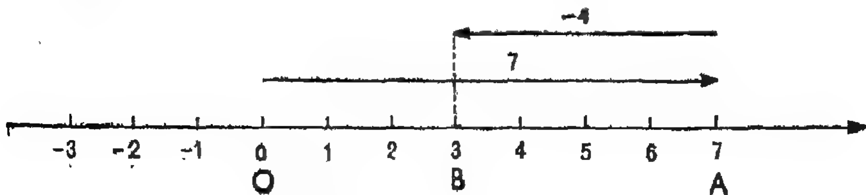
4.1 पूर्णांकों का योग

4.1.1 यदि दोनों पूर्णांक धनात्मक या शून्य हों तो हम पहले से ही जानते हैं कि इनको किस प्रकार जोड़ा जाता है। परन्तु यदि दोनों में से एक या दोनों ही ऋणात्मक हों तो क्या होगा? हम पूर्णांकों के योग के लिए ऐसा या ऐसे नियम जानना चाहेंगे जिनसे हमें अपने दैनिक जीवन के अनुभवों के अनुकूल उत्तर प्राप्त हों। उदाहरण के तौर पर हम जानते हैं कि 7 रु० के लाभ और उसके बाद 4 रु० की हानि से परिणामतः 3 रु० का शुद्ध लाभ होता है। अतः हमें यह आशा करनी चाहिए कि $7+(-4)=3$। इसी प्रकार 3 रु० के लाभ और उसके बाद 5 रु० की हानि से परिणामतः 2 रु० की शुद्ध हानि होगी। अतः हमें यह आशा करनी चाहिए कि $3+(-5)=-2$।

आइए अब संख्या रेखा को लें। आपको याद होगा कि, उदाहरणार्थ, $+3$ **दाईं** ओर को 3 पग चलना निरूपित करता है और -3, **बाईं** ओर को 3 पग चलना। इस ज्ञान के साथ आइए अब पूर्णांकों को जोड़ें।

उदाहरण 1: $+7$ और -4 को जोड़िए।

हल: हम संख्या रेखा पर O से प्रारम्भ करेंगे तथा पहले, **दाईं** ओर को 7 पग चलेंगे। माना हम इस प्रकार A पर आ जाते हैं। अब हम A से प्रारम्भ करते हुए **दाईं** ओर को 4 पग चलेंगे और इस प्रकार मान लीजिए हम B पर आ जाते हैं। (देखिए आकृति 4.1) इस प्रकार, $7+(-4)=3$।

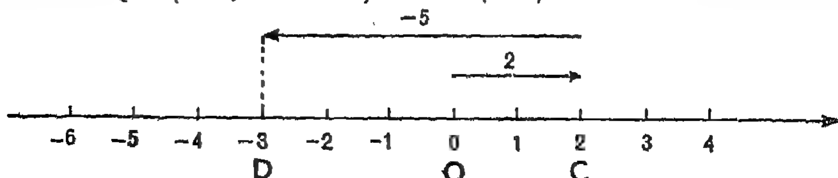


आकृति 4.1: $7+(-4)=3$

(यदि हम पहले 4 पग बाईं ओर को चलें तथा फिर 7 पग दाईं ओर को चलें तो क्या होता है ?)

उदाहरण 2 : $+2$ और -5 को जोड़िए।

हल : संख्या रेखा पर O से प्रारम्भ करके हम पहले 2 पग दाईं ओर को चलते है और, मान लीजिए, C पर आ जाते हैं। फिर, C से प्रारम्भ करके हम 5 पग बाईं ओर को चलते हैं और, मान लीजिए, D पर आ जाते हैं। (देखिए आकृति 4.2) तब $2+(-5)=-3$



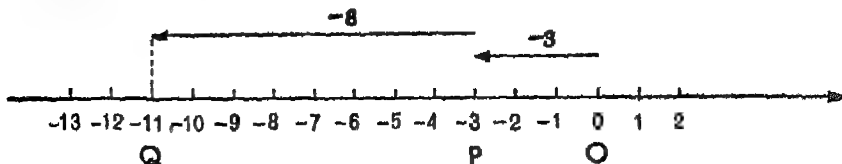
आकृति 4.2 : $2+(-5)=-3$

(यदि हम पहले 5 पग बाईं ओर को चलें तथा फिर 2 पग दाईं ओर को चलें तो क्या होता है ?)

उदाहरण 3 : -3 और -8 को जोड़िए।

हल : संख्या रेखा पर O से प्रारम्भ करके पहले हम 3 पग बाईं ओर को चलते हैं। मान लीजिए हन P पर आ जाते हैं।

अब हम P से प्रारम्भ करके बाईं ओर को 8 पग और चलते हैं तथा, मान लीजिए, Q पर आ जाते हैं। (दखिए आकृति 4.3) इस प्रकार, $(-3)+(-8)=-11$



आकृति 4.3 : $(-3)+(-8)=-11$

(यदि हम पहले 8 पग बाईं ओर को चलें तथा फिर 3 पग बाईं ओर को और चलें तो क्या होता है?)

प्रश्नावली 4.1

1. संख्या रेखा की सहायता से निम्न पूर्णांक युग्मों (*pairs of integers*) का योग ज्ञात कीजिए :

(i) $-7, 5$ (ii) $-3, -9$ (iii) $8, -11$
(iv) $6, 4$ (v) $-2, 2$

2. निम्न में से प्रत्येक ज्ञात कीजिए। जहां तक संभव हो संख्या रेखा का प्रयोग न कीजिए।

(i) $2+(-8)$ (ii) $-7+(-3)$

(iii) $-14+(-15)$ (iv) $-20+12$

(v) $-22+(-68)$ (vi) $49+(-12)$

3. निम्न योग सारणी पूरी कीजिए :

दूसरी संख्या

	+	−3	−2	−1	0	1	2	3
	−3							
	−2							
	−1							
पहली	0							
संख्या	1							
	2							
	3							

(क) क्या यह योग सारणी मुख्य विकर्ण के सापेक्ष सममित है ?

(ख) सारणी से जाँच कीजिए कि $-3+(-1)=-1+(-3)$। क्या यह सत्य है कि $3+(-2)=(-2)+3$?

(ग) उन पूर्णांकों के युग्म लिखिए जिनका योग शून्य है।

(घ) आप पहली संख्या 0 को तदनुरूपी पंक्ति में क्या देखते हैं ?

4.1.2 अनुच्छेद 4.1.1 के उदाहरणों से हम पूर्णांकों का योग ज्ञात करने के नियम निकाल सकते हैं। क्या आप बता सकते हैं कि ये नियम कौन से हैं ? हम देखते हैं कि

(क) यदि पूर्णांक समान चिन्हों (*like signs*) के हैं (अर्थात् दोनों धनात्मक हैं या दोनों ही ऋणात्मक हैं तो हम उनके निरपेक्ष मानों को जोड़ते हैं और योग में उभयनिष्ठ चिन्ह लगा देते हैं।

(ख) यदि पूर्णांक असमान चिन्हों (*unlike signs*) के हों तो हम

(i) उनके निरपेक्ष ज्ञात करते हैं,

(ii) छोटे निरपेक्ष मान को बड़े मान में से घटाते हैं, तथा

(iii) इस अंतर में बड़े निरपेक्ष मानवाले पूर्णांक का चिन्ह लगा देते हैं।

उदाहरण 4 : -257 और -63 को जोड़िए।

हल : हम देखते हैं कि दोनों पूर्णांक ऋणात्मक हैं। अत: हम नियम (क) का प्रयोग करते हैं।

$$|-257|=257$$
$$|-63|=63$$

हम निरपेक्ष मानों अर्थात् 257 और 63 को जोड़ते हैं और हमें 257+63=320 प्राप्त होता है।

अब हम इसमें उभयनिष्ठ चिन्ह लगा देते हैं। इस प्रकार,

$$-257+(-63)=-320$$

उदाहरण 5 : −39 और 81 को जोड़िए।

हल : पूर्णांक असमान चिन्हों के हैं। इसलिए हम यहाँ नियम (ख) का प्रयोग करते हैं।

$$|-39|=39$$
$$|81|=81$$

−39 का निरपेक्ष मान छोटा है। इसलिए हम 81 में से 39 घटाते हैं। इस प्रकार हमें 42 प्राप्त होता है।

अब इस अंतर अर्थात् 42 में, हम बड़े निरपेक्ष मान वाले पूर्णांक का चिन्ह लगा देते हैं जो कि '+' है।

इस प्रकार, $-39+81=+42$

उदाहरण 6 : 494 और −795 को जोड़िए।

हल : हमें नियम (ख) के प्रयोग की आवश्यकता है। (क्यों ?)

$$|494|=494$$
$$|-795|=795$$
$$795-494=301$$

बड़े निरपेक्ष मान वाले पूर्णांक का चिन्ह '−' है। इस प्रकार, $494+(-795)=-301$

प्रश्नावली 4.2

1. निम्न पूर्णांक युग्मों को जोड़िए :

(i) −140, 110 (ii) −80, 46

(iii) −800, 820 (iv) 240, −320

(v) 908, −8 (vi) 498, −312

(vii) 6666, −7777 (viii) −5894, −789

2. निम्न में से प्रत्येक में योग ज्ञात कीजिए :

(i) −494+(−795) (ii) 509+(−871)

(iii) −1280+(−45) (iv) −623+(623)

(v) 3003+(−999) (vi) 1816+(−1816)

4.1.3 अनुच्छेदों 4.1.1 और 4.1.2 के प्रश्नों में हम, उदाहरणार्थ, यह देख सकते हैं कि

$7+(-4)=(-4)+7,$

$2+(-5)=(-5)+2,$

$-3+(-8)=(-8)+(-3),$

$-257+(-63)=-63+(-257)$, तथा

$494+(-795)=(-795)+494$

दूसरे शब्दों में, **हम पूर्णांकों को किस क्रम में जोड़ते हैं इससे योग पर कोई प्रभाव नहीं पड़ता।** क्या आप इस गुण का नाम बता सकते हैं? हम कहते हैं कि **पूर्णांकों का योग क्रमविनिमेय है।** अर्थात्

यदि a और b कोई दो पूर्णांक हों, तो

$$a+b=b+a$$

यदि हमें तीन पूर्णांकों को जोड़ना हो तो क्या होगा? आइए -80, 46 और 110 को जोड़ें। यह सरलता से जाँच की जा सकती है कि

$$(-80+46)+110=-80+(46+110)$$

क्या आप इस गुण का नाम बता सकते हैं? हम कहते हैं कि **पूर्णांकों का योग सहचारी है।** अर्थात्

यदि a b और c कोई तीन पूर्णांक हों, तो

$$(a+b)+c=a+(b+c)$$

पहले ही की तरह हम, प्रायः इस समान योग को $a+b+c$ लिखते हैं।

दूसरे शब्दों में, **तीन पूर्णांकों का योग ज्ञात करने में इसका कोई प्रभाव नहीं पड़ता कि हम पहले कौन से दो पूर्णांक (लेकर) जोड़ते हैं और फिर योग ज्ञात करने के लिए उसमें अंतिम पूर्णांक जोड़ते हैं।**

शून्य और किसी पूर्णांक के योग के बारे में आप क्या सोचते हैं? हम देखते हैं कि

$0+0=0$

$0+(-20)=-20=-20+0$

$0+63=63=63+0$

दूसरे शब्दों में, **किसी पूर्णांक और शून्य का योग स्वयं वह पूर्णांक होता है।** संकेतों का प्रयोग कर हम कहते हैं कि

यदि a कोई पूर्णांक हो, तो

$$a+0=a=0+a$$

यह **शून्य का योज्य गुण** कहलाता है तथा 0, (पूर्णांकों के) **योग के लिए तत्समक अवयव** कहलाता है।

अंत में, हम **चार या उससे अधिक पूर्णांकों को किस प्रकार जोड़ते हैं?** पहले ही की तरह हम योग के क्रमविनिमेय और साहचर्य गुणों का, यदि आवश्यक हो तो कई बार, प्रयोग करेंगे और **उनके सरलतम समूह बनाकर योग ज्ञात करेंगे।**

उदाहरण 7 : -11, -19, 23, -32 और -15 का योग ज्ञात कीजिए।

हल : $-11+(-19)+23+(-32)+(-18)$

$=[-11+(-19)]+23+[(-32)+(-18)]$

$=(-30)+23+(-50)$

$=[-30+(-50)]+23$

$=-80+23=-57$

प्रश्नावली 4.3

1. निम्न योग ज्ञात करने के लिए संख्या रेखा का प्रयोग कीजिए :
   (i) $-6+3+(-4)$ (ii) $-11+8+(-1)$
   (iii) $-3+(-4)+(-5)$
2. योग ज्ञात कीजिए। [सबसे उपयुक्त संयोग (*association*) का प्रयोग कीजिए।]
   (i) 373, −245, −373 (ii) −391, −81, −9
   (iii) −982, 1934, −18, −2034 (iv) −4329, 8648, −4371

3. आइए (पूर्णांकों के लिए) एक संक्रिया '' ऐसी खोजें कि यदि a और b दो पूर्णांक हों तो

$$a*b=a+b+1$$

उदाहरणार्थ, $2*3=2+3+1=6,$

$4*(-3)=4+(-3)+1=2$

(क) $-2*(-2)$ का क्या अर्थ है? $0*7$ का क्या अर्थ है? $0*(-5)$ का क्या अर्थ है?

(ख) क्या $2*3=3*2$ है? क्या * एक क्रमविनिमेय संक्रिया है?

4.2 पूर्णांक का ऋणात्मक

धनात्मक और ऋणात्मक पूर्णांकों से सामान्य युग्मों उदाहरणार्थ 1 और −1, 2 और −2, 3 और −3, इत्यादि का संकेत मिलता है। हम देखते हैं कि प्रत्येक युग्म में योग शून्य है, अर्थात् $1+(-1)=0$, $2+(-2)=0$, इत्यादि। ऐसे किसी भी युग्म में प्रत्येक पूर्णांक दूसरे पूर्णांक का ऋणात्मक [(या योज्य प्रतिलोम (*additive inverse*)] कहलाता है। अतः 3 का ऋणात्मक −3 है तथा −3 का ऋणात्मक 3 है। इस प्रकार, प्रत्येक शून्येतर पूर्णांक 'a' के लिए एक पूर्णांक −'a' इस प्रकार होता है कि $a+(-a)=0$। '$-a$', a का ऋणात्मक (या योज्य प्रतिलोम) कहलाता है। शून्य का ऋणात्मक स्वयं शून्य ही है?

प्रश्नावली 4.4

1. निम्न में से पूर्णांकों और उनके ऋणात्मकों के युग्म चुनिए :
   $-17,\ 8,\ -6,\ -4,\ -3,\ 3,\ 6,\ 17,\ 4,\ -8$
2. निम्न में से प्रत्येक का ऋणात्मक लिखिए :
   $-200,\ -100,\ -65,\ -48,\ 84,\ 95,\ 0,\ -1$
3. संख्या रेखा पर निम्न में से प्रत्येक को '.' से तथा उसके ऋणात्मक को '×' से दर्शाइए :
   $3,\ -7,\ -2,\ 4,\ 8$
4. रिक्त स्थान भरिए :

   (i) $-4+\ldots=0$ (ii) $8+\ldots=0$
   (iii) $3+(-3)=\ldots$ (iv) $\ldots+(-5)=0$
   (v) $-7+\ldots=0$ (vi) $-2+2=\ldots$

4.3 पूर्णांकों का व्यवकलन

आपको याद होगा कि व्यवकलन, योग का प्रतिलोम होता है। उदाहरणार्थ 9 में से 5 घटाने का अर्थ वही है जो ऐसी संख्या ज्ञात करने का जिसे 5 में जोड़ने से 9 प्राप्त हो जाए।

उदाहरण के तौर पर 10 में से -8 घटाने के लिए हम इसी प्रकार का प्रश्न पूछते हैं कि '-8 में हम क्या जोड़ें कि 10 प्राप्त हो जाए?' स्पष्टतया इसका उत्तर 18 है।

(यदि आप संख्या रेखा पर मान लीजिए किसी बिंदु A पर हैं जो कि प्रारम्भिक बिंदु O से 8 पग बाईं ओर को है तो आपको एक ऐसे बिंदु तक आने में, जो कि O से 10 पग दाईं ओर को है, A से उसके दाईं ओर कितने पग चलने पड़ेंगे?)

अब आइए -8 का ऋणात्मक ज्ञात करें और उसे 10 में जोड़ें। हमें क्या प्राप्त होता है? -8 का ऋणात्मक 8 है जिसे 10 में जोड़ने पर हमें 18 प्राप्त होता है। इस प्रकार,

$$10-(-8)=10+8=18$$

दूसरे शब्दों में, 10 में से -8 घटाने के लिए हम 10 में -8 का ऋणात्मक (या योज्य प्रतिलोम) जोड़ते हैं। वास्तव में यह **पूर्णांकों के व्यवकलन का नियम** ही है।

यदि a और b कोई दो पूर्णांक हों तो a में से b को घटाने का अर्थ वही है जो a में b का ऋणात्मक (या योज्य प्रतिलोम) जोड़ने का। अर्थात्

$$a-b=a+(-b)$$

एक बार जब हमें b का ऋणात्मक ज्ञात हो जाए, तो हम वांछित परिणाम प्राप्त करने के लिए **पूर्णांकों के योग के नियम (क) या (ख) का प्रयोग कर सकते हैं।**

प्रश्नावली 4.5

1. घटाइए :
   (i) 8 में से -3 (ii) 7 में से -9
   (iii) -6 में से -5 (iv) -7 में से 2
2. घटाइए :
   (i) 3126 में से -812 (ii) -6 में से 8650
   (iii) -4109 में से -3987 (iv) 0 में से -236
   (v) 0 में से 236 (vi) 732 में से 0
   (vii) -732 में से 0 (viii) 40321 में से 83241
3. रिक्त स्थानों में उपयुक्त संकेत '<' या '>' भरिए :
   (i) $(-5)+(-8)\ldots(-5)-(-8)$
   (ii) $(-15)-(-15)\ldots(-15)+(-15)$
   (iii) $(-20)-(+20)\ldots(+20)-(+65)$
4. -27 में से 23 घटाइए। 23 में से -27 घटाइए।
   क्या $23-(-27)=-27-23$ है?

5. एक दिन दोपहर 12 बजे दिल्ली में तापमान 40°C था। उसी दिन सायं 4 बजे तक यह तापमान गिरकर 32°C रह गया। तापमान में गिरावट ज्ञात कीजिए।

6. दो स्थान A और B क्रमशः समुद्र सतह से 10 मीटर और 22 मीटर ऊपर हैं। A से B की कितनी ऊंचाई है?

7. रिक्त स्थान भरिए :
   (i) किसी पूर्णांक में से 7 घटाने के लिए, हम उस पूर्णांक में...जोड़ते हैं।
   (ii) किसी पूर्णांक में से -11 घटाने के लिए, हम उस पूणांक में...जोड़ते हैं।
   (iii) $-4+\ldots=12$
   (iv) $-8+\ldots=0$
   (v) $\ldots-215=-64$

8. दो पूर्णांकों का योग -396 है। यदि इनमें से एक 641 हो तो दूसरा ज्ञात कीजिए।

9. एक कक्षा के टैस्ट (*class-test*) में उत्तीर्ण होने के लिए न्यूनतम जितने अंकों की आवश्यकता है उससे शशि ने 25 अंक अधिक प्राप्त किए तथा उसकी बहिन रीता ने 3 अंक कम प्राप्त किए। शशि ने रीता से कितने अंक अधिक प्राप्त किए?

*10. ज्ञात कीजिए :
   $1-2+3-4+5-6+\ldots+49-50.$

*11. योग $1+(-1)+1+(-1)+1+(-1)+\ldots$पर विचार कीजिए। यह योग कितना होगा जबकि इसमें
   (i) 731 'एक' हों (ii) 262 'एक' हों।

4.4 पूर्णांकों का गुणन

4.4.1 हम पहले से ही जानते हैं कि यदि दोनों पूर्णांक धनात्मक या शून्य हों तो उनको किस प्रकार गुणा किया जाता है। आपको याद होगा कि गुणन केवल बार बार योग ही है। उदाहरण के तौर पर, $4\times 5=4+4+4+4+4$ अर्थात् 20 है।

यदि हम, मान लीजिए, -5 और 4 को गुणा करना चाहें तो हम क्या करेंगे? आइए एक वास्तविक जीवन से संबंधित स्थिति पर विचार करें तथा देखें कि -5 और 4 का योग ज्ञात करने में क्या व्यावहारिक ज्ञान (*common sense*) हमारी कुछ सहायता कर सकता है?

उदाहरण 1: एक घड़ी प्रतिदिन (अर्थात् प्रत्येक 24 घंटे की अवधि में) 5 मिनट सुस्त हो जाती है। वह 4 दिन में कितनी सुस्त हो जाएगी?

हल: स्पष्ट है कि घड़ी 4 दिन में 20 मिनट सुस्त हो जाएगी।

दूसरे शब्दों में,

$$(-5)\times 4=-20=(-5)+(-5)+(-5)+(-5)$$

अब आइए एक ऐसी स्थिति का अध्ययन करें जिससे हमें, उदाहरणार्थ, 4 और -5 का गुणा करने में सहायता मिलेगी।

उदाहरण 2: एक व्यक्ति जो कि पश्चिम से पूर्व की ओर 4 किलोमीटर प्रति घंटे की चाल से जा रहा है किसी बिंदु O तक आ जाता है और उसके आगे भी उसी चाल से और उसी दिशा में चलना जारी रखता है। O पर पहुंचने के 2 घंटे बाद वह कहाँ होगा? O पर पहुंचने से 5 घंटे पहले वह कहाँ था?

हल: स्पष्ट है, O पर पहुंचने के 2 घंटे **बाद** वह व्यक्ति O से 8 किलोमीटर **पूर्व** में होगा और O पर पहुंचने से 5 घंटे **पहले**, वह O से 20 किलोमीटर **पश्चिम** में था। यदि हम उसकी स्थितियों को एक संख्या रेखा पर निरूपित करें तो हम देखते हैं कि O पर पहुंचने के 2 घंटे बाद वह, बिंदु $+8$ पर होगा तथा O पर पहुंचने से 5 घंटे पहले वह, बिंदु -20 पर था।

इस प्रकार $4\times(+2)=8$ तथा $4\times(-5)=-20$

अंत में, हम दो ऋणात्मक पूर्णांकों, उदाहरणार्थ, -2 और -5 को किस प्रकार गुणा करते हैं? हम यहाँ ऋणात्मक पूर्णांकों को गुणा करने के नियम के बारे में कुछ संकेत प्राप्त करने के लिए प्रतिरूप विधि (*pattern approach*) का प्रयोग करेंगे?

आइए निम्न को देखें:

$$(-5)\times 6=-30$$
$$(-5)\times 5=-25$$
$$(-5)\times 4=-20$$
$$(-5)\times 3=-15$$
$$(-5)\times 2=-10$$
$$(-5)\times 1=-5$$
$$(-5)\times 0=0$$
$$(-5)\times(-1)=?$$

इस 'ऋण पाँच गुनी' सारणी में हम देखते हैं कि जब दूसरा पूर्णांक 1 घटता है तो गुणनफल 5 बढ़ जाता है। अतः $(-5)\times(-1)$ ज्ञात करने के लिए हमें इससे पहले गुणनफल को 5 बढ़ा देना चाहिए। इससे हमें $(-5)\times(-1)=+5$ प्राप्त होता है। इसी प्रकार $(-5)\times(-2)$ ज्ञात करने के लिए हमें इससे पहले गुणनफल को 5 बढ़ा देना चाहिए। इससे हमें $(-5)\times(-2)=10$ प्राप्त होता है। आइए ऐसे कुछ और गुणनफल लिखें।

$(-5)\times(-3)=15,$
$(-5)\times(-4)=20,$
$(-5)\times(-5)=25,$ इत्यादि

अतः दो पूर्णांकों के गुणन के लिए हमें निम्न नियम प्राप्त होते हैं :

(क) यदि पूर्णांक समान चिन्हों के हों तो उनका गुणनफल धनात्मक होता है।

(ख) यदि दोनों पूर्णांक असमान चिन्हों के हों तो उनका गुणनफल ऋणात्मक होता है, प्रत्येक स्थिति में, गुणनफल का निरपेक्ष मान, दोनों पूर्णांकों के निरपेक्ष मानों के गुणनफल के बराबर होता है।

उदाहरण 3 : -14 और 12 को गुणा कीजिए।

हल : $|-14|=14$, $|12|=12$, निरपेक्ष मानों का गुणनफल $14\times12=168$ है। अब दोनों पूर्णांक असमान चिन्हों के हैं, अतः नियम (ख) के अनुसार इनके गुणनफल का चिन्ह ऋणात्मक होगा।

इस प्रकार, $(-14)\times12=-168$

उदाहरण 4 : -25 और -3 को गुणा कीजिए।

हल : $|-25|=25$, $|-3|=3$

निरपेक्ष मानों का गुणनफल 25×3 अर्थात 75 है। साथ ही, चूंकि दिए हुए पूर्णांक समान चिन्हों के हैं अतः इनके गुणनफल का चिन्ह धनात्मक होगा।

इसप्रकार, $(-25)\times(-3)=75$

प्रश्नावली 4.6

1. गुणा कीजिए :

(i) -5 और 6 (ii) 11 और -13

(iii) -18 और 7 (iv) -12 और -6

(v) 0 और -5 (vi) -11 और 0

2. निम्न गुणन सारणी पूरी कीजिए :

दूसरी संख्या

	×	−3	−2	−1	0	1	2	3
	−3							
	−2							
	−1							
पहली संख्या	0							
	1							
	2							
	3							

(क) क्या गुणन सारणी मुख्य विकर्ण के सापेक्ष सममित है ?

(ख) निम्न में से प्रत्येक के पक्ष में दो दो उदाहरण दीजिए :

(i) दो धनात्मक पूर्णांकों का गुणनफल धनात्मक होता है;

(ii) दो ऋणात्मक पूर्णांकों का गुणनफल धनात्मक होता है;

(iii) एक धनात्मक पूर्णांक और एक ऋणात्मक पूर्णांक का गुणनफल ऋणात्मक होता है ?

(ग) सारणी से जाँच कीजिए कि $(-3)\times 2 = 2\times(-3)$ । क्या यह सत्य है कि
$(-3)\times(-2) = (-2)\times(-3)$?

(घ) उन पूर्णांक युग्मों की सूची बनाइए जिनके गुणनफल शून्य है।

3. निम्न गुणनफल ज्ञात कीजिए :

(i) $651\times(-3)$

(ii) $(-399)\times 9$

(iii) $(-465)\times(-25)$

(iv) $(-23067)\times 0$

4. (i) 16 और −1 का गुणा कीजिए। क्या आपको याद है कि 16 का ऋणात्मक क्या है ?

(ii) −27 और −1 का गुणा कीजिए। क्या आपको याद है कि −27 का ऋणात्मक क्या है ?

(हम देखेंगे कि **किसी पूर्णांक का ऋणात्मक प्राप्त करने का अर्थ वही है जो उस पूर्णांक को '−1' से गुणा करने का।**)

4.4.2 क्या इसका कुछ प्रभाव पड़ता है कि हम दो पूर्णांकों को किस क्रम में लेकर उनका गुणनफल ज्ञात करते हैं ? उदाहरण 1 में यह सरलता से जाँच की जा सकती है कि $(-5)\times 4 = 4\times(-5)$। इसी प्रकार उदाहरण 4 में यह सरलता से जाँच की जा सकती है कि

$$(-25)\times(-3) = (-3)\times(-25)$$

वास्तव में, यह किन्हीं भी दो पूर्णांकों के लिए सत्य है। हम कहते हैं कि **पूर्णांकों का गुणन क्रमविनिमेय है।** दूसरे शब्दों में, **दो पूर्णांकों के गुणा करने में इसका कोई प्रभाव नहीं पड़ता कि हम पूर्णांकों को किस क्रम में लेकर गुणा करते हैं।**

संकेतों का प्रयोग करने पर हम लिखते हैं कि

यदि a और b कोई दो पूर्णांक हों, तो

$a \times b = b \times a$

अब किसी पूर्णांक और शून्य के गुणनफल के बारे में आप क्या सोचते हैं? प्रश्नावली 4.6 के प्रश्न 1 के (v) और (vi) भागों में हम देखते हैं कि

$0 \times (-5) = 0$

$(-11) \times 0 = 0$

हम यह पहले से ही जानते हैं कि शून्य और किसी भी पूर्ण संख्या का गुणनफल सदैव शून्य होता है। इस परिणाम और उपर्युक्त दो उदाहरणों से हम देखते हैं कि **शून्य और किसी भी पूर्णांक का गुणनफल सदैव शून्य होता है।** संकेतों का प्रयोग करने पर हम कहते हैं कि

यदि a कोई पूर्णांक हो, तो

$a \times 0 = 0 \times a = 0$

आइए अब दो पूर्णांकों के गुणनफल का निरीक्षण करें जबकि इनमें से एक पूर्णांक 1 है। हम यह पहले से ही जानते हैं कि 1 और किसी पूर्ण संख्या का गुणनफल स्वयं वह पूर्ण संख्या होती है। हम यह भी देखते हैं कि

$1 \times (-1) = -1 = (-1) \times 1$

$1 \times (-2) = -2 = (-2) \times 1$

$1 \times (-3) = -3 = (-3) \times 1$

$1 \times (-4) = -4 = (-4) \times 1$

इस प्रकार, हम देखते हैं कि **1 और किसी पूर्णांक का गुणनफल सदैव स्वयं वह पूर्णांक होता है। यह 1 का गुणन गुण** (*multiplication property of* 1) कहलाता है तथा **पूर्णांकों के गुणन के लिए 1 तत्समक अवयव कहलाता है।** संकेतों का प्रयोग करने पर हम लिखते हैं कि

यदि a कोई पूर्णांक हो, तो

$$1 \times a = a \times 1 = a$$

आइए अब **तीन पूर्णांकों, उदाहरणार्थ, -45, -8 और 25 का आपस में गुणा करें।**

$[(-45) \times (-8)] \times 25 = 360 \times 25$

$= 9000$

$(-45) \times [(-8) \times 25] = (-45) \times (-200)$

$= 9000$

हम देखते हैं कि

$[(-45) \times (-8)] \times 25 = (-45) \times [(-8) \times 25]$

दूसरे शब्दों में, हम **किन्हीं भी दो पूर्णांकों** का संयोग करके उनके गुणनफल का तीसरे पूर्णांक से

गुणा कर सकते हैं। वास्तव में, हम देखते हैं कि उपर्युक्त उदाहरण में संयोग $[(-45)\times(-8)]$ की तुलना में संयोग $[(-8)\times 25]$ से सरलतम समूह प्राप्त होता है।

उपर्युक्त उदाहरण गुणन के एक अन्य महत्वपूर्ण गुण की व्याख्या करता है। वह यह कि **पूर्णांकों का गुणन सहचारी है।** अर्थात्

तीन पूर्णांकों का गुणनफल ज्ञात करने में इसका कोई प्रभाव नहीं पड़ता कि पहले हम कौनसे दो पूर्णांक (लेकर) गुणा करते हैं और फिर इस गुणनफल को अंतिम पूर्णांक से गुणा करते हैं।

संकेतों का प्रयोग करने पर, हम लिखते हैं कि **यदि** a, b **और** c **कोई तीन पूर्णांक हों तो,**

$$(a\times b)\times c=a\times(b\times c)$$

इस गुण के फलस्वरूप ही, **हम प्रायः इन समान गुणनफलों को** $a\times b\times c$ **(या केवल** abc**) लिखते हैं।** अंत में, हम **चार या अधिक पूर्णांकों का** किस प्रकार **गुणा** करते हैं। हम क्रमविनिमेय और साहचर्य गुणों का, **यदि आवश्यक** हो तो कई बार, प्रयोग करते हैं और **पूर्णांकों के सरलतम समूह बनाकर उनका गुणनफल ज्ञात करते हैं।**

उदाहरण 5 : -48, -4, 25 और -23 को गुणा कीजिए।

हल : $(-48)\times(-4)\times(25)\times(-23)$

$=[(-48)\times(-4)]\times[(25)\times(-23)]$

$=192\times(-575)=-110400$

या, हम इनके निम्न सरलतम समूह बना सकते हैं और इनका गुणनफल ज्ञात कर सकते हैं :

$(-48)\times(-4)\times(25)\times(-23)$

$=[(-48)\times(-23)]\times[(-4)\times 25]$

$=1104\times(-100)=-110400$

4.4.3 पूर्णांकों के लिए वितरण गुण

आइए, उदाहरणार्थ, $2\times[3+(-1)]$ को दो विभिन्न विधियों से ज्ञात करें :

$2\times[3+(-1)]=2\times 2=4$

साथ ही, $2\times 3+2\times(-1)=6+(-2)=4$

इस प्रकार, $2\times[3+(-1)]=2\times 3+2\times(-1)$

इसी प्रकार, उदाहरणार्थ, हम यह भी जाँच कर सकते हैं कि

$6\times[(-8)+4]=6\times(-8)+6\times 4$

या, $(-2)\times[7+(-5)]=(-2)\times(7)+(-2)\times(-5)$

या, $(-8)\times[(-6)+(-4)]=(-8)\times(-6)+(-8)\times(-4)$

वास्तव में, **यदि तीन पूर्णांक** a, b **और** c, **दिए हुए हों तो यह सरलता से जाँच की जा सकती है कि**

$$a\times(b+c)=a\times b+a\times c$$

या,
$$a(b+c)=ab+ac$$

हम कहते हैं कि **पूर्णांकों का गुणन, योग के ऊपर वितरणात्मक है।**

प्रश्नावली 4.7

1. निम्न ज्ञात कीजिए : (सबसे उपयुक्त संयोग का प्रयोग कीजिए)
   (i) $(-2) \times 36 \times (-5)$
   (ii) $(-8) \times (-43) \times 0$
   (iii) $(18) \times (-183) \times (-4)$
   (iv) $(-45) \times (55) \times (-10)$
2. निम्न के मान ज्ञात कीजिए :
   (i) $-125 \times (-8) \times 35 \times (-42)$
   (ii) $-2 \times (-3) \times (-4) \times (-5) \times (-6)$
   (iii) $(-48) \times (-8) \times 6 \times 10$
   (iv) $(-8) \times 0 \times 37 \times (-37)$
3. यदि हम निम्न का गुणा करें तो गुणनफल का क्या चिन्ह होगा ?
   (i) 1 ऋणात्मक और 3 धनात्मक पूर्णांक
   (ii) 2 ऋणात्मक और 5 धनात्मक पूर्णांक
   (iii) 8 ऋणात्मक और 1 धनात्मक पूर्णांक
   (iv) 21 ऋणात्मक और 1 धनात्मक पूर्णांक
4. पूर्णांकों -40, 16, -54, -68 और 0 में से प्रत्येक को दो पूर्णांकों के गुणा के रूप में लिखिए जब कि इनमें से एक पूर्णांक -1 हो।
5. सरल कीजिए : $(-8) \times [10-5-43+98]$
6. निम्न की तुलना कीजिए :
   (i) $(11+9) \times 10$ और $11+(9 \times 10)$
   (ii) $(41-3) \times 10$ और $41-(3 \times 10)$

4.5 पूर्णांकों का विभाजन

अब जबकि हमारे पास पूर्णांकों को गुणा करने के नियम हैं, उनके विभाजन के नियम ज्ञात करना सरल है। निस्संदेह, हम इस बात पर बल देंगे, जैसा कि हमने पूर्ण संख्याओं के लिए कहा था, कि **शून्य से विभाजन नहीं किया जा सकता।**

आपको याद होगा कि विभाजन, गुणन का प्रतिलोम है। उदाहरणार्थ, 18 को -6 से भाग देने के लिए हमें यह पूछना चाहिए, "हम 18 प्राप्त करने के लिए -6 को किससे गुणा करें ?" स्पष्ट है, इसका उत्तर -3 है। इस प्रकार,

$$18 \div (-6) = -3$$

या, उदाहरणार्थ, -64 को 8 से भाग देने के लिए हमें पूछना चाहिए, "-64 प्राप्त करने के लिए हम 8 को किससे गुणा करें ?" स्पष्ट है, इसका उत्तर, -8 है। इस प्रकार,

$$(-64) \div 8 = -8$$

इसीप्रकार, उदाहरणार्थ, -32 को -4 से भाग देने के लिए हमें यह पूछना चाहिए "-32 प्राप्त करने के लिए हम -4 को किससे गुणा करें ?" स्पष्ट है, इसका उत्तर 8 है। इस प्रकार,

$$(-32) \div (-4) = 8$$

इस प्रकार हमें दो पूर्णांकों के विभाजन के लिए निम्न नियम प्राप्त होते हैं :

(क) यदि पूर्णांक समान चिन्हों के हैं तो उनका भागफल धनात्मक होगा;

(ख) यदि पूर्णांक असमान चिन्हों के हैं तो उनका भागफल ऋणात्मक होगा; प्रत्येक स्थिति में भागफल का निरपेक्षमान भाज्य के निरपेक्ष मान को भाजक के निरपेक्ष मान से भाग करने से प्राप्त किया जाता है।

उदाहरण 1 : 68 को -17 से भाग दीजिए।

हल : $|68| = 68$, $|-17| = 17$

भाज्य के निरपेक्ष मान को भाजक के निरपेक्ष मान से भाग करने पर, हमें $68 \div 17$ अर्थात् 4 प्राप्त होता है।

अब पूर्णांक असमान चिन्हों के हैं, अतः नियम (ख) के अनुसार भागफल का चिन्ह ऋणात्मक होगा। इस प्रकार,

$$68 \div (-17) = -4$$

उदाहरण 2 : -78 को 13 से भाग दीजिए।

हल : $|-78| = 78$, $|13| = 13$ तथा $78 \div 13 = 6$

यहाँ पूर्णांक पुनः असमान चिन्हों के हैं अतः भागफल का चिन्ह ऋणात्मक होगा। इस प्रकार,

$$(-78) \div 13 = -6$$

उदाहरण 3 : -324 को -9 से भाग दीजिए।

हल : $|-324| = 324$, $|-9| = 9$ तथा $324 \div 9 = 36$

दोनों पूर्णांक समान चिन्हों के हैं, अतः नियम (क) से, भागफल का चिन्ह धनात्मक होगा। इस प्रकार,

$$(-324) \div (-9) = 36$$

प्रश्नावली 4.8

1. निम्न में से प्रत्येक में भागफल ज्ञात कीजिए :

(i) $36 \div (-9)$　　(ii) $-48 \div (-16)$

(iii) $-24 \div 8$　　(iv) $-56 \div (-4)$

(v) $0 \div (-7)$

2. रिक्त स्थानों को भरिए :

(i) $\ldots \div (-11) = -3$　　(ii) $\ldots \div 7 = -1$

(iii) $\ldots \div 60 = 2$　　(iv) $\ldots \div (-3) = -4$

(v) $\ldots \div (-5) = 2$

3. भाग दीजिए :

(i) 10 को 10 से　　(ii) 10 को -10 से

(iii) -10 को 10 से　　(iv) -10 को -10 से

विविध प्रश्नावली II

(एकक III और IV पर)

1. निम्न को संकेतों '<' या '>', का प्रयोग करते हुए दुबारा लिखिए :
   (i) -7, -17 से बड़ा है।
   (ii) 10 धनात्मक है।
   (iii) -3 ऋणात्मक है।
   (iv) -3, 3 से छोटा है।
2. दो पूर्णांकों का गुणनफल -1 है। पूर्णांक ज्ञात कीजिए।
3. निम्न को दो पूर्णांकों के गुणा के रूप में लिखिए जिनमें से एक पूर्णांक -1 हो :
   (i) 5 (ii) -13 (iii) 9 (iv) -1 (v) 1 (vi) 0
4. $|-3|$ और $-|3|$ के अर्थ बताइए।
5. सरल कीजिए :
   (i) $-|4|+|-4|-|-4|$
   (ii) $|(-4)\times(-2)|+|-4|-|-2|$
6. यदि तापमान $-4°C$ हो और फिर वह $20°C$ बढ़ जाए तो नया तापमान ज्ञात कीजिए।
7. $0\div 3$, $0\div(-3)$, $0\div 162$ और $0\div(-162)$ ज्ञात कीजिए।
   [संकेत : $0\times 3=0$, इत्यादि]
   हम देखेंगे कि शून्य को किसी शून्येतर पूर्णांक से भाग देने पर सदैव शून्य प्राप्त होता है।
8. निम्न को पूरा कीजिए :
   (i) -15 घटाने का अर्थ है, ... जोड़ना।
   (ii) 16 घटाने का अर्थ है, ... जोड़ना।
9. रिक्त स्थान भरिए :
   (i) $(-8)+(-5)=(-8)-(\ldots)$
   (ii) $7+5=7-(\ldots)$
   (iii) $(-9)-(+6)=(-9)+(\ldots)$
10. सरल कीजिए :
   (i) $18-(-3)(12)\div 6$
   (ii) $-20+(-60)(5)\div(-10)$
   (iii) $36\div(-3)+12$
11. निम्न में से प्रत्येक में एकक अंक (*unit's digit*) ज्ञात कीजिए :
   (i) $17\times(-27)\times 37\times 22$
   (ii) $12\times 25\times(-16)\times(-13)$
   (iii) $16\times(-26)\times(-36)\times(-46)$

12. 50 प्रश्नों के एक सत्य-असत्य टैस्ट में, प्रत्येक सही उत्तर के लिए विद्यार्थी को 2 अंक मिलते हैं तथा प्रत्येक गलत उत्तर के लिए -1 मिलता है। कोई उत्तर न देने पर उसे 0 मिलता है।

(क) एक विद्यार्थी 32 सही और 14 गलत उत्तर देता है तथा 4 प्रश्न नहीं करता है। वह कितने अंक प्राप्त करेगा?

(ख) उस विद्यार्थी को कितने अंक मिलेंगे जो कि 25 प्रश्न सही करता है और 25 गलत करता है?

13. ज्ञात कीजिए :

(i) $(-3)\times[5+(-8)+9]-[4\times\{9+8+(-7)\}]$
(ii) $[(-6)\times(9-12)]+[(-8+15)\times(-15)]$
(iii) $(-7)\times[(-5)+8(43-57)]$
(iv) $[(-15)\times(48-43+18-10-11)]+[(37-45)\times(-15)]$

14. आइए (पूर्णांकों के लिए) एक ऐसी संक्रिया '' खोजें जिसका अर्थ है दोनों पूर्णांकों को जोड़कर उनके योग में उनका गुणनफल जोड़ दे। दूसरे शब्दों में, यदि a और b कोई दो पूर्णांक हों तो

$$a*b=a+b+ab$$

इस प्रकार, $2*3=2+3+6=11$, $2*(-3)=2+(-3)+(-6)=-7$, $0*4=4$, इत्यादि।

(क) $4*5$, $0*3$, $(-3)*0$ और $(-2)*(-1)$ के क्या अर्थ हैं?
(ख) क्या $2*3=3*2$ है?

*15. एक मेंढक, जो 8 मीटर गहरे एक कुएं में गिर गया है छलांग लगाकर उससे बाहर निकलने का प्रयत्न करता है। प्रत्येक बार, मेंढक ऊपर को 70 सेमी की छलांग लगाता है और 20 सेमी पीछे को फिसल जाता है। प्रत्येक छलांग का वास्तव में क्या परिणाम रहता है? कुएं से बाहर निकलने के लिए मेंढक को कितनी छलांगें लगानी पड़ेंगी?

एकक V

पूर्णांकों की घातें

हम, इस एकक में, पूर्णांकों के वर्ग, घन और बड़ी पूर्णांकीय घातों (*integral powers*) का अध्ययन करेंगे। हम यह भी सीखेंगे कि उन धनात्मक पूर्णांकों का जो कि पूर्ण वर्ग (*perfect squares*) हैं, वर्गमूल किस प्रकार निकाला जाता है।

5.1 पूर्णांकों की घातें

जब किसी पूर्णांक को उसी से गुणा किया जाता है तो हम कहते हैं कि हमें पूर्णांक का **द्वितीय घात** (*second power*) या **वर्ग** (*square*) प्राप्त हो गया है। उदाहरणार्थ, 3×3, 3 का वर्ग है और इसे 3^2 लिखा जाता है। हम 3^2 को '**3 की द्वितीय घात**' या '**3 का वर्ग**' या '**3 पर घातांक** (*exponent*) **2**' (या केवल '**3-घात 2**') या **3-वर्ग** पढ़ते हैं। निस्संदेह, $3^2 = 9$

इसी प्रकार, $(-5)^2 = (-5) \times (-5) = 25$,

तथा, $10^2 = 10 \times 10 = 100$

ऊपर लिखा हुआ संख्यांक, **घातांक** (*exponent* या *index*) कहलाता है तथा 3×3 को 3^2 के रूप में लिखने को **घातांक संकेतन** (*exponential notation*) या **घात संकेतन** (*power notation*) में लिखना कहते हैं।

जब किसी गुणनफल में कोई पूर्णांक तीन बार आता है तो हम कहते हैं कि हमें पूर्णांक की **तृतीय घात** (*third power*) या **घन** (*cube*) प्राप्त हो गया है। उदाहरणार्थ $2 \times 2 \times 2$, 2 का घन है और इसे 2^3 लिखा जाता है। हम 2^3 को '**2 की तृतीय घात**' या '**2 का घन**' या '**2 पर घातांक 3**' या '**2-घन**' पढ़ते हैं। इस प्रकार, $2^3 = 2 \times 2 \times 2 = 8$,

इसी प्रकार, $(-6)^3 = (-6) \times (-6) \times (-6) = -216$,

$7^3 = 7 \times 7 \times 7 = 343$

तथा $10^3 = 10 \times 10 \times 10 = 1000$

पूर्णांकों की बड़ी घातों के विषय में भी उपर्युक्त प्रकार से विचार किया जा सकता है। उदाहरणार्थ, **2 की चतुर्थ घात** (*fourth power*) अर्थात् **2 पर घातांक** $4 = 2^4 = 2 \times 2 \times 2 \times 2 = 16$

− 3 की चतुर्थ घात $= (-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81$

− 5 घात $4 = (-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 625$

10 घात $4 = (10)^4 = 10 \times 10 \times 10 \times 10 = 10000$

2 की पाँचवीं घात अर्थात् 2 घात 5 $= 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

-3 की पाँचवीं घात $= (-3)^5 = (-3) \times (-3) \times (-3) \times (-3) \times (-3) = -243$

-5 घात 5 $= (-5)^5 = (-5) \times (-5) \times (-5) \times (-5) \times (-5) = -3125$

10 घात 5 $= (10)^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100000$

घातांक संकेतन के प्रयोग से हमें बड़ी संख्याओं को संक्षेप और सुन्दर प्रकार से लिखने में सहायता मिलती है। क्या आपको स्थानीय-मान की संकलपना में 10 की घातों के प्रयोग के बारे में याद हैं ?

यह सरलता से जाँच की जा सकती है कि

$$(-1)^{\text{विषम धनात्मक पूर्णांक}} = -1$$

$$(-1)^{\text{सम धनात्मक पूर्णांक}} = +1$$

यदि a कोई पूर्णांक हो तो a^2 भी एक पूर्णांक होता है और इसे एक **पूर्ण वर्ग** (*perfect square*) कहते हैं। पहले पाँच शून्येतर पूर्ण वर्ग पूर्णांक $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$ तथा $5^2 = 25$ हैं। अगले दो पूर्ण वर्ग पूर्णांक कौन से हैं ?

पुन: यदि a कोई पूर्णांक हो, तो a^3 भी एक पूर्णांक होता है और इसे एक **पूर्ण घन** (*perfect cube*) कहते हैं। पहले पाँच धनात्मक पूर्ण घन पूर्णांक $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$ तथा $5^3 = 125$ हैं। अगले दो पूर्ण घन पूर्णांक कौन से हैं ?

हम इसी प्रकार पूर्णांकों की चतुर्थ घात, पंचम घात, इत्यादि ज्ञात कर सकते हैं।

प्रश्नावली 5.1

1. निम्न में से प्रत्येक में वह संख्यांक लिखिए जो घातांक में है :
   (i) $(20)^8$ (ii) $(-7)^4$ (iii) 23 (iv) $(-5)^2$ (v) $(-1)^{644}$
2. ज्ञात कीजिए :
   (i) 16 का वर्ग
   (ii) -4 का घन
   (iii) -14 का वर्ग
   (iv) 3 की चतुर्थ घात
3. निम्न को घातांक संकेतन में लिखिए :
   (i) 7
   (ii) $(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)$
   (iii) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
   (iv) -26

4. ज्ञात कीजिए :

(i) 50^2 (ii) 4^3 (iii) 3^4 (iv) $(-1)^{20}$ (v) 1^{16}
(vi) $(-1)^{63}$ (vii) 1^{37} (viii) $(-2)^5$ (ix) 2^5 (x) 2^{10}

5. सरल कीजिए :

(i) $2^3 \times 3^2$ (ii) $(-2)^2 \times (-3)^3$
(iii) $(-1)^3 \times (-10)^2$ (iv) $2^3 \times (-3)^2 \times 8$
(v) $(5)^2 \times (10)^2 \times (-1)^3$ (vi) $(-5)^3 \times (-2)^3$
(vii) $(3)^2 \times (10)^4 \times (-1)^{10}$ (viii) $(-2)^4 \times (-3)^3 \times (-1)$

6. निम्न के घन ज्ञात कीजिए :

(i) -12 (ii) -25 (iii) -9 (iv) 100 (v) 20

7. प्रथम दस धनपूर्णांकों के वर्ग ज्ञात कीजिए। इनके एकक अंकों को देखिए। आप क्या देखते हैं ?

8. प्रथम नौ धनपूर्णांकों के घन ज्ञात कीजिए।

9. 10^2, 20^2, 30^2, 100^2, 200^2 तथा 1000^2 ज्ञात कीजिए। (हम देखेंगे कि **पूर्णांक के वर्ग में दाईं ओर शून्यों की संख्या, पूर्णांक में दाईं ओर शून्यों की संख्या की दुगुनी है।**)

5.2 वर्गमूल

हम अनुच्छेद 5.1 में देख चुके हैं कि किसी पूर्णांक का वर्ग उस पूर्णांक को उसी से गुणा करने प्राप्त होता है। इसकी विलोम प्रक्रिया अर्थात् ऐसा पूर्णांक ज्ञात करना जिसका वर्ग करने पर दिया हु पूर्णांक प्राप्त हो जाए, पूर्णांक का 'अवर्ग करने (*unsquaring*)' की प्रक्रिया या उसका **वर्गमूल** (*squa root*) ज्ञात करने की प्रक्रिया कहलाती है। उदाहरणार्थ 16 का वर्गमूल ज्ञात करने के लिए हमें ऐ पूर्णांक ज्ञात करने की आवश्यकता है जिसका वर्ग 16 हो। स्पष्ट है ऐसा पूर्णांक 4 या -4 इनमें से प्रत्येक 16 का वर्गमूल कहलाता है।

(इस एकक में **हम अपने को पूर्ण वर्गों के वर्गमूल निकालने तक ही सीमित रखेंगे।** साथ **हम केवल धनात्मक वर्गमूल ज्ञात करेंगे** क्योंकि फिर दूसरा वर्गमूल सरलता से लिखा जा सकता है। तरह हम केवल एक वर्गमूल की बात करेंगे। **धनात्मक वर्गमूल के लिए हम** $\sqrt{\ }$ संकेत का प्रयं करते हैं। उदाहरणार्थ, हम $\sqrt{16}=4$ तथा $-\sqrt{16}=-4$ लिखते हैं।)

उदाहरण 1 : 49 का वर्गमूल ज्ञात कीजिए।

हल : हम जानते हैं कि $7^2=49$। इस प्रकार, $\sqrt{49}=7$

उदाहरण 2 : 625 का वर्गमूल ज्ञात कीजिए।

हल : $625=5\times5\times5\times5$ अर्थात् 25×25

इस प्रकार, $\sqrt{625}=25$

उदाहरण 3 : 1764 का वर्गमूल ज्ञात कीजिए।

हल : पहली ही दृष्टि में हम यह नहीं कह सकते कि कौनसा ऐसा पूर्णांक है जिसका वर्ग 1764 है। अतः हम 1764 के गुणनखंड ज्ञात करते हैं। हम देखते हैं कि

$$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$$
$$= 2^2 \times 3^2 \times 7^2$$

इस प्रकार, $\sqrt{1764} = 2 \times 3 \times 7 = 42$

2	1764
2	882
3	441
3	147
7	49
	7

हम देखते हैं कि

किसी पूर्ण वर्ग का वर्गमूल ज्ञात करने के लिए, हम

(क) दिए हुए पूर्णांक के गुणनखंड ज्ञात करते हैं,

(ख) पूर्णांक के गुणनखंडों के प्रत्येक युग्म में से हम, वर्गमूल में सम्मिलित करने के लिए एक एक गुणनखंड चुन लेते हैं, और

(ग) यदि आवश्यक हो तो, चुने हुए गुणनखंडों का गुणा कर देते हैं।

यह विधि, प्रत्यक्ष कारणों से, **गुणनखंड विधि** (*factor method*) कहलाती है। इसका प्रयोग हम यह ज्ञात करने में कर सकते हैं कि कोई दिया हुआ पूर्णांक पूर्ण वर्ग है या नहीं। यदि दिए हुए पूर्णांक में कोई ऐसा गुणनखंड है जो एक युग्म में नहीं आता तो स्पष्ट है कि वह पूर्णांक पूर्ण वर्ग नहीं होगा।

उदाहरण 4 : 1089 का वर्गमूल ज्ञात कीजिए।

हल : हम देखते हैं कि

$$\sqrt{1089} = \sqrt{3^2 \times 11^2}$$

3	1089
3	363
11	121
	11

इस प्रकार, 1089 का एक वर्गमूल 3×11 अर्थात् 33 है। इसलिए दूसरा वर्गमूल -33 होगा। किसी पूर्णांक का **घनमूल** (*cube root*) वह पूर्णांक होता है जिसका घन दिया हुआ पूर्णांक हो। आप किसी पूर्णांक के **चतुर्थमूलों** (*fourth roots*), **पंचमूलों** (*fifth roots*) इत्यादि को किस प्रकार परिभाषित करेंगे। (घनमूल, चतुर्थमूल, पंचमूल, इत्यादि का अध्ययन इस पुस्तक की सीमा के बाहर है। हम अपने को केवल वर्गमूल ज्ञात करने तक ही सीमित रखेंगे।)

प्रश्नावली 5.2

1. निम्न में से प्रत्येक का वर्गमूल ज्ञात कीजिए :

   (i) 100 (ii) 256 (iii) 225 (iv) 169
   (v) 196 (vi) 324 (vii) 10000 (viii) 676
   (ix) 1000000

2. $\sqrt{289}$, $\sqrt{400}$, $\sqrt{3600}$ तथा $\sqrt{784}$ के मान बताइए।

3. ज्ञात कीजिए कि निम्न में से कौन कौन पूर्ण वर्ग हैं :

   7, 121, 144, 18, 11025

4. चूँकि $0 \times 0 = 0$, अतः हम $0^2 = 0$ लिखते हैं और कहते हैं कि शून्य का वर्ग शून्य है। शून्य का वर्गमूल क्या है ?

एकक VI

संख्याओं के गुण

पिछले एककों में हमने पूर्ण संख्याओं और पूर्णांकों पर संक्रियाओं के गुणों का अध्ययन किया था। अब हम संख्याओं, विशेषकर, पूर्ण संख्याओं के कुछ रोचक गुणों का पता लगाएंगे। हम संख्याओं के 2, 3, 4, 5 इत्यादि से विभाजित होने के नियमों का भी अध्ययन करेंगे।

6.1 गुणनखंड और गुणज

हम यह पहले से ही जानते हैं कि किसी संख्या के **गुणनखंड** (*factor*) का क्या अर्थ है। वास्तव में हम इनको, अनुच्छेद 5.2 में पूर्ण वर्ग पूर्णांकों के वर्गमूल ज्ञात करने में प्रयोग कर चुके हैं।

आपको याद होगा कि **वह संख्या जो 2 से (पूर्णतया) विभाजित हो जाए, सम संख्या** (*even number*) **कहलाती है।** 0, 2, 4, 6, इत्यादि सम संख्याओं के कुछ उदाहरण हैं। **वह संख्या जो 2 से (पूर्णतया) विभाजित न हो, विषम संख्या** (*odd number*) **कहलाती है।** 1, 3, 5, 7, इत्यादि विषम संख्याओं के उदाहरण हैं। हम देखते हैं कि यदि किसी संख्या को 2 से विभाजित करने पर शेष 1 प्राप्त हो तो वह संख्या विषम होती है। (क्यों ?)

संख्या 12 को लीजिए। हम, उदाहरण के तौर पर, इसे $12 = 3 \times 4$ लिख सकते हैं। हम कहते हैं कि 12, 3 का एक **गुणज** (*multiple*) है। 12, 4 का भी एक **गुणज** है। पूर्ण संख्याओं में, 3 के और भी **गुणज** हैं जैसे कि 0, 3, 6, 9, 15, 18, 21 इत्यादि। इसी प्रकार 4 के भी और गुणज हैं जैसे कि 0, 4, 8, 16, 20, 24, 28 इत्यादि। आप क्या देखते हैं? क्या यह सत्य नहीं है कि जब आपने अपनी गुणन सारणियाँ (*multiplication tables*) याद की थीं तो आप वास्तव में विभिन्न संख्याओं के क्रमागत गुणज ही सीख रहे थे?

प्रश्नावली 6.1

1. **(पुनर्निरीक्षण प्रश्न)** 60 के सभी गुणनखंड लिखिए।
   [याद रखिए कि इनमें 1 और 60 भी गुणनखंड हैं।]

2. (क) 1 से प्रारम्भ करके दो दो के अंतर पर संख्याएँ गिनिए। आपको किस प्रकार की **संख्याएँ** प्राप्त होती हैं?

(ख) 0 से प्रारम्भ करके दो दो के अंतर पर संख्याएँ गिनिए। आपको किस प्रकार की संख्याएँ प्राप्त होती हैं?

[देखिए कि सम संख्याएँ 2 का गुणज हैं।]

(ग) 0 से प्रारम्भ करके चार चार के अंतर पर संख्याएँ गिनिए। आपको किस प्रकार की संख्याएँ प्राप्त होती हैं?

3. रिक्त स्थानों को भरिए :

(i) 343, 7 का गुणज है। अत: 7, 343 का एक ... है।

(ii) 27, 36, 45, 54 और 63 ... के तथा ... के गुणज हैं।

(iii) 76 के सभी गुणनखंड ... हैं।

4. हम एक कमरे में 30 कुर्सियाँ इस प्रकार रखना चाहते हैं कि प्रत्येक पंक्ति में कुर्सियों की संख्या बराबर हो। **क्या ऐसा करने की केवल एक ही विधि है?** हम कितनी पंक्तियाँ बना सकते हैं?

6.2 अभाज्य और भाज्य संख्याएँ

3 के गुणनखंड क्या हैं? 5 के गुणनखंड क्या हैं? 7 के गुणनखंड क्या हैं? 11 के गुणनखंड क्या हैं? हम देखते हैं कि इनमें से प्रत्येक संख्या में 1 और स्वयं उस संख्या को छोड़कर इनका **अन्य कोई** गुणनखंड नहीं है। ऐसी संख्याएँ **अभाज्य संख्याएँ** (*prime numbers*) कहलाती हैं। 13, 17, 19, 23, इत्यादि अभाज्य संख्याओं के कुछ अन्य उदाहरण हैं। इस प्रकार,

यदि किसी संख्या के गुणनखंड केवल 1 और वह संख्या स्वयं ही हों तो वह संख्या, अभाज्य संख्या कहलाती है, अन्यथा वह संख्या भाज्य संख्या (*composite number*) **कहलाती है।**

4, 6, 8, 9, 10, 12, 14, 15, 16, इत्यादि भाज्य संख्याओं के कुछ उदाहरण हैं। **संख्या 1 न तो अभाज्य है और न ही भाज्य।**

हम देखते हैं कि 2 **सबसे छोटी अभाज्य संख्या है।** वास्तव में, **केवल 2 ही एक ऐसी अभाज्य संख्या है जो कि सम भी है।**

अब क्या हम, उदाहरणार्थ, 1 और 100 के मध्य सभी अभाज्य संख्याओं की सूची बना सकते हैं? एक यूनानी गणितज्ञ और खगोलशास्त्री (*astronomer*) इरेटोसथीन्स (*Eratosthenes*) [274 ई० पू०—194 ई० पू०] ने अभाज्य संख्याओं को छाँटने (छानने) की एक बहुत ही सरल विधि ज्ञात की। उसकी यह विधि **इरेटोसथीन्स की सीव** (*Sieve of Eratosthenes*) कहलाती है, परन्तु हम इसे केवल **सीव विधि** ही कहेंगे।

हम पहले 1 से लेकर 100 तक की संख्याओं को दस दस की 10 पंक्तियों में लिखते हैं। हम 1 को काट देते हैं। (क्यों?)

संख्या 2 अभाज्य है। इसलिए हम 2 को ऐसा ही रखते हैं तथा 4 से प्रारम्भ करके 2 के सभी गुणजों को काट देते हैं। (व्यावहारिक तौर पर, हम 4 से प्रारम्भ करके प्रत्येक **दूसरी** संख्या को काटते जाते हैं।)

आगे, संख्या 3 अभाज्य है। इसलिए हम 3 को ऐसा ही रखते हैं तथा 6 से प्रारम्भ करके 3 के सभी गुणजों को काट देते हैं। [पुन: हम यहाँ 6 से प्रारम्भ करके प्रत्येक **तीसरी** संख्या को काटते जाएँगे। हम देखेंगे कि इनमें से कुछ संख्याएँ पहले ही कटी हुई हैं। (क्यों ?)]

इरेटोसथोन्स की सीव

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

अब, 4 के गुणज पहले ही कट चुके हैं क्योंकि 4, 2 का गुणज भी है।

आगे, 5 एक अभाज्य संख्या है। इसलिए हम 5 को ऐसा ही रखते हैं और 10 से प्रारम्भ करके प्रत्येक **पाँचवीं** संख्या को काट देते हैं।

हम अभाज्य संख्याओं को क्रम से ऐसा ही रखते हुए और उनके गुणजों को काटते हुए इस क्रिया को तब तक जारी रखते हैं जब तक कि हमारे पास काटने के लिए कोई और संख्या न बचे।

हमारे पास काटने के लिए कब कोई संख्या नहीं बचेगी ? हम देखते हैं कि 7 के सभी गुणजों को काटने के बाद हमारे पास काटने के लिए कोई संख्या नहीं बचेगी। (क्यों ?)

इस प्रकार 1 से लेकर 100 के मध्य में निम्न अभाज्य संख्याएँ हैं :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

इरेटोसथीन्स ने शायद, संख्याओं को काटने के स्थान पर कागज़ में छेद किये थे। इसलिए कागज़ एक छलनी (*sieve*) की तरह लगता था। संभवतः इसी कारण से यह विधि सीव विधि कहलाती है।

हम देखते हैं कि क्रमागत अभाज्य संख्याओं से उनके विषय में कोई प्रतिरूप (*pattern*) स्पष्ट नहीं होता। गणितज्ञों ने कई शताब्दियों तक ऐसे सूत्रों को निकालने या खोजने के प्रयत्न किए जिनसे सभी अभाज्य संख्याएँ प्राप्त हो जाएँ। वे इसमें असफल रहे। अभाज्य संख्या ज्ञात करने की केवल एक ही विधि है वह यह कि प्रत्येक संख्या की जाँच की जाए कि, 1 और स्वयं उस संख्या को छोड़कर, उसके अन्य कोई गुणनखंड है या नहीं। निस्संदेह, अब कम्प्यूटरों (*computors*) के आने से हम बड़ी बड़ी संख्याओं की जाँच करने में समर्थ हो चुके हैं। क्या आप जानते हैं कि 39 अंकों की संख्या

170141183460469231731687303715884105727

एक अभाज्य संख्या है? कम्प्यूटर और बड़ी अभाज्य संख्याओं की खोज कर रहा है।

प्रश्नावली 6.2

1. एक सम संख्या बताइए जो कि अभाज्य भी हो।

2. क्या कोई भाज्य संख्या विषम हो सकती है? यदि हाँ, तो ऐसी छोटी से छोटी संख्या कौन सी है?

3. 100 से छोटी अभाज्य संख्याओं की सूची में हम देखते हैं कि युग्म 3, 5 में अभाज्य संख्याओं का अंतर 2 है। इसी प्रकार युग्मों 5, 7 और 11, 13 में भी अभाज्य संख्याओं का अंतर 2 है। अभाज्य संख्याओं के ऐसे युग्मों को **अभाज्य युग्म** (*twin primes*) कहते हैं। 100 से छोटे सभी अभाज्य युग्म लिखिए।

4. देखिए, हम $6=3+3$, $8=3+5$, $10=3+7$ या $5+5$, $12=5+7$, इत्यादि लिख सकते हैं। हम देखते हैं कि ये सब सम संख्याएँ हैं जिन्हें दो विषम अभाज्य संख्याओं (*odd primes*) के योग के रूप में लिखा गया है। 14, 16, 18, 20, 22, 24, 30 और 56 को दो विषम अभाज्य संख्याओं के योग के रूप में लिखिए। [सन् 1742 में गोल्डबैच (*Goldbach*) नामक एक गणितज्ञ ने अपने मित्र को एक पत्र में लिखा कि उसने एक अनुमान (*conjecture*) या (*guess*) लगाया है। वह इस अनुमान की कोई उपपत्ति (*proof*) नहीं दे सका। गोल्डबैच का अनुमान था कि

4 से बड़ी प्रत्येक सम संख्या को दो विषम अभाज्य संख्याओं के योग के रूप में लिखा जा सकता है।

उसका मित्र भी इसकी कोई उपपत्ति नहीं दे सका। वास्तव में, अभी तक कोई भी गणितज्ञ न तो इसकी उपपत्ति दे सका है और न ही कोई ऐसा उदाहरण देकर जिसमें गोल्डबैच का अनुमान सही न बैठता हो यह सिद्ध कर सका है कि गोल्डबैच का यह अनुमान असत्य है। गणित में यह अभी तक बिना हल हुई समस्या (*unsolved problem*) है और यह **गोल्डबैच कनजैक्चर** (*Goldbach Conjecture*) कहलाती है।

5. निम्न में कौन सी संख्याएँ अभाज्य हैं :
   (i) 957 (ii) 139 (iii) 204
6. ऐसी सात क्रमागत संख्याएँ लिखिए जिनमें से प्रत्येक **100 से छोटी** एक भाज्य संख्या हो।

6.3 अभाज्य गुणनखंडन

हम गुणनखंडों और अभाज्यों के विषय में पढ़ चुके हैं। उदाहरणार्थ,

$12 = 3 \times 4$ या $3 \times 2 \times 2 = 3 \times 2^2$

$15 = 3 \times 5$

$24 = 2 \times 12$ या 3×8 या 4×6 या $2 \times 2 \times 2 \times 3 = 2^3 \times 3$

हम देखते हैं कि एक संख्या का कई प्रकार से गुणनखंडन (*factorization*) हो सकता है। साथ ही हम देखते हैं कि 12 के गुणनखंडन $3 \times 2 \times 2$ में प्रत्येक गुणनखंड एक अभाज्य संख्या है। इसी प्रकार 24 के गुणनखंडन $2 \times 2 \times 2 \times 3$ में प्रत्येक गुणनखंड एक अभाज्य संख्या है। ऐसे गुणनखंडन, **अभाज्य गुणनखंडन** (*prime factorization*) कहलाते हैं। दूसरे शब्दों में,

कोई गुणनखंडन, अभाज्य होता है जबकि उसके सभी गुणनखंड अभाज्य हों।

हम देखते हैं कि हम किसी भी गुणनखंडन से प्रारम्भ करके अंत में अभाज्य गुणनखंडन पर आ सकते हैं। उदाहरणार्थ,

$112 = 7 \times 16 = 7 \times 2 \times 2 \times 2 \times 2$

या, $112 = 8 \times 14 = 2 \times 2 \times 2 \times 2 \times 7$

या, $112 = 28 \times 4 = 7 \times 2 \times 2 \times 2 \times 2$

हम यह भी देखते हैं कि प्रत्येक अभाज्य गुणनखंडन में, गुणनखंडों को तो भिन्न प्रकार से व्यवस्थित किया जा सकता है परन्तु वास्तव में, **अभाज्य गुणनखंडन अद्वितीय** (*unique*) **है**। यह गुण, अर्थात् **किसी भाज्य संख्या का केवल एक ही अभाज्य गुणनखंडन होता है, अभाज्य गुणनखंडन गुण** (*Prime Factorization Property*) **या अंकगणित की आधारभूत प्रमेय** (*Fundamental Theorem of Arithmetic*) कहलाती है।

(इस गुण की उपपत्ति, इस पुस्तक की सीमा के बाहर है।)

प्रश्नावली 6.3

1. निम्न का एक से अधिक प्रकार से गुणनखंडन कीजिए :
   54, 62, 72
2. निम्न के अभाज्य गुणनखंडन ज्ञात कीजिए :
   90, 108, 9000, 221, 7325, 8712, 13915
   [जहाँ संभव हो घातांक संकेतन प्रयोग कीजिए।]
3. 3 अंकों की छोटी से छोटी संख्या लिखिए और उसे अभाज्यों के गुणा के रूप में व्यक्त कीजिए।

6.4 संख्याओं से विभाज्य होने की जाँच

अभी तक यह ज्ञात करने के लिए कि कोई संख्या, उदाहरण के तौर पर, 2 या 3 या 5 या 9 से विभाजित है या नहीं अर्थात् विभाज्य (*divisible*) है या नहीं हम केवल यह विधि जानते हैं कि संख्या को 2 या 3 या 5 या 9 से विभाजित करें और देखें कि कुछ शेष बचता है या नहीं। परन्तु इसमें समय अधिक लगता है और यह अनावश्यक भी है। बहुत सी सरल विधियाँ उपलब्ध हैं जिनसे यह जाँच (*test*) की जा सकती है कि कोई संख्या कुछ अन्य संख्याओं से विभाजित है या नहीं।

6.4.1 आइए, उदाहरणार्थ, 10 के गुणज लिखें। ये 0, 10, 20, 30, 40, 50, इत्यादि हैं। हम क्या देखते हैं? प्रत्येक संख्या '0' पर समाप्त होती है। इससे हमें तुरन्त यह जाँच करने का नियम प्राप्त हो जाता है कि कोई संख्या 10 से विभाज्य है या नहीं।

कोई संख्या 10 से विभाज्य होती है जबकि वह '0' पर समाप्त होती हो।

6.4.2 5 से विभाजन के बारे में आप क्या सोचते हैं? आइए, 5 के गुणजों को देखें। ये 0, 5, 10, 15, 20, 25, 30, 35, इत्यादि हैं। प्रत्येक संख्या या तो 0 पर या 5 पर समाप्त होती है। हम कहते हैं कि

कोई संख्या 5 से विभाज्य होती है जबकि वह '0' या 5 पर समाप्त होती हो।

6.4.3 निस्संदेह यह बताना बहुत ही सरल है कि कोई संख्या 2 से विभाज्य है या नहीं। इसके लिए संख्या को सम होना चाहिए। अब, सम संख्याएँ किस प्रकार की दिखती हैं? ये 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, इत्यादि हैं। क्या अब आप 2 से विभाज्यता (*divisibility*) की कोई जाँच बता सकते हैं?

कोई संख्या 2 से विभाज्य होगी जबकि वह 0 या 2 या 4 या 6 या 8 पर समाप्त होती हो। वास्तव में, हम और भी अच्छा कर सकते हैं। हम केवल यह भी कह सकते हैं कि

कोई संख्या 2 से विभाज्य होती है जबकि वह किसी सम अंक (*even digit*) पर समाप्त होती हो।

इससे पहले कि हम विभाज्यता की अन्य जाँचों पर विचार करें, हम एक रोचक गुणधर्म सीखेंगे।

6.4.4 संख्याएँ 316 और 108 दोनों ही 2 से विभाजित हो जाती हैं। इनके योग के बारे में आप क्या सोचते हैं? $316+108=424$ जो कि एक सम अंक पर समाप्त होता है। इसलिए विभाज्यता की जाँच (*divisibility test*) के अनुसार, यह योग 2 से विभाज्य है। क्या इनका अंतर भी 2 से विभाज्य है?

3165 और 2625 दोनों ही 5 से विभाज्य हैं। इनके अंतर के बारे में आप क्या सोचते हैं? $3165-2625=540$। अब 540, 0 पर समाप्त होता है। इसलिए विभाज्यता की जाँच के अनुसार 540, 5 से विभाज्य है। क्या इनका योग भी 5 से विभाज्य है?

वास्तव में यह सरलता से सिद्ध किया जा सकता है कि **किसी दी हुई संख्या से विभाज्य संख्याओं का योग और अंतर भी उस संख्या से विभाज्य होता है।** परन्तु इसकी उपपत्ति इस पुस्तक की सीमा के बाहर है और इसे बाद में उपयुक्त स्थान पर बताया जाएगा।

6.4.5 अब हम 3 और 9 के लिए विभाज्यता के नियम खोजेंगे। आइए उदाहरणार्थ संख्या 5871 को लें और यह ज्ञात करने का प्रयत्न करें कि यह (i) 3, (ii) 9 से विभाज्य है या नहीं।

हम 5871 को प्रसारित संकेतन में लिखते हैं, अर्थात्

$5871 = 5 \times 1000 + 8 \times 100 + 7 \times 10 + 1 \times 1$

अब, $10 = 9 + 1$, इसलिए वितरण गुण के कारण $7 \times 10 = 7 \times (9 + 1) = 7 \times 9 + 7$

इसी प्रकार, $8 \times 100 = 8 \times 99 + 8$

तथा, $5 \times 1000 = 5 \times 999 + 5$

इस प्रकार,

$5871 = (5 \times 999 + 5) + (8 \times 99 + 8) + (7 \times 9 + 7) + 1$

अब, 9, 99, 999, इत्यादि में से प्रत्येक 3 और 9 दोनों से विभाज्य है। अतः 5871

(i) 3 से विभाज्य होगा यदि $5 + 8 + 7 + 1$, 3 से विभाज्य हो।

(ii) 9 से विभाज्य होगा यदि $5 + 8 + 7 + 1$, 9 से विभाज्य हो।

परन्तु यह $5 + 8 + 7 + 1$ क्या है? यह दी हुई संख्या के अंकों का योग है। हम कहते हैं कि कोई संख्या

(i) 3 से विभाज्य होती है यदि उसके अंकों का योग 3 से विभाज्य हो।

(ii) 9 से विभाज्य होती है यदि उसके अंकों का योग 9 से विभाज्य हो।

हम देखते हैं कि 5871, 3 से विभाज्य है परन्तु 9 से नहीं।

उदाहरण 1: ज्ञात कीजिए कि 267525 निम्न से विभाज्य है या नहीं:

(i) 3 (ii) 9

हल: दी हुई संख्या में अंकों का योग $2 + 6 + 7 + 5 + 2 + 5 = 27$ है जो कि 3 और 9 दोनों से विभाज्य है।

अतः स्वयं संख्या भी 3 और 9 से विभाज्य है।

6.4.6 अब हम 4 और 8 के लिए विभाज्यता के नियम खोजेंगे। हम जानते हैं कि 1 और 10, 4 से विभाज्य नहीं हैं, परन्तु 100, 1000, 10000, इत्यादि विभाज्य हैं। अब आइए एक संख्या 4728 लें। हम लिखते हैं कि

$4728 = 4 \times 1000 + 7 \times 100 + 2 \times 10 + 8 \times 1$

चूंकि 1000 और 100, 4 से विभाज्य हैं तथा 10 और 1 विभाज्य नहीं हैं इसलिए स्पष्ट है कि संख्या 4728, 4 से तब ही विभाज्य होगी जबकि $2 \times 10 + 8 \times 1$, 4 से विभाज्य हो।

परन्तु यह $2 \times 10 + 8 \times 1$ क्या है? यह 28 है अर्थात् दी हुई संख्या में अंतिम दो (एकक और दस के स्थानों के) अंक हैं।

हम कहते हैं,

कोई संख्या 4 से विभाज्य होती है यदि उसके अंतिम दो अंकों से (अंकों के, क्रम में ही) बनी संख्या 4 से विभाज्य हो।

हम देखते हैं कि 4728, 4 से विभाज्य है।

हम विद्यार्थियों के अभ्यास के लिए यह छोड़ रहे हैं कि वे जाँच करें कि

कोई संख्या 8 से विभाज्य होती है यदि उसके अंतिम तीन अंकों से (अंकों के क्रम में ही) बनी संख्या 8 से विभाज्य हो।

उदाहरण 2: क्या 12504, 8 से विभाज्य है?

हल : इसके अंतिम तीन अंकों से बनी संख्या 504 है जो कि **8 से विभाज्य है ।**

अतः 12504, 8 से विभाज्य है ।

6.4.7 अंत में हम किसी संख्या की 11 से विभाज्यता पर विचार करते हैं । आइए, उदाहरणार्थ देखें कि 296813, 11 से विभाज्य है या नहीं । हम लिखते हैं,

$$296813 = 2\times 100000 + 9\times 10000 + 6\times 1000 + 8\times 100 + 1\times 10 + 3\times 1$$

हम जानते हैं कि 11, 11 से विभाज्य है, इसलिए हम

$10 = 11 - 1$ लिखते हैं ।

हम यह भी जानते हैं कि 99, 11 से विभाज्य है, इसलिए हम

$100 = 99 + 1$ लिखते हैं ।

इन्हीं कारणों से, हम $1000 = 1001 - 1$, $10000 = 9999 + 1$ और $100000 = 100001 - 1$ लिखते हैं । इस प्रकार

$$296813 = 2(100001-1) + 9(9999+1) + 6(1001-1) + 8(99+1) + 1(11-1) + 3\times 1$$

स्पष्ट है, 296813, 11 से विभाज्य होगा, यदि

$-2 + 9 - 6 + 8 - 1 + 3$, 11 से विभाज्य हो ।

परन्तु यह $-2+9-6+8-1+3$ क्या है? हम इसे $3+8+9-(1+6+2)$ लिख सकते हैं और देख सकते हैं कि यह (एकक स्थान से प्रारम्भ करते हुए) विषम स्थानों के अंकों के योग तथा सम स्थानों के अंकों के योग का अंतर है ।

हम देखते हैं कि $-2+9-6+8-1+3 = 11$ है जो कि 11 से विभाज्य है । अतः 296813 11 से विभाज्य है । हम कहते हैं,

कोई संख्या 11 से विभाज्य होती है, यदि उसके (एकक स्थान से प्रारम्भ करते हुए) विषम स्थानों के अंकों के योग तथा सम स्थानों के अंकों के योग का अंतर 11 से विभाज्य हो ।

उदाहरण 3 : क्या 4932718, 11 से विभाज्य है ?

हल : इसके विषम स्थानों के अंकों का योग $= 8+7+3+4 = 22$ है ।

इसके सम स्थानों के अंकों का योग $= 1+2+9 = 12$ है ।

इनका अंतर $(22-12)$, 11 से विभाज्य नहीं है । अतः संख्या 4932718, 11 से विभाज्य नहीं है ।

पाठक को चाहिए कि वह विभिन्न विभाज्यता नियमों की सूची बनाएँ । इन नियमों की उपपत्तियाँ इस पुस्तक की सीमा के बाहर हैं और इनके विषय में बाद में बताया जाएगा ।

प्रश्नावली 6.4

1. बताइए निम्न में से कौन से कथन सत्य हैं :
   (i) यदि कोई संख्या 2 से विभाज्य है तो वह 4 से भी विभाज्य होती है ।
   (ii) यदि कोई संख्या 4 से विभाज्य है तो वह 2 से भी विभाज्य होती है ।

(iii) यदि कोई संख्या 3 से विभाज्य है तो वह 9 से भी विभाज्य होती है।

(iv) यदि कोई संख्या 9 से विभाज्य है तो वह 3 से भी विभाज्य होती है।

2. ऐसी संख्या का एक उदाहरण दीजिए जो कि

(i) 3 से विभाज्य है परन्तु 9 से नहीं।

(ii) 5 से विभाज्य है परन्तु 10 से नहीं।

3. यदि कोई संख्या 2 और 3 दोनों से विभाज्य हो तो वह 6 से भी विभाज्य होगी। अब 2 से विभाज्य होने के लिए, संख्या को किसी सम अंक पर समाप्त होना चाहिए। 3 से विभाज्य होने के लिए अंकों का योग, 3 से विभाज्य होना चाहिए। हम कहते हैं कि **कोई संख्या 6 से विभाज्य होती है, यदि उसके अंकों का योग 3 से विभाज्य हो तथा वह एक सम अंक पर समाप्त होती हो।**

ज्ञात कीजिए कि क्या निम्न संख्याएँ 6 से विभाज्य हैं :

(i) 72354 (ii) 40083

4. ज्ञात कीजिए कि निम्न में कौन कौन 2, 3, 5, 9, 10 से विभाज्य हैं :

(i) 390 (ii) 126 (iii) 567 (iv) 4566

(v) 7530 (vi) 715230 (vii) 325

5. क्या 1060301, 11 से विभाज्य है ?

6. क्या 980122, 11 से विभाज्य है ?

7. निम्न में से कौन कौन सी संख्याएँ 11 से विभाज्य हैं :

(i) 432311 (ii) 57860 (iii) 439

विविध प्रश्नावली III

(एकक V और VI पर)

1. 10^4, 26^2, 101^2, $(-11)^3$, 9^5, $(-1)^{123}$ और $(-1)^{462}$ के मान ज्ञात कीजिए।

2. 441, 15625 और 1024 के वर्गमूल ज्ञात कीजिए।

3. 90 से कम 12 के सभी गुणज लिखिए।

4. 8025 के अभाज्य गुणनखण्ड ज्ञात कीजिए।

5. पता लगाइए कि निम्न में से कौन सी संख्याएँ अभाज्य हैं :
   112, 323, 151, 135

6. 100 और 110 के मध्य की अभाज्य संख्याएँ ज्ञात कीजिए।

7. 819, 3105 और 153549 के अभाज्य गुणनखंडन ज्ञात कीजिए।

8. हम जानते हैं कि कोई भी सम संख्या 2 से विभाज्य होती है। दूसरे शब्दों में, हम किसी भी (पूर्ण) सम संख्या को किसी पूर्ण संख्या के दुगुने के रूप में व्यक्त कर सकते हैं। संकेतों का प्रयोग कर, हम लिखते हैं कि

$$a = 2\,m$$

जबकि 'a' सम संख्या को तथा 'm' पूर्ण संख्या को व्यक्त करता है। इस प्रकार हम, उदाहरणार्थ $8 = 2 \times 4$ या $612 = 2 \times 306$ लिख सकते हैं।

निम्न को उपर्युक्त रूप में लिखिए :

16, 234, 0, 82

9. चूँकि जब किसी विषम संख्या को 2 से विभाजित किया जाता है तो सदैव 1 शेष रहता है, इसलिए हम किसी भी विषम संख्या 'a' को निम्न प्रकार लिख सकते हैं।

$$a = 2m + 1$$

जबकि m पूर्ण संख्या है। उदाहरणार्थ,

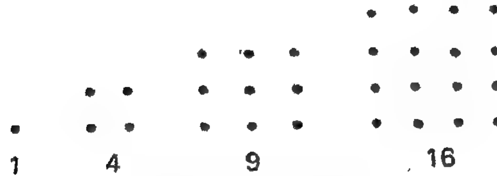
$1 = 2 \times 0 + 1$, $7 = 2 \times 3 + 1$, $981 = 2 \times 490 + 1$

निम्न को उपर्युक्त रूप में लिखिए :

5, 19, 325, 6081

10. ऐसी संख्या जो कि 1 सहित परन्तु स्वयं को छोड़कर अपने सभी गुणनखंडों के योग के बराबर होती है **संपूर्ण संख्या** (*perfect number*) कहलाती है। उदाहरणार्थ 6, एक संपूर्ण संख्या है **क्योंकि** $6 = 1 + 2 + 3$। 40 से छोटी एक और संपूर्ण संख्या है। उसे ज्ञात कीजिए।

11. बिंदुकित प्रतिरूप (*dot pattern*) में संख्या 4 को दो दो बिंदुओं की 2 पंक्तियों में, 9 को तीन तीन बिंदुओं को 3 पंक्तियों में, 16 को चार चार बिंदुओं की 4 पंक्तियों में लिखा जा सकता है। (देखिये आकृति 6.1) ऐसी संख्याएँ **वर्ग संख्याएँ** (*square numbers*) कहलाती हैं।

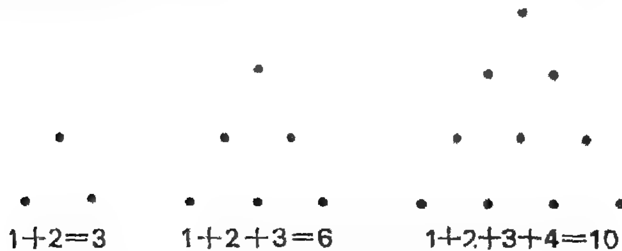


आकृति 6.1 : वर्ग संख्याएँ

(क्या आप इन संख्याओं का दूसरा नाम बता सकते हैं?)
अगली दो वर्ग संख्याओं के बिंदुकित प्रतिरूप बनाइए।

12. (क) जाँच कीजिए कि $(2\times3\times4\times5)+1$ एक पूर्ण वर्ग है।
(ख) जाँच कीजिए कि $(8\times9\times10\times11)+1$ एक पूर्ण वर्ग है।
(ग) कोई चार क्रमागत धनपूर्णांक लीजिए। इनको परस्पर गुणा करके, गुणनफल में 1 जोड़िए। जाँच कीजिए कि परिणामी संख्या एक पूर्ण वर्ग है।

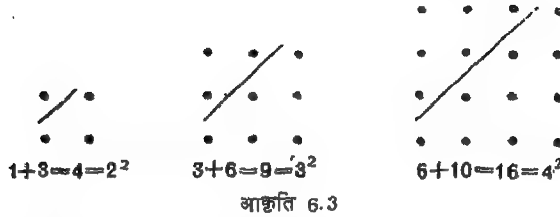
13. प्राचीन ग्रीस (*Greece*) देश में पाइथागोरस (*Pythagoras*) नाम का एक गणितज्ञ था। उसके जन्म की तारीख और स्थान दोनों ही अज्ञात हैं। परन्तु ऐसा अनुमान किया जाता है कि लगभग 580 और 568 ई० पू० के बीच में उसका जन्म हुआ था। उसने गणित का एक गुप्त स्कूल स्थापित किया और गणित में बहुत कुछ योगदान दिया। इस स्कूल के सदस्य पाइथागोरियन्स (*Pythagoreans*) कहे जाते हैं। पाइथागोरियन्स ने संख्याओं 1, $1+2=3$, $1+2+3=6$, $1+2+3+4=10$, $1+2+3+4+5=15$, इत्यादि को **त्रिभुजाकार संख्याएँ** (*triangular numbers*) कहा। इन संख्याओं को त्रिभुजाकार बिंदुकित प्रतिरूपों (*triangular dot patterns*) आकृति 6.2 की तरह दिखाया जा सकता है।



आकृति 6.2 : त्रिभुजाकार संख्याएँ

अगली चार त्रिभुजाकार संख्याएँ लिखिए और उनके बिंदुकित प्रतिरूप बनाइए।

14. यदि किन्हीं दो आसन्न त्रिभुजाकार संख्याओं को जोड़ें तो हमें एक वर्ग संख्या प्राप्त होती है। उदाहरणार्थ, $1+3=4=2^2$, $3+6=9=3^2$, $6+10=16=4^2$ इत्यादि। (देखिये आकृति 6.3)



आकृति 6.3

(i) चौथी और पाँचवीं त्रिभुजाकार संख्याओं का क्या योग है ?
(ii) पाँचवीं और छठी त्रिभुजाकार संख्याओं का क्या योग है ?
(iii) क्या आप सत्ताइसवीं और अट्ठाइसवीं त्रिभुजाकार संख्याओं का योग बता सकते हैं ?

एकक VII

पूर्णांकीय गुणांकों के बीजीय व्यंजक

हम संख्याओं को निरूपित करने के लिए अक्षरों (*letters*) का प्रयोग पहले ही, उदाहरणार्थ, पूर्ण संख्याओं या पूर्णांकों पर संक्रियाओं के गुणों को व्यक्त करते समय, कर चुके हैं। अब, हम इस एकक में एक क्रमबद्ध रूप में अंकगणित (*arithmetic*) से बीजगणित (*algebra*) की ओर गमन करेंगे। हम यह सीखेंगे कि इन अक्षर-संख्याओं के जोड़ने, घटाने और गुणा करने में इनका किस प्रकार प्रयोग किया जाता है।

7.1 अंकगणित से बीजगणित

एक महान जर्मन गणितज्ञ फलिक्स क्लीन (*Felix Klein*) [1849-1925] ने एक बार कहा था, 'वास्तविक गणित अक्षरों पर संक्रियाओं से, प्रारम्भ होती है।' हम इस पुस्तक और अपनी पिछली कक्षाओं में अक्षरों (*letters*) का संख्याओं को व्यक्त करने में पहले ही प्रयोग कर चुके हैं। क्या आप को याद है कि, उदाहरणार्थ, किसी आयत के क्षेत्रफल का सूत्र लिखने के लिए हमने अक्षरों जैसे A, l, w का प्रयोग किया था और लिखा था कि $A=l\times w$? इस सूत्र में A, आयत का क्षेत्रफल [के मात्रकों (*units*) की संख्या], l, लम्बाई (के मात्रकों की संख्या) तथा w, चौड़ाई (के मात्रकों की संख्या) व्यक्त करते हैं। इन अक्षरों के प्रवेश से हम और अधिक व्यापक पदों में विचार कर सकते हैं।

$A=l\times w$ से हम **किसी** भी आयत का क्षेत्रफल निकाल सकते हैं। उदाहरणार्थ, किसी आयत के लिए जिसमें l, मान लीजिए, 12 सेमी और w, मान लीजिए, 8 सेमी है, हम $A=l\times w$ में l और w के मान रखकर उसका क्षेत्रफल ज्ञात करते हैं। **अक्षर A, l और w केवल संख्याएँ व्यक्त करते हैं।** यह व्यापक पदों में विचार करने में समर्थ होने के लिए संख्याएँ निरूपित करने में अक्षरों का प्रयोग, अंकगणित से बीजगणित की ओर गमन (*transition*) है।

हम, प्रत्यक्ष कारणों से, इन अक्षरों को **अक्षर संख्याएँ** (*literal numbers*) कहेंगे। **चूंकि ये संख्याएँ निरूपित करते हैं अतः स्पष्ट है कि इनको संख्याओं के योग, व्यवकलन, गुणन और विभाजन के सभी नियमों (और चिन्हों) तथा साथ ही इन संक्रियाओं के सभी गुणों का पालन करना चाहिए।**

कोई संख्या या आधारभूत संक्रियाओं (*fundamental operations*) के प्रयोग से बना संख्याओं का कोई समूह या संयोग (*combination*) **बीजीय व्यंजक** (*algebraic expression*) कहलाता है। $y-3x$, $a+b-c$, $4pq-4qr-4rs+4st$, 23, $2y$, $5n$, lwh इत्यादि बीजीय व्यंजकों के कुछ उदाहरण हैं। जब व्यंजक में '+', '−' के दो या दो से अधिक चिन्ह आते हैं तो वह कई भागों में पृथक हो जाता है। **अपने चिन्ह सहित प्रत्येक भाग व्यंजक का एक पद** (*term*) कहलाता है। प्रायः व्यंजक

के पहले पद के धन चिन्ह को छोड़ दिया जाता है अर्थात् उसका धन चिन्ह लिखा नहीं जाता। y और $-3x$ व्यंजक $y-3x$ के पद हैं; a, b और $-c$ व्यंजक $a+b-c$ के पद हैं।

जब किसी व्यंजक में एक ही पद हो तो वह **एकपदी** (*monomial*) कहलाता है। 23, $2y$, $5a$, twh, $4z$, इत्यादि एकपदियों के उदाहरण हैं। वह व्यंजक जिसमें दो पद हों **द्विपद*** (*binomial*) कहलाता है। $y-3x$, $23-z$, $2l+2w$ इत्यादि द्विपदों के कुछ उदाहरण हैं। **त्रिपद** (*trinomial*) क्या होता है? क्या आप बता सकते हैं? त्रिपदों के कुछ उदाहरण दीजिए।

किसी पद, उदाहरणार्थ $18xy$, में 18, x और y इस पद के **गुणनखंड** कहलाते हैं। x और y **अक्षर गुणनखंड** (*literal factors*) हैं; 18 एक **संख्यात्मक गुणनखंड** (*numerical factor*) है। इनमें से कोई भी एक गुणनखंड शेष गुणनखंडों के गुणनफल का **गुणांक** (*coefficient*) कहलाता है। इस प्रकार, पद $18xy$ में y, $18x$ का गुणांक है, पद $-4qr$ में $-q$, $4r$ का गुणांक है, पद $18xy$ में, 18, xy का गुणांक है। हम, कभी कभी, संख्यात्मक गुणनखंड को **पद का गुणांक** (*coefficient of the term*) भी कहते हैं। इस प्रकार 18 को पद $18xy$ का गुणांक भी कहा जा सकता है। जब किसी पद का गुणांक $+1$ या -1 होता है तो प्रायः '1' लिखा नहीं जाता। उदाहरणार्थ हम $1x$ को x तथा $-1x$ को $-x$ लिखते हैं।

जब किन्हीं पदों में अक्षर गुणनखंड एक से हों तो वे **समान पद** (*like terms*) कहलाते हैं, अन्यथा वे **असमान पद** (*unlike terms*) कहलाते हैं। उदाहरणार्थ, व्यंजक $2xy-3x+7xy+4x$ में $2xy$ और $7xy$ समान पद हैं, $-3x$ और $4x$ समान पद हैं। परन्तु व्यंजक $a+b-c$ या $y-3x$ या $4pq-4qr-4rs+4st$ में सभी पद, असमान पद हैं।

प्रश्नावली 7.1

1. बताइए कि निम्न में से कौन एकपदी हैं, कौन द्विपद हैं तथा कौन त्रिपद हैं। अन्य के नामों के लिए सुझाव दीजिए।
(नीचे की **टिप्पणी** देखिए)

(i) $4x-3y$ (ii) x^2
(iii) $4p^2q-4qp^2+r$ (iv) $3abc$
(v) $x+y+z+w$ (vi) $7-x+y$
(vii) $5x^3-2x+4$

---

*'द्वि,' '2' व्यक्त करता है, अतः **द्विपद** का अर्थ है **दो संख्याएं**। 'त्रि', '3' व्यक्त करता है, अतः **त्रिपद** का अर्थ है **तीन संख्याएं**। **एक पदी** का अर्थ है **एक संख्या**।

टिप्पणी : हम संख्याओं के वर्ग, घन और बड़ी घातों के संकेतन से पहले से ही परिचित हैं। चूंकि ये अक्षर भी संख्याएँ ही व्यक्त करते हैं, इसलिए इसी संकेतन को इन अक्षर संख्याओं के लिए भी सुविधाजनक रूप से प्रयोग किया जा सकता है।

अतः $x^2=x\times x$, $x^3=x\times x\times x$, इत्यादि।

2. निम्न में से कौन कौन समान पद हैं :
   (i) $3x, -7x$ (ii) $11x, 11y$ (iii) $14xy, -21xy$ (iv) $15ab, -4b$
3. निम्न में से प्रत्येक व्यंजक में x का गुणांक लिखिए :
   $-3xy$, $4x-3y$, $7-x+y$, mx, $17xyz$

7.2 बीजीय व्यंजकों का योग और व्यवकलन

अब हम यह जानते हैं कि किसी व्यंजक में समान और असमान पद हो सकते हैं। अतः बीजीय व्यंजकों को जोड़ने (या घटाने) के लिए हमें समान पद संग्रहित (*collect*) करके उन्हें जोड़ (या घटा) लेना चाहिए। अब हम समान पदों को किस प्रकार जोड़ते (या घटाते) हैं? उदाहरण के तौर पर $3x$ और $7x$ को लीजिए। मान लीजिए हम $3x+7x$ ज्ञात करना चाहते हैं। क्या हम इसे $3x+7x=(3+7)x$ नहीं लिख सकते? हाँ! क्यों? क्या आपको याद है कि वितरण गुण क्या है?

अतः, $3x+7x=(3+7)x=10x$

इसी प्रकार, $18xy-3xy+6xy=(18-3+6)xy=21xy$

अतः समान पदों को जोड़ने या घटाने का क्या नियम है?

कई समान पदों का योग (या अंतर), एक अन्य समान पद होता है जिसका गुणांक इन समान पदों के योग (या अंतर) के बराबर होता है।

उदाहरण 1 : $3pq, -2pq$ तथा $-11pq$ को जोड़िए।

हल : इनका योग, एक अन्य समान पद होगा जिसका गुणांक $3-2-11=-10$ है। इस प्रकार,

$$3pq-2pq-11pq=-10pq$$

वैकल्पिक विधि : यदि हम उपर्युक्त नियम याद नहीं रखना चाहते तो हम वितरण गुण का प्रयोग करके लिख सकते हैं कि

$$3pq-2pq-11pq=(3-2-11)pq=-10pq$$

उदाहरण 2 : $8ab^2$ में से $24ab^2$ घटाइए।

हल : $8ab^2-24ab^2=(8-24)ab^2=-16ab^2$

उदाहरण 3 : समान पद संग्रहित कीजिए और व्यंजक
$-7x^2+3x+x^2-8-5x+9x^2-4$ को सरल कीजिए।

हल : हम पुनर्व्यवस्थित करके समान पदों को संग्रहित करते हैं। इससे हमें निम्न प्राप्त होता है :

$$-7x^2+x^2+9x^2+3x-5x-8-4$$
$$=(-7+1+9)x^2+(3-5)x-12$$
$$=3x^2-2x-12$$

उदाहरण 4 : व्यंजकों $3x+4y-5z$, $5y+2x$, $7x-8y$ और $4x-9y-5z$ को जोड़िए।

हल : इन व्यंजकों को जोड़ने के लिए हमें इनके समान पदों को जोड़ने की आवश्यकता है। सुविधा

की दृष्टि से हम इन व्यंजकों को इस प्रकार लिखेंगे कि इनके समान पद एक स्तंभ* (*column*) में हों जैसा कि नीचे दिखाया गया है :

$$\begin{array}{r} 3x+4y-5z \\ 2x+5y \quad\;\; \\ 7x-8y \quad\;\; \\ 4x-9y-5z \\ \hline 16x-8y-10z \end{array}$$

उदाहरण 5 : $15xy+6yz+7zx$ में से $12xy-5yz-9zx$ को घटाइए।

हल : $15xy+6yz+7zx-(12xy-5yz-9zx)$

$=15xy-12xy+6yz-(-5yz)+7zx-(-9zx)$

$=(15-12)xy+[6-(-5)]yz+[7-(-9)]zx$

$=3xy+11yz+16zx$

[आपको याद होगा कि एक ऋणात्मक संख्या का ऋणात्मक, धनात्मक होता है, इसलिए $[6-(-5)]=6+5$ अर्थात् 11 होगा, इत्यादि। किसी धनात्मक संख्या का ऋणात्मक क्या होता है? अतः क्या हम यह नहीं कह सकते कि एक व्यंजक में से दूसरे को घटाने के लिए हम उस व्यंजक के, जो कि घटाया जाना है, प्रत्येक पद का चिन्ह बदलें ('+' से '−' या '−' से '+') और फिर दोनों व्यंजकों को जोड़ लें? हम उदाहरण 4 ही की तरह समान पदों को एक स्तंभ में रख लेंगे। **उस व्यंजक के जो कि घटाया जाना है प्रत्येक पद के चिन्ह परिवर्तन को मूल (*original*) चिन्ह के नीचे लिखकर दर्शाया जाता है।**]

अब हम इस विधि से $15xy+6yz+7zx$ में से $12xy-5yz-9zx$ को घटाते हैं।

$$\begin{array}{rrr} 15xy+ & 6yz+ & 7zx \\ 12xy- & 5yz- & 9zx \\ - & + & + \\ \hline 3xy+ & 11yz+ & 16zx \end{array}$$

उदाहरण 6 : $3x^2-8x+11$, $-2x^2+12x$ और $-4x^2+17$ के योग में से x^2-x-1 को घटाइए।

हल : हम व्यंजकों को इस प्रकार लिखते हैं कि इनके समान पद एक ही स्तंभ में रहें। निस्संदेह, हम अंतिम व्यंजक के प्रत्येक पद का चिन्ह बदल कर लिखेंगे। **क्यों ? अब हमें निम्न प्राप्त होता है :**

$$\begin{array}{rrr} 3x^2- & 8x+ & 11 \\ -2x^2+ & 12x & \\ -4x^2 & & +17 \\ x^2- & x- & 1 \\ - & + & + \\ \hline -4x^2+ & 5x+ & 29 \end{array}$$

*इसके लिए व्यंजकों में पदों के क्रम को यदि आवश्यक हो तो, बदला जा सकता है जैसा कि उपर्युक्त उदाहरण 4 में दूसरे व्यंजक में किया गया है।

प्रश्नावली 7.2

1. जोड़िए :
   (i) $7x^2y,\ -3x^2y,\ 14x^2y$
   (ii) $y^3,\ -2y^3,\ -3y^3, 4y^3$
   (iii) $-abc,\ 13abc,\ 5abc$
2. घटाकर सरल कीजिए :
   (i) $3y^2-18y^2$ (ii) $-12ab-6ab$
   (iii) $23a^3-17a^3$
3. प्रत्येक व्यंजक को समान पद संग्रहित करके सरल कीजिए :
   (i) $-x^2+4x^2-8x^2+11x^2$
   (ii) $12b-7b-3b$
   (iii) $3x^2+y+7-6x^2-5y-11+2y$
   (iv) $2b-7a+8a-5b+3c-c$
   (v) $10m^2-9m+7m-3m^2-5m-8$
4. जोड़िए :
   (i) $3x+4y-15z,\ 6x+7y,\ 12y-7z-9x$
   (ii) $x^2y-3x+4,\ -8x^2y+3x-4$
   (iii) $13x^3-7x^2,\ 10x^2+8x^3,\ -5x^3,\ 4x^2-3x^3$
   (iv) $15a+11b-13c-17,\ 18-12c-7b-3a$
5. घटाइए :
   (i) $c^3+2a^3-b^3+abc$ में से $3abc-a^3-b^3$
   (ii) $-2x^2+4xy-5y^2$ में से $x^2-3xy-2y^2$
   (iii) $3m^2-3mn+8$ में से $-m^2+3mn$
6. $3a-5b+3c$ और $2a+4b-5c$ के योग में से $4a-b-c+3$ को घटाइए।
7. $2x^2+3xy$ प्राप्त करने के लिए x^2+xy+y^2 में क्या जोड़ना चाहिए?
8. $11x-16y+7a$ प्राप्त करने के लिए $-13x+5y-8a$ में से क्या घटाना चाहिए ?
9. $2x^2+3xy, -x^2-xy+y^2$ और $xy+2y^2$ के योग में से $3x^2-y^2$ और $-x^2+xy+y^2$ का योग घटाइए।
10. $6m-7n-5p, -4m+6p-9n$ और $5m-4n+3p$ के योग में से $13m-11n+9p$ और $-7p+3m-5n$ का योग घटाइए।

7.3 समूहन संकेतों का प्रयोग

बहुधा, यह आवश्यक हो जाता है कि दो या दो से अधिक पदों वाले व्यंजक को एक संख्या ही माना जाए। उदाहरणार्थ $2x$ को व्यंजक $3x-5y$ से गुणा करने के लिए हमें व्यंजक $3x-5y$ को एक ही

संख्या मानना चाहिए। ऐसे व्यंजकों को अलग दर्शाने के लिए समूहन संकेतों (*grouping symbols*) '(   )', '[   ]' और '{   }' का प्रयोग किया जाता है। इन संकेतों को क्रम से **छोटा कोष्ठक, बड़ा कोष्ठक** और **मंझला कोष्ठक** कहते हैं। अतः इनमें से किसी भी कोष्ठक में व्यंजक $3x-5y$ को रखकर तथा $2x\times(3x-5y)$ या केवल $2x(3x-5y)$ लिखकर हम $2x$ और $3x-5y$ का गुणन दर्शा सकते हैं।

इस प्रकार, व्यंजकों पर (बीजीय) संक्रियाएँ करते समय हमें समूहन संकेतों को लगाने या हटाने की आवश्यकता पड़ेगी।

उदाहरणार्थ, अंतर $(3x^2-2x+7)-(2x^2-4x-3)$ को ज्ञात करने के लिए, हम प्रत्येक व्यंजक में से पहले छोटे कोष्ठक हटाएंगे। पहले व्यंजक के कोष्ठक से पहले कौनसा चिन्ह है? धन चिन्ह। अतः हम कोष्ठक हटा देते है और पहले व्यंजक को बिना कुछ परिवर्तन किए लिख देते है। दूसरे व्यंजक के कोष्ठक के पहले कौनसा चिन्ह है? ऋण चिन्ह। निस्संदेह, इसका अर्थ है कि हमें दूसरे व्यंजक को पहले व्यंजक में से घटाना है। अतः हम कोष्ठक हटाकर दूसरे व्यंजक के प्रत्येक पद का चिन्ह परिवर्तित कर देते हैं। इस प्रकार हमें निम्न प्राप्त होता है:

$$\begin{aligned}(3x^2-2x+7)-(2x^2-4x-3) &= 3x^2-2x+7-2x^2+4x+3\\ &= x^2+2x+10\end{aligned}$$

अतः हम देखते हैं कि

(1) **यदि किसी समूहन संकेत के पहले '+' चिन्ह लगा हो, तो इस संकेत को बिना पदों के चिन्ह में कोई परिवर्तन किए हटाया जा सकता है ;**

(2) **यदि किसी समूहन संकेत के पहले '−' चिन्ह लगा हो, तो प्रत्येक पद का चिन्ह परिवर्तित करके इस संकेत को हटाया जा सकता है ;**

(3) **यदि किसी व्यंजक में एक से अधिक समूहन संकेत हों, तो हम सबसे अंदर वाले संकेत को पहले हटाते हैं और यदि कोई समान पद हों तो उन्हें संग्रहित करके जोड़ते हैं। हम यह प्रक्रिया बाहर की ओर तब तक जारी रखते हैं जब तक कि सब समूहन संकेत न हट जाएं।**

उदाहरण 1: $6a-(7b-c)$ को सरल कीजिए।

हल: हम नियम 2 का प्रयोग करते हैं और इस प्रकार हमें $6a-7b+c$ प्राप्त होता है।

उदाहरण 2: $[2x^2-\{3x-(7x^2+4x-2)\}]$ को सरल कीजिए।

हल: हम सबसे अंदर वाले संकेत से बाहर की ओर चलते हैं। **इस प्रकार**

$$\begin{aligned}[2x^2-\{3x-(7x^2+4x-2)\}] &= [2x^2-\{3x-7x^2-4x+2\}]\\ &= [2x^2-\{-x-7x^2+2\}]\\ &= [2x^2+x+7x^2-2]\\ &= 9x^2+x-2\end{aligned}$$

अब देखें कि हम व्यंजकों में समूहन संकेत किस प्रकार लगाते हैं? इसके लिए उपर्युक्त नियमों (1) और (2) जैसे नियमों का ही प्रयोग किया जाता है। **यदि कोई ऐसा समूहन संकेत लगाना हो जिसके पहले '+' चिन्ह हो तो हम व्यंजक में केवल वह संकेत लगा देते हैं।** इस स्थिति में हम व्यंजक के पदों के चिन्हों में कोई परिवर्तन नहीं करते। परन्तु, **यदि कोई ऐसा समूहन संकेत लगाना हो**

जिसके पहले '—' चिन्ह हो तो हम व्यंजक के प्रत्येक पद का चिन्ह परिवर्तित कर देते हैं और व्यंजक में '—' चिन्ह वाला संकेत लगा देते हैं।

उदाहरण 3 : व्यंजक $3a-4b-2c$ में ऐसा समूहन संकेत लगाइए जिसके पहले '—' चिन्ह लगा हो।

हल : हम तीनों कोष्ठकों में से किसी भी कोष्ठक का प्रयोग कर सकते हैं। आइए बड़े कोष्ठक का प्रयोग करें। हमें निम्न प्राप्त होता है।

$$3a-4b-2c=-[-3a+4b+2c]$$

उदाहरण 4 : व्यंजक $-a^2-8a+6b-3c+8d$ के अंतिम दो पदों को कोष्ठक में रखिए जिसके पहले ऋण चिन्ह लगा हो।

हल : हम देखते हैं कि

$$-a^2-8a+6b-3c+8d=-a^2-8a+6b-(3c-8d)$$

प्रश्नावली 7.3

1. सरल कीजिए :

(i) $(2x-3y)-(x+2y)$
(ii) $(-8l+3m)-(5l-11m)$
(iii) $(3a-5b)-(-6a+2b)$
(iv) $(x^2+3x-2)-(4x-2x^2-2)$
(v) $x+(x-y-3)-(2x+y-4)$
(vi) $2l-[l-(3m-2l)+m]$
(vii) $-x+[-(3x+2y)+(x-4)]$
(viii) $(3x^2-4y+3x)-[x^2-(x^2-y)-3y+4]$
(ix) $[4-2a+5b-(a-b)+3]-(5a+4b-3c)$
(x) $-\{5x^3+x^2-[3x^4-(1-2x-x^3)-3x^3]+1\}$
(xi) $6ab-\{-(2ab-4a)+[3b-(a+ab)+7ab]\}$
(xii) $-\{3x-4y-[-8y-6+3x+(2y-5x-3)-6]+8x-3y\}$

2. निम्न में से प्रत्येक व्यंजक के अंतिम दो पदों को कोष्ठक में रखिए जिसके पहले '—' चिन्ह लगा हो।

(i) $9x+6z-4y-8$
(ii) $2x^3+4y^3-3z+9$
(iii) $a-b-4d-5$

7.4 बीजीय व्यंजकों का गुणन

7.4.1 आइए पहले **दो या अधिक एक पदीयों को गुणा करें।** उदाहरणार्थ, मान लीजिए ये $2x$, $-3y$ और $4z$ हैं। चूँकि अक्षर संख्याएँ, संख्याएँ व्यक्त करती हैं अतः यह सरलता से देखा जा सकता है कि $(2x)(-3y)(4z) = 2\times x\times(-3)\times y\times 4\times z$

$$= 2\times(-3)\times 4\times x\times y\times z \text{ (क्यों ?)}$$
$$= -24xyz$$

अतः हम देखते हैं कि **दो या अधिक एक पदीयों का गुणनफल उनके गुणांकों के गुणनफल का अक्षर गुणनखंडों के साथ गुणनफल है।**

उदाहरण 1 : $12x$, -8, $3x^2y$ और $4y^3$ को गुणा कीजिए।

हल : नियम का प्रयोग करने पर, हमें निम्न गुणनफल प्राप्त होता है :

$$12\times(-8)\times 3\times 4\times x\times x^2\times y\times y^3$$
$$= -1152\times x\times x^2\times y\times y^3$$
$$= -1152x^3y^4 \text{ (नीचे टिप्पणी देखिए)}$$

7.4.2 अब आइए **एक द्विपद और एक पदी का गुणा करें।** उदाहरणार्थ, हम $2a$ और $4b+3c$ लेते हैं।

हम गुणन का वितरण गुण प्रयोग कर सकते हैं। इस प्रकार हम लिखते हैं कि

$$2a(4b+3c) = (2a)(4b)+(2a)(3c)$$
$$= 8ab+6ac$$

उदाहरण 2 : $(-3x^2y)$ और (x^2+4y^2) का गुणा कीजिए।

हल : $-3x^2y(x^2+4y^2)$

$$= -3x^2yx^2-12x^2yy^2 = -3x^4y-12x^2y^3$$

हम एकपदी और त्रिपद का किसप्रकार गुणा करेंगे ? आइए निम्न उदाहरण लें।

उदाहरण 3 : $2x^2$ और $(-4x^2+4y^2-xy)$ का गुणनफल ज्ञात कीजिए।

हल : $2x^2(-4x^2+4y^2-xy) = 2x^2(-4x^2)+2x^2(4y^2)-2x^2(xy)$

$$= -8x^4+8x^2y^2-2x^3y$$

क्या आप वह नियम बता सकते हैं जिसका हमने एकपदी और त्रिपद का गुणा करने में प्रयोग किया है ?

7.4.3 अंत में, आइए **दो द्विपदों को गुणा करें।** उदाहरणार्थ, हम $(3a-2b)$ और $(5a-4b)$ लेते हैं। यहाँ हमें गुणन के वितरण गुण का दो बार प्रयोग करना पड़ेगा। $(5a-4b)$ को एक संख्या मानिए (आपको याद होगा कि समूहन संकेतों के प्रयोग से ऐसा हम कर सकते है) और तब,

$$(3a-2b)(5a-4b) = 3a(5a-4b)-2b(5a-4b)$$
$$= 3a(5a)-3a(4b)-2b(5a)-2b(-4b)$$
$$= 15a^2-12ab-10ab+8b^2$$
$$= 15a^2-22ab+8b^2$$

टिप्पणी : $x\times x^2 = x\times x\times x = x^3$ इसी प्रकार, $y\times y^3 = y^4$

इसप्रकार हम देखते हैं कि दो द्विपदों का गुणा करने के लिए हम एक द्विपद के पदों को क्रम से दूसरे द्विपद के पदों से गुणा करते हैं। और फिर इस परिणाम को जोड़ते (या घटाते) हैं।

उदाहरण 4 : $(x+y)$ और $(x+y)$ का गुणनफल ज्ञात कीजिए।

हल : हम, प्रत्यक्ष कारणों से, इस गुणनफल को $(x+y)^2$ लिखते हैं।

तब,

$$(x+y)^2 = x(x+y)+y(x+y)$$
$$= x^2+2xy+y^2$$

अत:, $(x+y)^2 = x^2+2xy+y^2$

दूसरे शब्दों में, किसी द्विपद का वर्ग पहले पद के वर्ग, दूसरे पद के वर्ग तथा दोनों पदों के गुणा के दुगुने के योग के बराबर होता है।

$(x-y)^2$ कितना है? शब्दों में, $(x-y)$ और $(x-y)$ के गुणा करने का एक नियम बताइए।

उदाहरण 5 : $(3x+2y)^2$ ज्ञात कीजिए।

हल : नियम का प्रयोग कर हम लिख सकते हैं कि

$$(3x+2y)^2 = (3x)^2+(2y)^2+2(3x)(2y)$$
$$= 9x^2+4y^2+12xy$$

उदाहरण 6 : $(a-3b^2)^2$ ज्ञात कीजिए।

हल : हम देखते हैं कि

$$(a-3b^2)^2 = [a+(-3b^2)]^2$$
$$= (a)^2+(-3b^2)^2+2(a)(-3b^2)$$
$$= a^2+9b^4-6ab^2$$

प्रश्नावली 7.4

1. गुणा कीजिए :
   (i) $x^3(x^4)$
   (ii) $x^2(x^3)$
   (iii) $x^2y^3(x^3y^3)$
   (iv) $x^5y(y^3)$
   (v) $13(-4x^2y)$
   (vi) $(-18a)(4bx)$
   (vii) $(-6y)(-3y^2x)$
   (viii) $(3a^2b)(-4a^2b^3)$
   (ix) $(-3mn)(2m^2)(-n^2)$

(x) $(-3a^2b^2)(-6ab^4)(b^3)$

(xi) $(-2ab^2c)(3a^2bc)(-4ab^2c^2)(5abc^2)$

2. निम्न गुणनफल ज्ञात कीजिए :

(i) $-7x(2x-y)$

(ii) $ab(a^2-3b^2)$

(iii) $-b^2(2a^3-ab)$

(iv) $-8x^2y(4x-y^2)$

(v) $13x(x^2y+y^2x-3xy)$

(vi) $2b(a^2-2by+5y^2)$

(vii) $-4m^2n(3n+6mn-3)$

(viii) $-8x^3(2x^2-x^3-5+2x)$

3. निम्न गुणन कीजिए :

(i) $(3a-2)(a+5)$

(ii) $(m-3n)(2m+n)$

(iii) $(4a-3b)(3a+4b)$

(iv) $(2x^2+3)^2$

(v) $(2x^2-3)^2$

(vi) $(x^2+1)(x^2+2)$

4. निम्न व्यंजकों को सरल कीजिए :

(i) $2x(5y+2z)-3x(-y+2z)$

(ii) $2a(3a+7b)-6a^2-4ab$

(iii) $(a+b)^2-2ab$

(iv) $(a+b)^2-(a-b)^2$

(v) $(x+3y)(3x+y)-(3x^2+9xy-3y^2)$

(vi) $x^2+x(x+1)-x(x-1)$

(vii) $(a+b)^2-a^2$

(viii) $(a+b)^2-b^2$

(ix) $(a+2b)^2+a(a+b)-b(a+b)-ab$

7.5 व्यंजक का मान निकालना

इस एकक में कई स्थानों पर हमने कहा है कि अक्षर-संख्याएँ, संख्याएँ निरूपित करती है। अतः यदि कोई व्यंजक $2l+2b$ दिया हो तथा हमें l और b ज्ञात हों, तो हम $2l+2b$ का (संख्यात्मक) मान निकाल सकते हैं। यदि उपर्युक्त उदाहरण में l, 10 हो तथा b, 6 हो तो व्यंजक का मान $2(10)+2(6)$ अर्थात् 32 होगा। अक्षरों के स्थान पर उनके मान रखना **प्रतिस्थापन** (*substitution*) कहलाता है।

उदाहरण 1 : यदि $y = -2$ हो, तो $2y^3 - 3y^2 + y - 1$ का मान ज्ञात कीजिए।

हल : हम व्यंजक में $y = -2$, प्रतिस्थापित करते हैं जिससे हमें निम्न प्राप्त होता है :

$$2(-2)^3 - 3(-2)^2 + (-2) - 1$$
$$= 2(-2)(-2)(-2) - 3(4) - 2 - 1$$
$$= -16 - 12 - 2 - 1$$
$$= -31$$

प्रश्नावली 7.5

1. यदि $x = 1, y = 2$ तथा $z = -1$ हो तो निम्न व्यंजकों के मान ज्ञात कीजिए :

(i) $x^2 - y^2$ (ii) $z^2 - x^2$

(iii) $xy + yz - 2$ (iv) $2xy^2 - 3x^2y + z^2$

(v) $y^2 - z^2 + x^2$ (vi) $4x^3 + 2y^3$

(vii) $(z + x)^2 - 2y$ (viii) $(x^3 - y^2)(3y - 2z)$

एकक VIII

समीकरणों का परिचय

इस एकक में हम देखेंगे कि कुछ ऐसी समस्याओं को, जिनमें ज्ञात और अज्ञात संख्याओं के परस्पर संबंध निहित हों, समीकरण के पदों में दुबारा व्यक्त करने में हम किस प्रकार अक्षर संख्याओं का प्रयोग कर सकते हैं। समीकरण की एक तुला (*balance*) से तुलना की गई है। समीकरणों को हल करने के नियम निकालने के लिए तुला का प्रोत्साहन (*motivation*) के रूप में प्रयोग किया गया है।

8.1 अज्ञात राशियाँ व्यक्त करने में अक्षरों का प्रयोग

हम संख्याएँ व्यक्त करने में अक्षरों का प्रयोग पहले ही कर चुके हैं। यहाँ हम इनका कुछ भिन्न संदर्भ में प्रयोग करेंगे। आइए निम्न उदाहरण को देखें।

उदाहरण 1 : एक दी हुई संख्या और उसके दुगुने का योग 162 है। वह संख्या क्या है?

यहाँ दी हुई संख्या **अज्ञात** है। हम सबसे पहले कुछ संख्याएँ लेकर जाँच करेंगे। आइए, उदाहरणार्थ, संख्या 10 लें। क्या 10 दी हुई संख्या हो सकती है? नहीं। $10+2(10)$ क्या है? आइए एक दूसरी संख्या, उदाहरणार्थ, 18 लें। क्या 18 दी हुई संख्या हो सकती है? नहीं। (क्यों?) हम अब भी 162 से बहुत अधिक दूर हैं। व्यावहारिक ज्ञान कहता है कि हम कोई बड़ी संख्या, उदाहरणार्थ, 50 लेकर जाँच करें। क्या 50 दी हुई संख्या हो सकती है? पुनः नहीं।

यदि हम संख्या ज्ञात भी कर लें (54 लेकर जाँच कीजिए) फिर भी प्रयत्न और भूल (*trial and error*) की इस प्रक्रिया में समय अधिक लगता है और यह अनावश्यक भी है जैसा कि आप देखेंगे, हम संख्या को सरलता और शीघ्रता से ज्ञात करने के लिए बीजगणित के साधनों का प्रयोग कर सकते हैं।

आइए, अज्ञात राशि अर्थात् दी हुई संख्या को एक अक्षर संख्या, मान लीजिए, x से व्यक्त करें अब हम दी हुई समस्या को गणित की भाषा में परिवर्तित कर सकते हैं। हम कहते हैं कि हमें ऐसा **x ज्ञात करना है** जिससे कि

$$x+2x=162 \text{ या } 3x=162$$

उपर्युक्त **समीकरण** (*equation*) का एक उदाहरण है। x, समीकरण की **अज्ञात** (*unknown*) राशि कहलाती है। हम अज्ञात राशि को अक्षरों y या z या u, इत्यादि से भी व्यक्त कर सकते थे। इसके लिए हम अंग्रेजी वर्णमाला के बाद वाले अक्षरों को प्राथमिकता देते हैं।

हम देखते हैं कि

समानता का वह कथन जिसमें एक अज्ञात* राशि निहित हों प्रतिबन्धित समीकरण (*conditional equation*) या केवल **समीकरण** (*equation*) **कहलाता है।** एक समीकरण के दो पक्ष (*sides*) होते हैं। पहला वाम पक्ष (बाईं ओर वाला) तथा दूसरा, दक्षिण पक्ष (दाईं ओर वाला)। उपर्युक्त उदाहरण में हम देख सकते हैं कि $3x$ समीकरण $3x = 162$ का वाम पक्ष है तथा 162 दक्षिण पक्ष है। यदि किसी संख्या को समीकरण में अज्ञात के स्थान पर प्रतिस्थापित करने से वाम पक्ष और दक्षिण पक्ष बराबर हो जाएँ तो कहा जाता है कि वह संख्या समीकरण को संतुष्ट (*satisfy*) करती है। इस संख्या को समीकरण का एक **हल** (*solution*) या एक **मूल** (*root*) कहते हैं। समीकरण के मूल ज्ञात करने की विधि, समीकरण का **हल करना** (*solving*) कहलाती है।

गणितज्ञ, बहुत प्राचीन समय से ही ऐसी समस्याएँ जिन्हें समीकरण के रूप में परिवर्तित किया जा सकता है प्रस्तावित तथा हल करके अपना मनोविनोद करते चले आए हैं। उदाहरणार्थ, लगभग 1800 ई० पू० में अहम्स (*Ahmes*) नामक एक मिस्रवासी ने निम्न समस्या प्रस्तावित तथा हल की :

एक संख्या और उसका दो तिहाई और उसका आधा और उसका सातवां भाग 37 के बराबर है। संख्या ज्ञात कीजिए।

हिन्दू गणितज्ञों ने लगभग चौथी **शताब्दी** में विभिन्न प्रकार की समीकरणों में अज्ञातों का प्रयोग किया और उनको हल करने की विधियाँ ज्ञात कीं। हमें, उदाहरणार्थ, बारहवीं **शताब्दी** के एक हिन्दू गणितज्ञ भास्कराचार्य की कृति लीलावती में निम्न समस्या मिलती है।

"भँवरों के एक झुंड का पांचवा भाग खिले हुए कदम्ब पर जा बैठा, एक तिहाई शीलीन्ध्र के फूल पर और इन संख्याओं के अंतर का तिगुना कुटज के बौर की ओर उड़ गया। शेष बचा हुआ एक भँवरा, चमेली और केतकी की मनमोहक सुगन्ध से प्रलोभित होकर हवा में इधर उधर मंडराता रहा। हे सुमुखी! मुझे बताओ, भँवरों की कुल संख्या कितनी थी?"

अहम्स और भास्कराचार्य—दोनों की समस्याएं इस पुस्तक की सीमा के बाहर है। हम इनके बारे में बाद में पढ़ेंगे।

अज्ञात राशियों को व्यक्त करने के लिए संकेतों के प्रयोग का श्रेय प्राचीन हिन्दू गणितज्ञों को दिया गया है। उन्होंने अज्ञातों के कई नाम जैसे कि यावत् तावत् (इसका अर्थ है इतना कि), वर्ना, बिजा, इत्यादि रखे और उन्हें व्यक्त करने के लिए रंगों के पहले अक्षर जैसे कि का, नी, पी, पा, या, इत्यादि का प्रयोग किया। लगभग 300 ई० पू० में ये अक्षर तथा इनकी घात और मूल ज्ञात करने की विधियाँ बहुत प्रचलित थीं।

सत्रहवीं **शताब्दी** में ही, एक फ्रांसीसी गणितज्ञ डेकारट्स (*Descartes*) [1596–1650] ने सबसे पहले अज्ञातों को व्यक्त करने के लिए अक्षरों x, y, z, इत्यादि का और इनकी घातों के लिए संकेतों x^2, x^3 इत्यादि का प्रयोग किया।

---

*हम केवल एक अज्ञात राशि वाली समीकरणों का ही अध्ययन करेंगे।

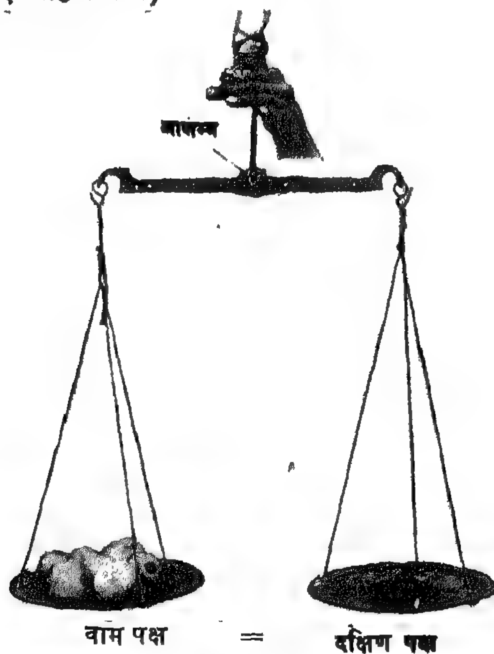
प्रश्नावली 8.1

1. प्रयत्न और भूल विधि से निम्न समीकरणों को हल कीजिए :

(i) $x+8=13$ (ii) $y-2=2$
(iii) $2m=6$ (iv) $10-x=6$
(v) $y+3=7$ (vi) $z-1=-3$
(vii) $2x=9-x$

8.2 समीकरण हल करना

समीकरण को एक तुला से तुलना की जा सकती है। इसके दोनों पक्षों (*sides*) को दो पलड़े माना जा सकता हैं तथा समानता का संकेत '=' यह दर्शाता है कि दोनों पलड़े संतुलन की स्थिति में हैं। (देखिए आकृति 8.1)



आकृति 8.1

निश्चय ही, आपने तुला देखी होगी। हम तुला की स्थिति में बिना कोई परिवर्तन किए उसके पलड़ों में क्या क्या कर सकते हैं? हम उसके दोनों पलड़ों में बराबर राशियाँ जोड़ (और इसलिए

गुणा कर) सकते हैं या हम उसके दोनों पलड़ों में से बराबर राशियां घटा (और इसलिए भाग दे) सकते हैं और ऐसा करने पर तुला की स्थिति में कोई परिवर्तन नहीं आएगा अर्थात् दोनों पलड़े अपनी पहली स्थिति में ही रहेंगे। ठीक यही हम समीकरणों में करने जा रहे हैं। दूसरे शब्दों में, हम

(1) समीकरण के दोनों पक्षों में एक ही संख्या जोड़ सकते है,

(2) समीकरण के दोनों पक्षों में से एक ही संख्या घटा सकते हैं,

(3) समीकरण के दोनों पक्षों को एक ही संख्या से गुणा कर सकते हैं, तथा

(4) समीकरण के दोनों पक्षों को एक ही संख्या से भाग दे सकते हैं।

अब हम इन नियमों का प्रयोग करेंगे और कुछ समीकरणों को हल करेंगे।

उदाहरण 1: $x-3=11$ को हल कीजिए।

हल : हम समीकरण के दोनों पक्षों में 3 जोड़ते हैं। (नियम 1)

इससे हमें निम्न प्राप्त होता है :

$x-3+3=11+3$

अर्थात्, $x=14$

[आइए $x=14$ को दी हुई समीकरण में प्रतिस्थापित करें। हम देखते हैं कि

वाम पक्ष$=14-3=11$

दक्षिण पक्ष$=11$

अर्थात् $x=14$ के लिए वाम पक्ष=दक्षिण पक्ष है।]

इस प्रकार, $x=14$, दी हुई समीकरण का एक मूल (*root*) या हल (*solution*) है।

उदाहरण 2: $2y=y+3$ को हल कीजिए।

हल : हम समीकरण के दोनों पक्षों में से y घटाते हैं। (नियम 2)

इस प्रकार, $2y-y=y+3-y$

अर्थात $y=3$

[अब आइए दी हुई समीकरण में $y=3$ प्रतिस्थापित करें। हम देखते हैं कि

वाम पक्ष$=2(3)=6$

दक्षिण पक्ष$=3+3=6$

अर्थात् $y=3$ के लिए वाम पक्ष=दक्षिण पक्ष है।]

इस प्रकार, $y=3$ दी हुई समीकरण का हल है।

उदाहरण 3: $10\div y=2$ को हल कीजिए।

हल : हम दी हुई समीकरण के दोनों पक्षों को y से गुणा करते हैं। (नियम 3)

इस प्रकार, $(10\div y)\times y=2y$

या $10=2y$

अब हमें $10=2y$ के दोनों पक्षों को 2 से भाग देना चाहिए। (नियम 4)

इस प्रकार, $10\div 2=(2y)\div 2$

अर्थात्, $y=5$

[आइए, $y=5$ को दी हुई समीकरण में प्रतिस्थापित करें। हम देखते हैं कि

वाम पक्ष$=10\div 5=2$

दक्षिण पक्ष $= 2$

अर्थात् $y = 5$ के लिए, वाम पक्ष = दक्षिण पक्ष है।]

इस प्रकार, $y = 5$, दी हुई समीकरण का हल है।

हम देखते हैं कि समीकरण हल करने में हम उपर्युक्त चारों नियमों में से एक या अधिक का प्रयोग करते हैं और ऐसे चरण तक आने का प्रयत्न करते हैं जिसमें **स्वयं अज्ञात संख्या समीकरण के एक पक्ष के रूप में प्रकट हो जाए।**

प्रश्नावली 8.2

निम्न में से प्रत्येक समीकरण हल कीजिए और समीकरण में प्रतिस्थापित करके अपने उत्तर की जाँच कीजिए।

1. $3x + 4 = 19$
2. $11x - 2 = 20$
3. $8y - 16 = 0$
4. $3x + 8 = 5x + 2$
5. $6y - 5 = 19$
6. $7 + 4y = -5$
7. $18 - 7x = -3$
8. $5y - 3 = 3y - 5$
9. $12 \div x = 6$
10. $100 \div z = 10$
11. $5y + 10 - 4y = -10$
12. $6m - 4 = 2m - 8$
13. $2y = 3(5 - y)$
14. $3(x - 3) = 5(2x + 1)$
15. $3x - 4 = 4 - (8 + 3x)$
16. $5x = 16(1 - 4x) + 4 + 17(2 + 3x)$
17. $6x - 9 - 2(1 - x) = x + 3$

8.3 समस्याएँ हल करने में समीकरणों का प्रयोग

अब हम देखेंगे कि ऐसी अनेक समस्याओं को जिनमें ज्ञात और अज्ञात संख्याओं के परस्पर संबंध निहित हों, पुनः समीकरणों के पदों में निरूपित किया जा सकता है। इन संबधों का अधिकतर शब्दों में वर्णन किया जाता है और इसी कारण हम इन समस्याओं को **शाब्द समस्याएं या प्रश्न** (*word problems*) कहते हैं। आइए कुछ उदाहरण लें।

उदाहरण 1 : दो संख्याओं का योग 52 है। इनमें से एक संख्या दूसरी से 10 अधिक है। संख्याएँ ज्ञात कीजिए।

हल : पहली दृष्टि में ऐसा लगता है कि इस प्रश्न में दो अज्ञातों की आवश्यकता है। परन्तु, जैसा कि हम देखेंगें कि, हम एक अज्ञात मानकर भी अपना लक्ष्य पूरा कर सकते हैं।

दूसरे वाक्य में कहा गया है कि एक संख्या, दूसरी से 10 अधिक है। इस प्रकार, यदि हम एक संख्या को x मानें तो दूसरी संख्या $x + 10$ होगी।

अब पहले वाक्य में कहा गया है कि इनका योग 52 है।

अतः $x+(x+10)=52$

इस प्रकार, हमने शाब्द प्रश्न को एक समीकरण के रूप में निरूपित कर दिया जिसे हम सरलता से हल कर सकते हैं। इस प्रकार,

$x+(x+10)=52$

या, $2x+10=52$

या, $2x+10-10=52-10$

या, $2x=42$

या, $x=21$

अतः दोनों संख्याएँ 21 और 31 है।

[हम देख सकते हैं कि इनका योग 52 है। साथ ही, 31, 21 से 10 अधिक है।]

उदाहरण 2: सुरेश के पिता की आयु, सुरेश की दो वर्ष पहले की आयु की तिगुनी है। आज उन दोनों की आयु का योग 62 है। प्रत्येक की आयु ज्ञात कीजिए।

हल: माना दो वर्ष पहले सुरेश की आयु x वर्ष थी। तब, आज

सुरेश के पिता की आयु $3x$ वर्ष है

तथा सुरेश की आयु $(x+2)$ वर्ष है।

उनकी आयु का योग 62 है। दूसरे शब्दों में,

$3x+(x+2)=62$

इस प्रकार हमने शाब्द प्रश्न को एक समीकरण के रूप में निरूपित कर दिया जिसे सरलता से हल किया जा सकता है। इस प्रकार,

$3x+(x+2)=62$

या, $4x+2=62$

या, $4x=60$

या, $x=15$

अतः सुरेश की आयु 17 वर्ष है तथा उसके पिता की 45 वर्ष।

[हम देख सकते हैं कि $45+17=62$। साथ ही, 45, 15 का, जो कि 2 वर्ष पहले सुरेश की आयु थी, तिगुना है।]

उदाहरण 3: एक आयत की लम्बाई उसकी चौड़ाई के दुगुने से 8 मीटर कम है। आयत का परिमाप 56 मीटर है। उसकी विमाएं (*dimensions*) अर्थात् लम्बाई और चौड़ाई ज्ञात कीजिए।

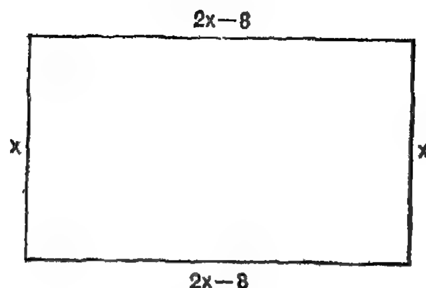
हल: चूंकि लम्बाई, चौड़ाई के पदों में दी हुई है, इसलिए हम अज्ञात चौड़ाई को x मीटर मान लेते हैं। (देखिये आकृति 8.2)

तब, लम्बाई $=(2x-8)$ मीटर (क्यों?)

अब, परिमाप $=(2x-8)+x+(2x-8)+x$

परन्तु, परिमाप 56 मीटर दिया है। अतः

$(2x-8)+x+(2x-8)+x=56$



आकृति 8.2

या, $6x-16=56$

या, $6x=72$

या, $x=12$

अतः, लम्बाई $=2(12)-8$ अर्थात् 16 मीटर

अतः आयत की विमाएं 16 मीटर और 12 मीटर है।

[हम देख सकते हैं कि 16 मीटर लम्बाई, चौड़ाई के दुगुने से 8 मीटर कम है। साथ ही परिमाप $=16+12+16+12$ अर्थात् 56 मीटर है।]

हम देखते हैं कि किस प्रकार तीन भिन्न प्रकारों के शाब्द प्रश्नों को समीकरणों के पदों में निरूपित करके हम उनका हल ज्ञात करने में समर्थ हो सके। इन प्रश्नों को समीकरणों के पदों में निरूपित करने की कोई निश्चित विधि नहीं है। परन्तु निम्न से कुछ उपयोगी संकेत अवश्य मिल सकते हैं :

1. प्रश्न को बार बार पढ़िए जब तक कि आप यह न समझ लें कि क्या दिया है और क्या ज्ञात करना है।
2. अज्ञात को किसी अक्षर x, या y या z इत्यादि से व्यक्त कीजिए।
3. (क) प्रश्न को धीरे धीरे, एक एक वाक्य अनुसार गणित की भाषा में परिवर्तित कीजिए।
   (ख) वह राशिएँ निर्धारित कीजिए जो कि बराबर हैं और उनसे एक समीकरण बनाइए।
4. अज्ञात के लिए, समीकरण को हल कीजिए।
5. जाँच कीजिए कि प्राप्त उत्तर प्रश्न में दिए हुए प्रतिबन्धों को संतुष्ट करता है या नहीं।

प्रश्नावली 8.3

1. किसी संख्या के दुगुने में 7 जोड़ने से 49 प्राप्त होता है। संख्या ज्ञात कीजिए।
2. किसी संख्या के तिगुने में से 22 घटाने पर 68 प्राप्त होता है। संख्या ज्ञात कीजिए।
3. ऐसी दो संख्याएँ ज्ञात कीजिए जिनमें से एक, दूसरी से 11 अधिक हो तथा इनका योग 53 हो।

4. ऐसी दो संख्याएँ ज्ञात कीजिए कि जिनमें से एक, दूसरी की तिगुनी है तथा इनका अंतर 28 है।

5. तीन क्रमागत पूर्णांकों का योग 24 है। पूर्णांक ज्ञात कीजिए।

6. एक आयत की लम्बाई, उसकी चौड़ाई से 4 मीटर अधिक है। आयत का परिमाप 84 मीटर है। उसकी विमाएं ज्ञात कीजिए।

7. एक आयत की लम्बाई, उसकी चौड़ाई की तिगुनी है। उसका परिमाप 96 मीटर है। उसकी लम्बाई ज्ञात कीजिए।

8. एक पर्स (*purse*) में जितने 25 पैसे के सिक्के हैं उससे दुगुने 10 पैसे के सिक्के हैं। यदि पर्स में इन दो प्रकार के सिक्कों में कुल 9.00 रु० हों तो प्रत्येक प्रकार के सिक्कों की संख्या ज्ञात कीजिए।

9 .3000.00 रु० की एक लाटरी (*lottery*) में कुल 63 इनाम दिए जाते हैं। एक इनाम या तो 100.00 रु० का है या 25.00 रु० का। प्रत्येक प्रकार के इनाम की संख्या ज्ञात कीजिए।

10. 15 वर्ष बाद, शीला की आयु उसकी वर्तमान आयु की चौगुनी हो जाएगी। उसकी वर्तमान आयु ज्ञात कीजिए।

विविध प्रश्नावली IV

(एकक VII और VIII पर)

1. जहाँ संभव हो वहाँ सरल कीजिए और बताइए कि निम्न में से कौन एक पदी, कौन द्विपद और कौन त्रिपद हैं ?

(i) -4

(ii) $3x^2-x+1$

(iii) xy^2-4y^2

(iv) $x^2-(x-y)$

(v) $x^2-(x^2+y)+4x$

2. निम्न में से प्रत्येक व्यंजक में x का गुणांक लिखिए :

(i) $(2-y)x$

(ii) y^2x-y

(iii) $7y^2-7xy$

*(iv) $4a^2x+2x$

3. निम्न को जोड़िए :

(i) $a+b-c,\ b+c-a,\ c+a-b$

(ii) $x^2-y^2-z^2,\ y^2-z^2-x^2,\ z^2-x^2-y^2$

(iii) $x-3xy,\ 3xy-y,\ y+1$

4. घटाइए :

(i) y^2x-x^2-z में से x^2-y^2x+z

(ii) $-a-b-c$ में से $a+b-c$

(iii) $-2x^2+4x+10$ में से $-2x+1$

5. दो द्विपदों का योग $4x-y$ है। यदि इनमें से एक $x-4y$ हो, तो दूसरा ज्ञात कीजिए।

6. सरल कीजिए :

(i) $3x^2-4xy+y^2-2x^2-2xy-4y^2-10x$

(ii) $xy^2-y^3+x^3+xy^2-4y^3-x^2-7$

7. एक मेज़ की लम्बाई, उसकी चौड़ाई की दुगुनी है। यदि चौड़ाई 'b' एकक हो, तो मेज़ की परिमाप ज्ञात कीजिए।

8. सरल कीजिए :

(i) $[(x^2-y^2)-(x^2+y^2)]-[-2x^2-(x^2-4y^2)]$

(ii) $[-a-(b-a)]-[-b-(a-b)]$

(iii) $[2b-a-b-\{2c-b+a-(a-b-c)\}]$

9. यदि $x=1$ और $y=2$ हो तो निम्न व्यंजकों के मान ज्ञात कीजिए :

(i) $x+y$

(ii) $x-3y+2$

(iii) $-4x+5y-7$

(iv) $x+4$

(v) $y-10$

10. यदि $x=0$ और $y=-1$ हो, तो निम्न के मान ज्ञात कीजिए :

(i) x^2-y+2

(ii) $x+y^2+8$

(iii) x^2+y^2

(iv) x^2y^2

(v) xy^3-x^2y+x

11. निम्न गुणन ज्ञात कीजिए :

(i) $x(x^5)$

(ii) $(x+1)x$

(iii) $(x-2)x$

(iv) $x^2(5-x)$

(v) $x^3(-1+x^2)$

12. सरल कीजिए :

(i) $(x-y)\ (x+y)$

(ii) $(x+y)\ (x+1)$

(iii) $(x-3y)\ (x-2y)$

(iv) $(y+2x)\ (y-x)$

13. निम्न में से प्रत्येक में x का गुणांक ज्ञात कीजिए :

(i) $(3x-2)-(4-x)$

(ii) $xy^2-(-3-3xy^2+4y^2)$

(iii) $[y-x-(x-y)]-[x-2y-(y-4x)]$

14. $3x-y+z$ और $-y-z$ के योग में से $3x-y-z$ घटाइए। इस परिणाम में x का गुणांक क्या है ?

15. गुणा कीजिए :

(i) $(a+b)$ और $(a+b)$, दिखाइए कि $(a+b)^2=a^2+2ab+b^2$

(ii) $(a-b)$ और $(a-b)$, दिखाइए कि $(a-b)^2=a^2-2ab+b^2$

16. एक तख्ते की लम्बाई और चौड़ाई क्रमश : $(a+b)$ और $(a-b)$ एकक है। उसका परिमाप ज्ञात कीजिए :

17. दो व्यंजकों का योग $x^2-y^2-2xy-2x+y-7$ है। यदि इनमें से एक $2x^2+3y^2-7y+1$ हो, तो दूसरा ज्ञात कीजिए।

18. यदि $a=1$, $b=0$ और $c=-1$ हो तो निम्न व्यंजकों के मान ज्ञात कीजिए :

(i) $(a^2-3ac+a-3)b-(a-b^2-2ab)$

(ii) $[2a^2-\{a^2-2c^2-b^2-(a+b+c)\}][c^2-2ab(b-a)]$

19. **(खेल के लिए एक प्रश्न)** यदि शब्द *mathematics* के अक्षर एक बीजीय व्यंजक के भाग हों तो हम इस शब्द को $m^2a^2t^2heics$ लिखते हैं। ऐसे संकेतन में,

(क) निम्न को किस प्रकार लिखेंगे ?

(i) *properties*

(ii) *commutativity*

(iii) *cancellation*

(ख) निम्न को कौन से (अर्थ पूर्ण) शब्दों को संक्षिप्त करने के लिए प्रयुक्त किया जा सकता है ?

(i) $p^2r^3o^3\ tin$ (ii) ad^2i^2ton (iii) sy^2m^2etr

20. निम्न समीकरणें हल कीजिए और दी हुई समीकरण में प्रतिस्थापित करके अपने उत्तर की जाँच कीजिए।

(i) $2x-7=19$ (ii) $3x+10=37$

(iii) $9x+1=-8$ (iv) $17-x=3$

(v) $2x+1=x-3$ (vi) $3x+2=-6-x$

(vii) $3x-10-2[4x+1-3(x+2)]=0$

(viii) $4x+2-2[4-x-3(2x-1)]=-x-(1-2x)+6$

(ix) $8-x=2-x+\{2x-4-(x-1)\}$

21. एक संख्या अपने तिगुने से 50 कम है। संख्या ज्ञात कीजिए।

*22. राम, श्याम और जीवन एक परीक्षा में बैठते हैं। राम ने श्याम द्वारा प्राप्त अंकों के तिगुने से 80 अंक कम प्राप्त किए जबकि जीवन ने श्याम द्वारा प्राप्त अंकों के दुगुने से 30 अंक अधिक प्राप्त किए। यदि तीनों व्यक्तियों द्वारा प्राप्त अंकों का योग 550 हो, तो उनके अलग अलग अंक ज्ञात कीजिए।

23. तीन दिन में 830 किलोग्राम सेब बेचे गए। दूसरे दिन, पहले दिन से 30 किलोग्राम कम बिक्री हुई तथा तीसरे दिन, दूसरे दिन से दुगुनी बिक्री हुई। पहले दिन कितने किलोग्राम सेब बेचे गए ?

24. एक संख्या, दूसरी संख्या से सात गुनी है। यदि इन संख्याओं का अंतर 516 हो, तो संख्याएँ ज्ञात कीजिए।

25. एक त्रिभुज ABC का परिमाप 94 सेमी है। AB, BC से 15 से मी छोटी है तथा AC, AB से 22 से मी लम्बी है। भुजा BC की लम्बाई ज्ञात कीजिए।

*26. एक हवाई जहाज़ की चाल, एक कार की चाल से 16 गुनी है। साथ ही, यदि कार की चाल, हवाई जहाज़ की चाल से 750 किलो मीटर प्रतिघंटा कम हो तो कार द्वारा 6 घंटों में चली हुई दूरी ज्ञात कीजिए।

एकक IX

अनुपात, प्रतिशतता और उनके अनुप्रयोग

इस एकक में हम अनुपातों (*ratios*) और प्रतिशतता (*percentages*) की संकल्पनाओं का उल्लेख करेंगे तथा इनके कुछ विषयों जैसे कि लाभ और हानि, साधारण ब्याज (*simple interest*) और अन्य दैनिक जीवन से संबंधित समस्याओं में अनुप्रयोग (*applications*) देखेंगे।

9.1 अनुपात

एक परीक्षा में बिन्दु ने 20 अंक प्राप्त किए तथा उसकी सहेली कमल ने 10 अंक प्राप्त किए। हम इन संख्याओं की दो प्रकार से तुलना कर सकते हैं।

(i) हम कह सकते हैं कि बिंदु ने कमल से (20–10) अर्थात् 10 अंक अधिक प्राप्त किए। यह विधि **अंतर द्वारा तुलना** (*comparison by difference*) कहलाती है। साथ ही, इसको यह भी कहा जा सकता है कि कमल ने बिन्दु से 10 अंक कम प्राप्त किए।

(ii) हम यह भी कह सकते हैं कि बिन्दु ने कमल से दुगुने अंक प्राप्त किए। यहाँ हम 20 को 10 से विभाजित करते हैं तथा इस विधि को **विभाजन द्वारा तुलना** (*comparison by division*) कहते हैं।

जब हम एक संख्या से दूसरी संख्या को विभाजित करके, दो संख्याओं की तुलना करते हैं तो हम कहते हैं कि हमने दोनों संख्याओं का एक **अनुपात** (*ratio*) बना लिया है। इस प्रकार 20 और 10 का एक अनुपात $20 \div 10$ है, 25 और 20 का एक अनुपात $25 \div 20$ है।

हम अक्षरों का संख्याएँ व्यक्त करने में पहले ही प्रयोग कर चुके हैं। हम कह सकते हैं कि

यदि a और b दो संख्याएँ हो तो a का b से अनुपात $a \div b$ है तथा इसे $a:b$ लिखा जाता है। a और b, अनुपात के पद (*terms of the ratio*) **कहलाते हैं।**

प्रत्यक्ष है कि a पहला पद है तथा b दूसरा पद है। तब, स्पष्टतया b का a से अनुपात $b \div a$ है तथा इसे $b:a$ लिखा जाता है। निस्संदेह हमें यह नहीं भूलना चाहिए कि हम किसी संख्या को शून्य से विभाजित नहीं कर सकते।

हम प्रायः **अनुपात को उसके सरलतम रूप** (*simplest form*) **में, अर्थात् उस रूप में जिसमें पदों में 1 को छोड़कर कोई अन्य गुणनखंड उभयनिष्ठ न हो, व्यक्त करते हैं।**

अतः हम, उदाहरणार्थ, 50 के 40 से अनुपात को 5 का 4 से अनुपात लिख सकते हैं तथा इसी प्रकार अनुपात 32:24 को सरलतम 4:3 लिख सकते हैं।

उदाहरण 1 : निम्न अनुपातों को सरलतम रूप में व्यक्त कीजिए :

(i) $25 \div 40$ (ii) $27 : 9$

हल : (i) $25 \div 40 = 25 : 40$

परन्तु अनुपात के पदों में 5 उभयनिष्ठ है।

अत: प्रत्येक पद को 5 से भाग देने पर, हमें निम्न प्राप्त होता है :

$25 : 40 = 5 : 8 = 5 \div 8$

(ii) $27 : 9$

अब, 9 दोनों पदों में उभयनिष्ठ है।

अत: $27 : 9 = 3 : 1$

यह स्पष्ट है कि 20 किलोग्राम और 5 किलोग्राम का अनुपात 4:1 है; 30 रु० और 60 रु० का अनुपात 1 : 2 है; परन्तु क्या इसका कोई अर्थ निकलेगा कि यदि हम, मान लीजिए, 40 पैसे और 2.00 रु० का अनुपात ज्ञात करें? नहीं! हम 2.00 रु० को 200 पैसे में बदलेंगे और फिर इनका अनुपात 40 : 200 या, सरलतम रूप में व्यक्त करते हुए, 1 : 5 ज्ञात करेंगे।

उदाहरण 2 : 3 घंटे का 75 मिनट से अनुपात ज्ञात कीजिए। इसे सरलतम रूप में व्यक्त कीजिए।

हल : 3 घंटे = 180 मिनट

अत:, वांछित अनुपात $= 180 : 75$

$= 12 : 5$

प्रश्नावली 9.1

1. निम्न को अनुपातों की भाषा में व्यक्त कीजिए :।

(i) चाय बनाने के लिए, 8 प्याले (*cups*) पानी के लिए, एक प्याला दूध की आवश्यकता है।

(ii) इस स्कूल में, चार कक्षाओं को पढ़ाने का कार्य, पाँच शिक्षकों को सौंपा गया है।

(iii) एक आयत की लम्बाई उसकी चौड़ाई से दुगुनी है।

2. निम्न अनुपातों को दैनिक जीवन की भाषा में व्यक्त कीजिए :

(i) भारत में, गाँवों की संख्या का शहरों की संख्या से अनुपात 2000 : 1 है।

(ii) आक्सीजन (*oxygen*) और हाइड्रोजन (*hydrogen*) को, आयतन के अनुसार (*by volume*), 1 : 2 के अनुपात में मिलाने से पानी बनता है।

(iii) एक परीक्षा में उत्तीर्ण हुए विद्यार्थियों की संख्या का परीक्षा में बैठने वाले विद्यार्थियों की कुल संख्या से अनुपात 4 : 5 है।

(iv) हमारे शहर में, ताँगों की संख्या का टैक्सियों (*taxies*) की संख्या से अनुपात 4 : 1 है।

3. निम्न में से प्रत्येक अनुपात को उसके सरलतम रूप में व्यक्त कीजिए :

(i) 4 : 12	(ii) 50:30	(iii) 340 : 510
(iv) 21 : 35	(v) 300 मी : 2 किमी	(vi) 75 सेमी : 1 मी
(vii) 3 किग्रा : 750 ग्रा	(viii) 75 पैसे : 3.00 रु०	(ix) 2 घंटे : 30 मिनट

4. एक खेत की लम्बाई और चौड़ाई क्रमश: 108 मीटर और 72 मीटर हैं। खेत की लम्बाई का उसकी चौड़ाई से अनुपात ज्ञात कीजिए।

5. किसी नक्शे (*map*) का स्केल प्राय: नक्शे के एक कोने पर वह अनुपात लिखकर जो कि 'नक्शे पर दूरी' का तदनुरूपी 'भूमि पर दूरी' से होता है, दर्शाया जाता है। यह अनुपात नक्शे का **प्रतिरूप भिन्न** (*Representative Fraction* या संक्षेप में R. F.) कहलाता है।

एक नक्शे का स्केल 1 : 100000 है। नक्शे पर 4 सेमी की दूरी वास्तव में कुल कितनी दूरी निरूपित करेगी? [**संकेत** : स्केल से हमें पता लगता है कि नक्शे पर 1 सेमी की दूरी वास्तव में 100000 सेमी की दूरी निरूपित करती है। आपको याद होगा कि 100 सेमी = 1 मी, 1000 मी = 1 किमी।]

6. एक फैक्टरी में बनी खराब पेंसिलों की संख्या का अच्छी पेंसिलों की संख्या से अनुपात 1 : 9 है। एक घंटे में एक मशीन 180 अच्छी पेंसिलें बनाती हैं। खराब पेंसिलों की संख्या ज्ञात कीजिए।

7. एक अम्ल (*acid*) को हल्का (*dilute*) करने के लिए विद्यार्थियों से कहा गया कि वे अम्ल और पानी को, आयतन के अनुसार, 2 : 5 के अनुपात में मिलाएँ। यदि कोई विद्यार्थी 10 घन सेंटीमीटर अम्ल लेता है तो उसे उसमें कितना पानी मिलाना चाहिए?

8. एक दिन किसी कक्षा में अनुपस्थित लड़कों की संख्या का उपस्थित लड़कों की संख्या से अनुपात 2 : 17 था। यदि कक्षा में 34 लड़के उपस्थित थे तो बताइए कि उस दिन कितने लड़के अनुपस्थित थे?

9. किसी स्कूल में खेलकूद (*sports*) में भाग ले रहे विद्यार्थियों की संख्या का कुल विद्यार्थियों की संख्या से अनुपात 5 : 16 है। यदि 250 विद्यार्थी खेलकूद में भाग ले रहे हों, तो स्कूल में कुल विद्यार्थियों की संख्या ज्ञात कीजिए।

9.2 समानुपात

मान लीजिए हम एक दुकान पर कोई कपड़ा खरीदने जाते हैं और मान लीजिए कपड़े का मूल्य 20.00 रुपये प्रति मीटर है। यदि हम 5 मीटर कपड़ा खरीदें तो हमें 100.00 रु० देने पड़ेंगे। यदि हम 8 मीटर कपड़ा खरीदें तो हमें 160.00 रु० देने पड़ेंगे। अब 5 मीटर का 8 मीटर से क्या अनुपात है? यह 5 : 8 है। और 100.00 रु० का 160.00 रु० से क्या अनुपात है? यह 100 : 160 है अर्थात् सरलतम रूप में व्यक्त करने पर यह 5 : 8 है। इस प्रकार हम देखते है कि

$$5 : 8 = 100 : 160$$

दो अनुपातों की ऐसी समिका (*equality*) एक समानुपात (*proportion*) कहलाती है। हम कहते हैं कि संख्याएँ 5, 8, 100 और 160 समानुपात में (*proportion*) हैं।

उदाहरण 1: क्या 30, 40, 45 और 60 समानुपात में है?

हल: हमें जांच करनी है कि क्या 30 : 40 = 45 : 60 है? अब 30 : 40, 3 : 4 के बराबर हैं क्योंकि इस अनुपात के पदों में 10 एक उभयनिष्ठ गुणनखंड है।

इसी प्रकार, 45 : 60 = 3 : 4

अतः, 30, 40, 45 और 60 समानुपात में हैं।

संख्याओं के स्थान पर अक्षरों का प्रयोग करते हुए हम कहते हैं कि

यदि $a : b = c : d$ है, तो a, b, c और d समानुपात में हैं। a, b, c और d **समानुपात के पद** (*terms of the proportion*) कहलाते हैं। ये क्रमशः पहले, दूसरे, तीसरे और चौथे पद हैं। प्रत्यक्ष है कि a और d **सिरों के पद** (*extreme terms*) हैं तथा b और c **मध्य के पद** (*middle terms*) हैं।

उदाहरण 2: जाँच कीजिए कि 75 : 60 = 20 : 16 तथा 60 : 16 = 75 : 20

हल: आइए प्रत्येक अनुपात को सरलतम रूप में परिवर्तित करें।

75 : 60 = 5 : 4। साथ ही, 20 : 16 = 5 : 4

अतः स्पष्ट है कि 75 : 60 = 20 : 16

अब, 60 : 16 = 15 : 4 तथा 75 : 20 = 15 : 4

इसलिए, पुनः 60 : 16 = 75 : 20

हम देखते हैं कि हम किसी समानुपात के पदों की स्थितियाँ इस प्रकार बदल सकते हैं कि हमें फिर बराबर अनुपात प्राप्त हो जाएँ। इस प्रकार हमें एक दूसरा समानुपात प्राप्त हो जाता है। वास्तव में, यदि कोई समानुपात दिया हुआ हो तो हम इस समानुपात के पदों से तीन और समानुपात बना सकते हैं।

उदाहरण 3: यदि $a : b = c : d$ हो तो **सिद्ध कीजिए कि**

$$a : c = b : d$$

हल: चूंकि $a : b = c : d$ इस लिए $a \div b = c \div d$

समानुपात के दोनों पक्षों को bd से गुणा करने पर,

$$(a \div b) \times bd = (c \div d) \times bd$$

अर्थात् $ad = cb$

अब इस समिका के दोनों पक्षों को dc से भाग देने पर,

$$ad \div (dc) = cb \div (dc)$$

अर्थात् $a \div c = b \div d$

दूसरे शब्दों में,

$$a : c = b : d$$

[उदाहरण 2 में हमने चार विशेष संख्याएँ लेकर जो जाँच की थी उसे इस उदाहरण में किन्हीं भी चार संख्याओं के लिए सिद्ध कर दिया गया है।]

अब आइए हम संख्याओं 3, 15, 15, 75 को लें। क्या ये समानुपात में हैं? हम देखते हैं कि 3 : 15 = 1 : 5, साथ ही 15 : 75 = 1 : 5

इस प्रकार वास्तव में 3 : 15=15 : 75 है अर्थात् 3, 15, 15, 75 समानुपात में हैं। हम देखते हैं कि मध्य पद 15 दुबारा आ रहा है। इस स्थिति में हम कहते हैं कि संख्याएँ 3, 15, 75 **समानुपात में हैं।** दूसरे शब्दों में,

यदि $a:b=b:c$ **हो तो हम कहते हैं कि** a, b, c **समानुपात में हैं।** प्रत्यक्ष है कि a और c **सिरों के पद** है तथा b **मध्य पद** है :

अब हम समानुपातों के एक रोचक गुण का अध्ययन करेंगे। आइए निम्न उदाहरणों को लें :

(i) 1, 2, 3, 6 समानुपात में हैं।
(ii) 5, 6, 20, 24 समानुपात में हैं।
(iii) 15, 25, 30, 50 समानुपात में हैं।

(i) में 1 और 6 सिरों के पद हैं तथा 2 और 3 मध्य पद हैं। (ii) में 5 और 24 सिरों के पद हैं तथा 6 और 20 मध्य पद हैं। (iii) में 15 और 50 सिरों के पद हैं तथा 25 और 30 मध्य पद हैं।

प्रत्येक उदाहरण में सिरों के पदों का गुणनफल क्या है? मध्य पदों का गुणनफल क्या है? हम देखते हैं कि **एक समानुपात में सिरों के पदों का गुणनफल, मध्य पदों के गुणनफल के बराबर होता है।** दूसरे शब्दों में

यदि $a:b=c:d$ **हो, तो** $ad=bc$

[3, 15, 75 समानुपात में हैं। इसमें सिरों के पदों का गुणनफल क्या है? मध्य पद का वर्ग क्या है?]

उदाहरण 4 : किसी समानुपात के पहले तीन पद क्रमशः 25, 10 और 15 हैं। चौथा पद ज्ञात कीजिए।

हल : आइए अज्ञात चौथे पद को x से व्यक्त करें। तब, 25, 10, 15, x, समानुपात में हैं।

अब हम पहले बताए गए गुण कि सिरों के पदों का गुणनफल मध्य पदों के गुणनफल के बराबर होता है, का प्रयोग करते हैं।

इस प्रकार, $25x=150$

यह एक समीकरण है जिसे सरलता से हल किया जा सकता है। हम दोनों पक्षों को 25 से भाग देते हैं और इस प्रकार हमें $x=6$ प्राप्त होता है।

अतः समानुपात में चौथा पद 6 है।

[विद्यार्थी को चाहिए कि वह जाँच करे कि 25, 10, 15, 6 वास्तव में समानुपात में हैं। दूसरे शब्दों में, यह कि क्या 25 : 10=15 : 6 है?]

उदाहरण 5 : यदि 12, x, 8, 14 समानुपात में हो तो x ज्ञात कीजिए।

हल : समानुपात के लिए, निश्चय ही,

$$12\times 14=x\times 8$$

या $$168=8x$$

दोनों पक्षों को 8 से भाग देने पर, हमें $x=21$ प्राप्त होता है। [जाँच कीजिए कि क्या 12 : 21=8 : 14 है?]

प्रश्नावली 9.2

1. जाँच कीजिए कि निम्न में से प्रत्येक में सिरों के पदों का गुणनफल, मध्य पदों के गुणनफल के बराबर हैं:

(i) $35 : 15 = 14 : 6$
(ii) $12 : 18 = 14 : 21$
(iii) $4 : 8 = 8 : 16$

2. क्या निम्न संख्याओं के समुच्चय समानुपात में हैं?

(i) 24, 45, 18, 30
(ii) 5, 6, 20, 18
(iii) 49, 45, 14, 10
(iv) 100, 150, 50, 75
(v) 33, 44, 66, 88
(vi) 150, 200, 250, 300

3. (क) क्या $12 : 18 = 14 : 21$ है?
(ख) क्या $63 : 45 = 28 : 20$ है?

4. निम्न समानुपातों के पदों का ही प्रयोग करते हुए प्रत्येक के लिए कम से कम एक अन्य समानुपात लिखिए:

(i) $30 : 45 = 16 : 24$
(ii) $15 : 45 = 45 : 135$

5. यदि 6, 18, x, 15 समानुपात में हों, तो x ज्ञात कीजिए।

6. किसी समानुपात के दूसरे, तीसरे और चौथे पद क्रमशः 44, 6 और 8 हैं। पहला पद ज्ञात कीजिए।

7. एक स्कूल के पुस्तकालय (*library*) में भौतिकी (*physics*) की पुस्तकों की संख्या का गणित (*mathematics*) की पुस्तकों की संख्या से वही अनुपात है जो गणित की पुस्तकों की संख्या का रसायन (*chemistry*) की पुस्तकों की संख्या से अनुपात है। यदि वहाँ गणित की 480 पुस्तकें और रसायन की 640 पुस्तकें हों, तो भौतिकी की पुस्तकों की संख्या ज्ञात कीजिए।

8. यदि $8 : 3 = 24 : x$ हो, तो x ज्ञात कीजिए।

9. यदि $y : 14 = 3 : (-2)$ हो, तो y ज्ञात कीजिए।

9.3 अनुक्रमानुपात

निश्चय ही आपने ऐसे कथन सुने होंगे कि 'जितना कोई व्यक्ति भारी होगा उतना ही अधिक उसको दिल का दौरा पड़ने की संभावना है', 'अधिक कमाओगे तो अधिक खर्च करोगे', 'जितना तुम अधिक कार्य करोगे उतनी ही अधिक देश की उन्नति होगी'। उपर्युक्त कथनों को निश्चित रूप से सदैव ठीक नहीं कहा जा सकता। परन्तु प्रत्येक कथन में दो राशियां संबद्ध हैं। ये हैं व्यक्ति का भार, दिल का दौरा पड़ने की संभावना; आय, व्यय; इत्यादि जो कि इस प्रकार आचरण करती हैं कि या तो ये साथ साथ बढ़ती हैं या साथ साथ घटती हैं।

आइए एक रेलगाड़ी पर विचार करें जो कि 50 किलोमीटर प्रतिघंटे की समान चाल से चली जा रही है। अब हम देखते हैं कि इस उदाहरण में दो राशियाँ समय और दूरी किस प्रकार संबंधित है। हमारे पास समय और दूरी की निम्न सारणी है :

घंटों में समय	1	2	3	4	5	6	7
किमी में दूरी	50	100	150	200	250	300	350

हम देखते हैं कि

(1) तय की गई किन्हीं दो दूरियों का अनुपात वही है, जो तदनुरूपी समयों का है।
उदाहरणार्थ, 50 : 100 = 1 : 2 या 50 : 250 = 1 : 5 या
150 : 350 = 3 : 7.

(2) तय किए गए किलोमीटरों की संख्या और घंटों की संख्या का अनुपात सदैव समान रहता है।

(3) दोनों राशियाँ, दूरी और समय, एक साथ बढ़ती या घटती हैं। दूसरे शब्दों में, ये एक एक साथ परिवर्तित होती हैं।

हम कहते हैं कि दोनों राशियाँ दूरी और समय **अनुक्रमानुपाती** (*vary directly*) हैं या यह कि ये **अनुक्रमानुपात** में (*indirect variation*) हैं। हम कभी कभी यह भी कहते हैं कि तय की गई दूरी समय के **अनुक्रमानुपाती** है या यह कि दूरी का समय से **अनुक्रमानुपात** (*direct proportion*) है। इस प्रकार, **दो राशियां x और y अनुक्रमानुपात में कहलाती हैं यदि x और y के तदनुरूपी मानों के लिए अनुपात $x : y$ एक ही रहे।** हम यह भी कहते हैं कि **x, y के अनुक्रमानुपाती है** या यह कि **x का y से अनुक्रमानुपात** है। हम इसे $x = Ky$ लिखते हैं जबकि यहाँ K **समानुपातकता स्थिरांक** (*constant of proportionality*) है। एक दिए प्रश्न के लिए, हम K का मान, प्रश्न में दिए हुए ब्यौरे से ज्ञात कर सकते हैं। जैसा कि नीचे उदाहरण 1 में दिखाया जा रहा है।

उदाहरण 1 : एक तार के टुकड़े से बहने वाली धारा (*current*), सिरों के विभावांतर (*potential difference*) के अनुक्रमानुपाती है। यदि विभावांतर 120 वोल्ट (*volts*) है तो धारा 6 ऐम्पियर (*amperes*) है। 10 ऐम्पियर की धारा के लिए कितने विभावांतर की आवश्यकता होगी ?

हल : माना c ऐम्पियर में धारा व्यक्त करता है तथा p, वोल्ट में विभवांतर। साथ ही, माना K समानुपातकता स्थिरांक है। तब, प्रश्न के पहले वाक्य के अनुसार $c = Kp$। दूसरे वाक्य में यह ब्यौरा किया है कि जब $p = 120$ वोल्ट है तो $c = 6$ ऐम्पियर है। इस ब्यौरे से हमें K ज्ञात करने में सहायता मिलती है। इस प्रकार,

$$6 = K \times 120$$

$$\text{या, } K = 6 \div 120$$

अतः अब c और p के संबंध को निम्न प्रकार लिखा जा सकता है :

$$c = (6 \div 120)p$$

हमें p ज्ञात करना है जबकि $c=10$ दिया हुआ है।
इस प्रकार, $10=(6\div 120)p$
या, $p=200$ वोल्ट

प्रश्नावली 9.3

1. नीचे दी हुई सारणी में हाइड्रोजन गैस से भरे एक गुब्बारे द्वारा हवा में चढ़ी गई ऊँचाइयाँ और इन ऊँचाइयों तक पहुँचने में लिए गए समय दिखाए गए हैं। यह मानते हुए कि ऊँचाई, समय के अनुक्रमानुपाती है, '*' के स्थानों पर उपयुक्त संख्याएं भरिए।

समय (मिनट में)	2	3	*	25	*
ऊँचाई (मीटर में)	*	36	84	*	1860

2. टेरीकोट के 3 मीटर कपड़े का मूल्य 78·00 रु० है। एक ऐसी सारणी बनाइए जिसमें से 4 मीटर, 7 मीटर, 15 मीटर और 20 मीटर कपड़े के मूल्य पढ़े जा सकें। मान लीजिए कि मूल्य, लम्बाई के अनुक्रमानुपाती हैं।

3. यदि 4 व्यक्तियों के एक परिवार में एक मास में 52 किलोग्राम अनाज की खपत (*consumption*) होती है, तो 3, 5, 6 और 10 सदस्यों के परिवार में क्रमशः अनाज की कितनी मात्रा की आवश्यकता होगी? मान लीजिए कि अनाज की खपत परिवार के साइज़ (*size*) के अनुक्रमानुपाती है।

4. x, y के अनुक्रमानुपाती है तथा जब $y=4$ हो तो $x=12$ हो जाता है। जब $y=-2$ हो तो x का मान ज्ञात कीजिए।

5. एक पर्यटकों (*tourists*) की कार 3 घंटे में 135 किलोमीटर की समान चाल से हैदराबाद से बम्बई (600 किमी की दूरी) की ओर जा रही है। कुल चली गई दूरी और लिए गए समय का अनुपात ज्ञात कीजिए। कार को 360 किलोमीटर की दूरी तय करने में कितना समय लगेगा? कार 2 घंटे में कितनी दूरी तय करेगी?

6. पुस्तकों को एक पंक्ति में व्यवस्थित करने पर ज्ञात हुआ कि 50 पुस्तकें एक 2 मीटर लम्बी शैल्फ (*shelf*) में आ पाती है। ऐसी 450 पुस्तकों रखने के लिए कितनी लम्बाई की आवश्यकता होगी?

7. किंग हैनरी अष्टम् (*8th*) के 6 पत्नियाँ थीं। हैनरी चतुर्थ (*4th*) के कितनी पत्नियां थीं? (पता लगाइए!)

9.4 व्युत्क्रमानुपात

पुन: आपने ऐसे कथन सुने होंगे जैसे कि 'जितनी चाल तेज होगी एक दी हुई दूरी तय करने में उतना ही कम समय लगेगा', 'जितना मूल्य ज्यादा होगा, उतनी ही माँग कम होगी', इत्यादि। इन सब कथनों को भी प्रत्येक स्थिति में निश्चित रूप से ठीक नहीं कहा जा सकता। परन्तु प्रत्येक कथन में दो राशियां संबद्ध हैं जैसे कि चाल, समय; मूल्य, माँग; इत्यादि। ये राशियां इस प्रकार आचरण करती हैं कि यदि एक बढ़ती है तो दूसरी घटती है या यदि एक घटती है तो दूसरी बढ़ती है।

मान लीजिए कि गोपाल के पास कुर्सियां खरीदने के लिए 300.00 रु० है। आइए देखें कि इस उदाहरण में दो राशियां कुर्सियों की संख्या और मूल्य किस प्रकार संबंधित हैं। हम मूल्यों और कुर्सियों की संख्या की निम्न सारणी बना सकते हैं :

एक कुर्सी का मूल्य (रुपयों में)	25	30	50	60	75	100
कुर्सियों की संख्या	12	10	6	5	4	3

हम देखते हैं कि जैसे जैसे एक कुर्सी का मूल्य बढ़ता है, गोपाल द्वारा कुर्सियां खरीद सकने की संख्या इस प्रकार घटती जाती है कि दोनों संख्याओं (कुर्सियों की संख्या और एक कुर्सी का रुपयों में मूल्य) का गुणनफल सदैव 300 अर्थात् एक ही रहता है।

हम कहते हैं कि ये राशियां **व्युत्क्रमानुपात में** (*in inverse proportion*) हैं या यह कि एक राशि, दूसरी के **व्युत्क्रमानुपाती** (*varies inversely*) है।

यदि दो राशियां x और y इस प्रकार संबंधित हों कि उनका गुणनफल सदैव समान रहे तो इनमें से प्रत्येक राशि, दूसरी के व्युत्क्रमानुपाती कही जाती है। हम इसे $xy=K$ लिखते हैं जबकि पुन: K **समानुपातकता स्थिरांक** है। एक दिए हुए प्रश्न के लिए हम K का मान प्रश्न में दिए हुए ब्यौरे से ज्ञात कर सकते हैं जैसा नीचे उदाहरण 1 में दिखाया गया है :

उदाहरण 1 : जब $y=2$ तो $x=5$ है तथा x, y के व्युत्क्रमानुपाती है। जब $x=1$ हो, तो y का मान ज्ञात कीजिए।

हल : x, y के व्युत्क्रमानुपाती है, अत:

$$xy=K$$

प्रश्न के ब्यौरे के अनुसार, जब $y=2$, तो $x=5$ इस प्रकार,

$$5(2)=K$$

अर्थात् $K=10$

अत: हम उपर्युक्त संबंध को

$xy=10$ लिख सकते हैं।

अब जब $x=1$ हो तो यह सरलता से ज्ञात किया जा सकता है कि $y=10$

प्रश्नावली 9.4

1. निम्न सारणियों में '*' के स्थान पर उपयुक्त संख्या भरिए जबकि प्रत्येक में x, y के व्युत्क्रमानुपाती है।

(i)

x	36	72	*	*
y	48	*	16	12

(ii)

x	50	75	*	150	*
y	300	*	150	*	75

2. x, y के व्युत्क्रमानुपाती है तथा जब $y=240$ तो $x=12$ है।

(i) जब $x=36$ हो तो y ज्ञात कीजिए।

(ii) जब $y=48$ हो तो x ज्ञात कीजिए।

3. यदि 40 किलोमीटर प्रति घंटा की चाल से एक नियत दूरी को तय करने में 7 घंटे लगते हैं तो उस दूरी को 5 घंटे में तय करने के लिए हमें किस चाल से चलना चाहिए?

4. एक विद्यार्थी अपने जेब खर्च में से 12 हास्य पुस्तिकाएं, जिनमें से प्रत्येक का मूल्य 2.00 रु० है, खरीदने के लिए पर्याप्त धन राशि बचा लेता है। अब वह इनके स्थान पर कुछ उपन्यास खरीदने का निश्चय करता है। यदि प्रत्येक उपन्यास का मूल्य 3.00 रु० हो तो वह कितने उपन्यास खरीद सकता है?

5. एक ठेकेदार यह अनुमान लगाता है कि वह किसी काम को 7 आदमियों की सहायता से 6 दिन में पूरा कर सकता है। बाद में उसे ज्ञात होता है कि उसे इस काम के लिए 3 आदमी ही मिल सकते हैं। अब उसे इस काम को पूरा करने के लिए कितने दिन लगेंगे?

6. 5 आदमी एक मेड़ अर्थात् चहारदीवारी 14 दिन में बना सकते हैं। यदि इस काम के लिए 7 आदमी नियुक्त किए गए हों तो ज्ञात कीजिए कि चहारदीवारी कितने दिन में बन जाएगी?

7. 6 नल एक टंकी को 2 घंटे में खाली कर सकते हैं। यह ज्ञात होता है कि इनमें से 4 नल खराब पड़े हैं। शेष बचे हुए 2 नलों को टंकी खाली करने में कितना समय लगेगा?

8. भौतिकी में हम पढ़ेंगे कि खींची गई डोरी की कम्पन आवृत्ति (*vibration frequency*) उसकी लम्बाई के व्युत्क्रमानुपाती है। 100 सेमी लम्बी एक डोरी को खींचने से उसमें 240 कम्पन प्रति सेकन्ड की आवृत्ति होती है। यह मानते हुए कि प्रत्येक दशा में डोरी को खींचने के लिए एक सा बल लगाया जाता है, ज्ञात कीजिए कि

(i) इसी प्रकार की 80 सेमी लम्बी डोरी की कितनी आवृत्ति होगी?

(ii) 400 कम्पन प्रति सेकन्ड की आवृत्ति के लिए इस प्रकार की कितनी लम्बी डोरी की आवश्यकता होगी?

9.5 प्रतिशतता

मनी ने अपनी अर्धवार्षिक परीक्षा में विभिन्न विषयों में निम्न अंक प्राप्त किए :

	अंग्रेजी	पंजाबी	गणित	भौतिक विज्ञान	सामाजिक विज्ञान
अधिकतम अंक	50	100	150	90	80
प्राप्तांक	35	82	90	72	76

क्या हम देखकर यह बता सकते हैं कि मनी ने किस विषय में सबसे अच्छे अंक प्राप्त किए तथा किस विषय में सबसे खराब अंक प्राप्त किए ? नहीं!

आइए प्रत्येक विषय में प्राप्तांक का अधिकतम अंक से अनुपात लिखें। अब हमें 7 : 10, 41 : 50, 3 : 5, 4 : 5 तथा 19 : 20 प्राप्त होता है। क्या अब देखकर मनी के विभिन्न विषयों के पारस्परिक कार्यों के बारे में कुछ कहना संभव है। पुनः, नहीं !

आइए अब प्रत्येक अनुपात को थोड़ा दूसरे रूप में लिखें। हम 7 : 10 को 70 : 100, 41 : 50 को 82 : 100, 3 : 5 को 60 : 100, 4 : 5 को 80 : 100, 19 : 20 को 95 : 100 लिखते हैं। यह देखकर ही सरलता से बताया जा सकता है कि मनी ने सामाजिक विज्ञान में सबसे अच्छा काम किया, गणित में सबसे खराब काम किया; तथा यह कि उसका दूसरा अच्छा कार्य पंजाबी में था।

अतः हम देखते हैं कि यदि कई अनुपातों का दूसरा पद 100 हो तो इनकी बड़ी सरलता से तुलना की जा सकती है। **वह अनुपात जिसका दूसरा पद 100 हो, प्रतिशतता कहलाता है। हम, उदाहरणार्थ, 70 : 100 को 70 प्रतिशत* (*per cent*) या 70% लिखते हैं।**

इस प्रकार 15% = 15 : 100 या 3 : 20।

उदाहरण 1 : निम्न प्रतिशतों को सरलतम रूप के अनुपातों में व्यक्त कीजिए :

(i) 48% (ii) 105%

हल : (i) 48% = 48 : 100 अर्थात् 12 : 25

(ii) 105% = 105 : 100 अर्थात् 21 : 20

उदाहरण 2 : निम्न का मान ज्ञात कीजिए :

(i) 50 रु० का 12% (ii) 10 मीटर का 30%

हल : (i) 50 रु० का 12% = (12 : 100) × 50 रु०
= (12 ÷ 100) × 50 रु० = 6 रु०

(ii) 10 मी का 30% = (30 ÷ 100) × 10 मी
= 3 मीटर

---

*शब्द 'पर सेंट' लेकिन भाषा के शब्द 'पर सेंटम' (*percentum*) का संक्षिप्त रूप है जिसका अर्थ है प्रति सैकड़ा। प्रतिशत व्यक्त करने के लिये संकेत '%' का प्रयोग किया जाता है।

प्रश्नावली 9.5

1. निम्न प्रतिशतों को सरलतम रूप के अनुपातों में व्यक्त कीजिए :
   (i) 20% (ii) 30% (iii) 24%
   (iv) 95% (v) 150%
2. निम्न का मान ज्ञात कीजिए :
   (i) 700 रु० का 20%
   (ii) 800 का 75%
   (iii) 200 गैलन का 35%
   (iv) 20 मीटर का 120%
   (v) 70 का 30%

3. अपने माल को निकालने के लिये एक दुकानदार 5% की छूट की घोषणा करता है। 40.00 रु० के माल पर कुल छूट ज्ञात कीजिए।

4. एक बीमा ऐजेन्ट जितना प्रारम्भिक प्रीमियम इकट्ठा करता है उस पर उसे 8% का कमीशन मिलता है। यदि उसने एक छः मास के काल में 4500 रु० इकट्ठा किए तो उसका कमीशन ज्ञात कीजिए।

5. एक स्कूल में सन् 1975 की अपेक्षा 1976 में विद्यार्थियों की संख्या 12% बढ़ गई। 1975 में स्कूल में कुल 1400 विद्यार्थी थे। बताइए कि 1976 में कितने विद्यार्थी हैं ?

*6. एक आदमी 25000 रु० छोड़कर मरता है। इसका 30% दान में तथा शेष उसके दो बच्चों में समान रूप से बांट दिया जाता है। दान में तथा उसके प्रत्येक बच्चे को दी जाने वाली धन राशियां ज्ञात कीजिए।

7. सन् 1971 की जनगणनानुसार, भारत के 2640 शहरों में से 5% की जनसंख्या कम से कम 1 लाख थी। ऐसे शहरों की संख्या ज्ञात कीजिए। 96

8. किसी वर्ष विशेष में वैज्ञानिक अनुसंधान (*scientific research*) के 300 करोड़ रुपयों के कुल बजट में से (लगभग) 36 करोड़ रुपये परमाणु ऊर्जा विभाग (*Atomic Energy Department*) को दिए गए। दी गई इस धन राशि को वैज्ञानिक अनुसंधान के कुल बजट के प्रतिशत के रूप में व्यक्त कीजिए।

9.6 लाभ और हानि

दुकानदार या तो सीधा निर्माता (*manufacturer*) से या फिर किसी ठोक विक्रेता (*wholesaler*) की मार्फत माल खरीदता है। वह इस माल का कुछ मूल्य देता है। इस मूल्य को उसका **क्रय मूल्य** (*cost price*) कहते हैं। फिर वह इस माल को ग्राहक को बेचता है। वह जिस मूल्य पर माल बेचता है वह उसका **विक्रय मूल्य** (*selling price*) कहलाता है।

यदि विक्रय मूल्य, क्रय मूल्य से अधिक हो तो दुकानदार को लाभ (*profit*) होता है, परन्तु यदि विक्रय मूल्य, क्रय मूल्य से कम हो तो दुकानदार को हानि (*loss*) होती है।

दूसरे शब्दों में,

लाभ = विक्रय मूल्य – क्रय मूल्य

हानि = क्रय मूल्य – विक्रय मूल्य

लाभ या हानि प्रायः क्रय मूल्य के प्रतिशत रूप में व्यक्त की जाती है। उदाहरणार्थ, यदि कोई दुकानदार 10.00 रु० में एक पुस्तक खरीदकर उसे 12.00 रु० में बेचे तो वह 10.00 रु० लगाकर 2.00 रु० का लाभ प्राप्त कर लेगा। अतः क्रय मूल्य के प्रतिशत के रूप में यह लाभ $(2 \div 10) \times 100\%$ अर्थात् 20% है। आइए कुछ और उदाहरण लें।

उदाहरण 1 : एक दुकानदार ने एक रेडियो सेट, जिसका क्रय मूल्य 500.00 रु० था, 550.00 रु० में बेचा। क्रय मूल्य के प्रतिशत के रूप में उसका लाभ ज्ञात कीजिए।

हल : लाभ = विक्रय मूल्य – क्रय मूल्य = 50.00 रु०

अतः उसने 500.00 रु० के क्रय मूल्य पर 50 रु० का लाभ प्राप्त किया।

क्रय मूल्य के प्रतिशत के रूप में व्यक्त करने पर, यह लाभ $(50 \div 500) \times 100\%$ अर्थात् 10% है।

दूसरे शब्दों में, दुकानदार ने 10% का लाभ प्राप्त किया।

उदाहरण 2 : एक पुस्तक विक्रेता ने एक पुस्तक की 400 प्रतियां 20% के लाभ पर बेचीं। यदि उसे एक पुस्तक की लागत 10.00 रु० आई हो, तो 400 प्रतियों का विक्रय मूल्य ज्ञात कीजिए।

हल : लाभ = 10.00 रु० का 20%

$= (20 \div 100) \times 10.00$ रु०

$= 2.00$ रु०

इस प्रकार, विक्रय मूल्य = 10.00 रु० + 2.00 रु० = 12.00 रु०

अतः 400 प्रतियों का विक्रय मूल्य = (12.00 रु०) × 400

= 4800.00 रु०

दूसरे शब्दों में, पुस्तक विक्रेता ने 400 प्रतियां 4800.00 रु० में बेची।

उदाहरण 3 : एक मेज 20% के लाभ पर 480.00 रु० में बेची गई। मेज का क्रय मूल्य ज्ञात कीजिए।

हल : माना रुपयों में मेज का क्रय मूल्य x है।

तब $480 = x + x$ का 20%

या, $480 = x + (20 \div 100) \times x$

अर्थात्, $480 = x + (1 \div 5)\, x$

दोनों पक्षों को 5 से गुणा करने पर,

$480 \times 5 = 5x + 5\,(1 \div 5)\, x$

या, $2400 = 5x + x = 6x$

इस प्रकार, $x = 400$

दूसरे शब्दों में, मेज का क्रय मूल्य 400.00 रु० था।

प्रश्नावली 9.6

1. एक फल विक्रेता ने एक फलों की टोकरी, जिसकी लागत 200.00 रु० थी, 5%के लाभ पर बेची। उसका विक्रय मूल्य ज्ञात कीजिए।

2. एक पुस्तक की 100 प्रतियाँ, जिनमें से प्रत्येक का मूल्य 8.00 रु० है, 864.00 रु० में बेची जाती हैं। उन पर लाभ या हानि ज्ञात कीजिए और उसे क्रय मूल्य के प्रतिशत के रूप में व्यक्त कीजिए।

3. एक व्यापारी ने एक पुरानी टाइप की मशीन 1000.00 रु० में खरीदी। उसने, उसकी मरम्मत पर 200.00 रु० व्यय किए और फिर उसे 10% के लाभ पर बेच दिया। टाइप की मशीन का विक्रय मूल्य ज्ञात कीजिए।
[**संकेत :** मरम्मत भी लागत का ही भाग है।]

4. एक कपड़ा विक्रेता ने 20 साड़ियाँ 1120.00 रु० में बेची और इस प्रकार 12% का लाभ प्राप्त किया। प्रत्येक साड़ी का क्रय मूल्य ज्ञात कीजिए।

5. एक दुकानदार ने कुछ माल 6% के लाभ पर 4240.00 रु० में बेचा। उसका कुल लाभ ज्ञात कीजिए।

6. कोई फर्नीचर 1320.00 रु० में बेचा गया। दुकानदार ने बताया कि उसने इस पर 10% का लाभ प्राप्त किया है। फर्नीचर का क्रय मूल्य कितना है?

*7. एक स्वर्णकार ने थोक विक्रेता से 100 ग्राम सोना 5400.00 रु० में खरीदा जिससे थोक विक्रेता को 8% का लाभ हुआ। फिर स्वर्णकार ने यह सोना 10% के लाभ पर बेच दिया। ज्ञात कीजिए :

(i) प्रति 10 ग्राम, स्वर्णकार का विक्रय मूल्य।

(ii) स्वर्णकार और थोक विक्रेता के क्रमशः प्रति 10 ग्राम विक्रय मूल्य और क्रय मूल्य का अंतर।

8. एक व्यक्ति ने एक भूमिखंड 15% के लाभ पर बेचा। यदि कुल लाभ 4500.00 रु० हो, तो भूमिखंड का विक्रय मूल्य ज्ञात कीजिए।

9.7 साधारण ब्याज

जब हम, मान लीजिए, किसी बैंक से रुपया उधार लेते हैं तो हम निर्दिष्ट अवधि के बाद न केवल उधार लिया हुआ रुपया ही वापिस करते हैं बल्कि बैंक के रूपयों का प्रयोग करने के बदले में कुछ **अतिरिक्त धन राशि** भी देते हैं। जो रुपया हम उधार लेते हैं वह **मूलधन** (*principal*) कहलाता है। जो अतिरिक्त धन राशि दी जाती है वह **ब्याज** (*interest*) कहलाती है और यह **एक वार्षिक दर प्रतिशत** (*rate per cent per annum*) जैसे कि 5% वार्षिक, के आधार पर दी जाती है। इसका अर्थ है कि एक वर्ष के लिए उधार लिए गए मूलधन 100 रु० पर ब्याज मूलधन का 5% अर्थात् 5.00 रु० होगा। निर्दिष्ट अवधि के बाद हम जो कुल रुपया वापिस करते हैं वह **मिश्रधन** (*amount*) कहलाता है। हम देखते हैं कि उपर्युक्त उदाहरण में मिश्रधन 105.00 रु० है। स्पष्ट है कि

मिश्रधन = मूलधन + ब्याज

अर्थात् $A = p + I$

अव, हम I किस प्रकार ज्ञात करते हैं ?

आइए (ब्याज की) दर को r से तथा वर्षों में समय (रुपया उधार लेनी की अवधि) को t से व्यक्त करें। तब, 1 वर्ष के लिए $I = pr$, अतः t वर्षों के लिए I, prt होगा। इस प्रकार, हमें I ज्ञात करने के लिए एक सूत्र (*formula*) $I = prt$ प्राप्त हो जाता है।

अव हम उपर्युक्त सूत्रों का कुछ उदाहरणों में प्रयोग करेंगे।

उदाहरण 1 : 1000.00 रु० का 5% की दर से 3 वर्ष का ब्याज ज्ञात कीजिए।

हल : यहाँ, $p = 1000.00$ रु०, $r = 5\% = (5 \div 100)$ $t = 3$, $I = ?$

इस प्रकार, $I = (1000) \times (5 \div 100) \times 3$ अर्थात् 150.00 रु०

उदाहरण 2 : 3000.00 रु० के मूलधन पर एक वर्ष का 240.00 रु० ब्याज दिया गया। ब्याज की दर ज्ञात कीजिए।

हल : यहाँ $p = 3000$ रु०, $I = 240$ रु०, $t = 1$, $r = ?$

माना ब्याज की दर r, $x\%$ वार्षिक है।

तब, $240 = 3000 \times (x \div 100)$

अर्थात् $x = 8$ (क्यों ?)

अतः $r = 8\%$

उदाहरण 3 : यदि 10% वार्षिक की दर से एक वर्ष का ब्याज 400.00 रु० हो, तो मूलधन ज्ञात कीजिए।

हल : यहाँ, $p = ?$ $I = 400.00$ रु०, $r = 10\% = (10 \div 100)$, $t = 1$

इस प्रकार, $400 = p(10 \div 100)$

अतः $p = 4000.00$ रु०

प्रश्नावली 9.7

1. 500.00 रु० का एक ऋण (*loan*) एक वर्ष बाद वापिस कर दिया जाता है। यदि ब्याज की दर 5% वार्षिक हो तो मिश्रधन ज्ञात कीजिए।

2. 6% वार्षिक ब्याज की दर से लगाई हुई 1200.00 रु० की धन राशि का वार्षिक ब्याज ज्ञात कीजिए। 5 वर्ष का कितना ब्याज होगा ?

3. किसी व्यक्ति ने एक स्कूल को 1500.00 रु० दान में दिए जिसके ब्याज से प्रतिवर्ष समान मूल्य के चार पुरस्कार दिये जाते हैं। यदि इस दान पर 12% वार्षिक ब्याज मिलता हो तो प्रत्येक पुरस्कार का मूल्य ज्ञात कीजिए।

4. 2000.00 रु० का एक ऋण 10%वार्षिक की दर से लिया जाता है। 3 वर्ष बाद कितने रुपये वापिस करने पड़ेंगे ?

[**संकेत :** पहले वार्षिक ब्याज ज्ञात कीजिए।]

5. एक कंपनी अपने यहाँ जमा धन राशि पर, जबकि यह जमा न्यूनतम 5 वर्ष के लिए हो, 14% ब्याज देती है। 2500.00 रु० की जमा धन राशि पर वार्षिक ब्याज ज्ञात कीजिए। यदि कोई व्यक्ति 2500.00 रु०, 7 वर्ष के लिए जमा कराए तो उसे कितना मिश्रधन मिलेगा ?

6. 3500.00 रु० के ऋण पर ब्याज दर ज्ञात कीजिए जबकि एक वर्ष बाद मिश्रधन 3850.00 रु० हो जाता है।

एकक—X

तल में आपतन गुण

10.1 भूमिका :

बिंदु (*points*), रेखाएँ (*lines*) और तल (*planes*) ज्यामिति के आधारभूत अवयव हैं। आप पिछली कक्षाओं में इनके विषय में पढ़ चुके हैं। इस एकक में हम इस प्रकार के प्रश्नों पर विचार करेंगे जैसे कि : एक दिए हुए बिंदु से होकर कितनी रेखाएं खींची जा सकती हैं? दो दिए हुए बिंदुओं से होकर कितनी रेखाएँ खींची जा सकती हैं? दो दी हुई रेखाओं में कितने बिंदु उभयनिष्ठ (*common*) हैं? इन प्रश्नों के उत्तर देने में हमें बिंदुओं और रेखाओं में जो सम्बन्ध प्राप्त होते हैं, **वे तल में आपतन गुण** (*Incidence properties in the plane*), कहलाते हैं।

आइए पहले तल, रेखाओं और बिंदुओं की कुछ महत्वपूर्ण विशेषताओं का पुनरावलोकन करें।

10.2 तल, रेखाएं और बिंदु

10.2.1 **तल की मुख्य कल्पना यह है कि यह एक सपाट (*flat*) सतह है जो सभी दिशाओं में असीमित रूप से विस्तृत है।** किसी मेज की ऊपरी सतह, कागज का सपाट पन्ना इत्यादि तलों के कुछ उदाहरण हैं। कमरे का फर्श, निश्चल अवस्था में तालाब में पानी की ऊपरी सतह आदि तलों के अन्य उदाहरण हैं। इनमें से प्रत्येक उदाहरण में यह कल्पना की जाती है कि सतह असीमित रूप से विस्तृत है। साथ ही, **तल की लम्बाई और चौड़ाई तो होती है परन्तु कोई मोटाई नहीं होती।** क्योंकि कोई भी तल असीमित रूप से विस्तृत होता है, अत: किसी आकृति में पूर्ण तल को दिखाना संभव नहीं है। और इसी कारण हम किसी तल को उसका एक सीमित भाग जैसे आयत या समांतर चतुर्भुज, आदि खींचकर दर्शाते हैं। तल को उस पर स्थित तीन या अधिक बिंदुओं द्वारा नामांकित किया जाता है जैसे कि ABC अथवा $ABCD$।

10.2.2 **रेखा में मुख्य कल्पना है उसका सीधापन (*straightness*) तथा यह कि उसका लम्बाई के अनुदिश दोनों दिशाओं में असीमित रूप से विस्तार है।**

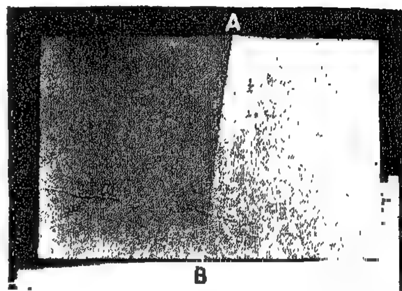
रेखा की केवल लम्बाई होती है परन्तु उसकी न तो कोई चौड़ाई होती है और न ही मोटाई। दो व्यक्तियों द्वारा सिरों से पकड़कर, कसकर खींची गई कोई पतली डोरी (देखिए आकृति 10.1) रेखा के एक (सीमित) भाग का उदाहरण है।





आकृति 10.1: रेखा के उदाहरण के रूप में तनी हुई डोरी

आइए, एक कागज़ का पन्ना लें और उसे मोड़कर दोनों भागों को परस्पर दबाएं। कागज़ पर मोड़ का निशान (*crease*) पड़ जाता है। यदि अब हम कागज़ को खोल दें तो हमें यह मोड़ का निशान दिखाई देता है।(देखिए आकृति 10.2) यह रेखा के भाग का एक अन्य उदाहरण है।



आकृति 10.2 रेखा का उदाहरण

चूंकि रेखा की लम्बाई असीमित है, हम इसे किसी आकृति में पूर्णतया नहीं दिखा सकते। इसलिए हम रेखा को इसका एक सीमित भाग खींचकर दर्शाते हैं। जैसा कि आकृति 10.3 में दिखाया गया है।



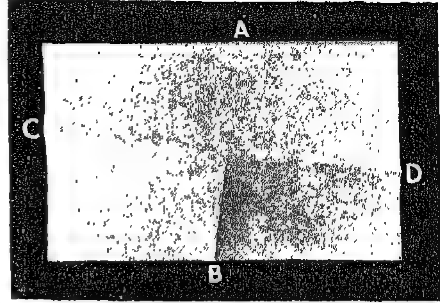
आकृति 10.3 रेखा AB

रेखा को उस पर कोई भी दो बिंदु लेकर या एक छोटे अक्षर द्वारा नामांकित किया जाता है जैसे कि AB, l या m

10.2.3 बिंदु में मुख्य कल्पना यह है कि न तो इसकी कोई लम्बाई होती है, न कोई चौड़ाई और न ही कोई मोटाई। बिंदु को एक बड़े अक्षर जैसे कि A या B या P इत्यादि द्वारा नामांकित किया जाता है किसी नुकीली पेंसिल से कागज पर बनाया गया चिन्ह, किसी पिन से कागज में किया गया छेद, आदि बिंदुओं के उदाहरण हैं।

यदि हम किसी कागज के पन्ने को मोड़कर दोनों भागों को परस्पर दबा दें तो एक मोड़ का निशान बन जाता है। मान लीजिए यह AB है।

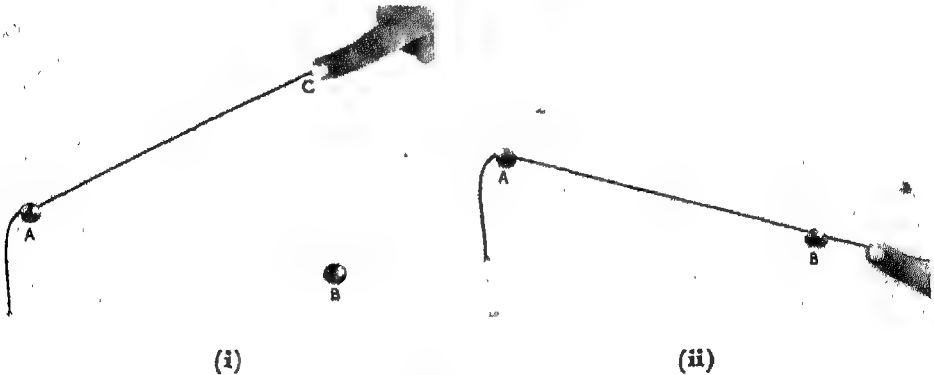
अब यदि हम पहले मोड़ के निशान की विपरीत अवस्था में एक दूसरा मोड़ का निशान बनाएँ तो दोनों मोड़ के निशान एक दूसरे को माना P पर काटते हैं (देखिए आकृति 10.4)। P, बिंदु का एक अन्य उदाहरण है।



आकृति 10.4 बिंदु का चित्र

10.3 पहला आपतन गुण

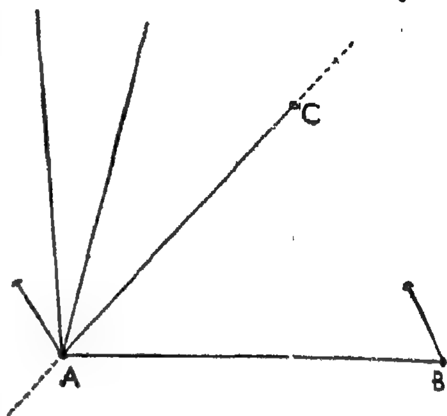
आइए, एक दीवार के तल में दो नुकली कीलें ठोकें अथवा एक ड्राइंग बोर्ड पर दो ड्राइंग पिन लगाएँ। माना इन कीलों (पिनों) के पाद A और B पर है। हम यह मान लेते हैं कि A और B दीवार अथवा ड्राइंगबोर्ड के तल में दो बिंदु निरूपित करते हैं। आइए, अब, एक पतली डोरी लें और उसके एक सिरे को A पर स्थित कील (अथवा पिन) से बाँध दें। अब हम डोरी को तल के जितने समीप संभव हो सके रखते हैं और किसी एक दिशा में खींचते हैं। माना यह दिशा AC है। [देखिए आकृति 10.5 (i)]



आकृति 10.5

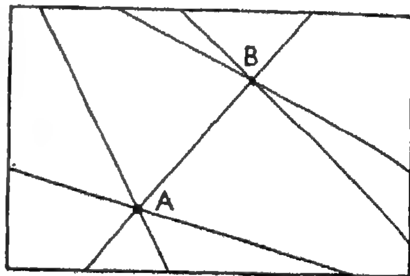
आइए,कल्पना करें कि डोरी का दोनों दिशाओं में असीमित विस्तार है जैसा कि आकृति 10.5 (iii) में बिंदुकित चिन्हों द्वारा दर्शाया गया है। तब यह एक रेखा निरूपित करती है जिसमें बिंदु A अंतर्निहित है अर्थात, जो बिंदु A से होकर जाती है। स्पष्ट है कि डोरी की ऐसी कितनी भी स्थितियां हो सकती हैं। अतः **हम बिंदु A से होकर कितनी भी रेखाएँ खींच सकते हैं।**

अब हम डोरी को तना हुआ रखते हुए उसे A पर स्थित कील के चारों ओर तब तक घुमाते हैं जब तक कि वह B पर स्थित कील को न स्पर्श कर ले देखिए आकृति 10.5 (ii)]। डोरी की ऐसी कितनी स्थितियाँ संभव हैं? स्पष्ट है कि केवल एक स्थिति संभव है। आइए डोरी को B पर बांधकर इस प्रयोग को दोहराएँ। हम डोरी को तना हुआ रखते हुए B के चारों ओर तब तक घुमाते हैं जब तक कि वह A को स्पर्श न कर ले। हमें पुनः डोरी की एक ही ऐसी स्थिति प्राप्त होती है। वास्तव में, A और B के बीच में डोरी का तना हुआ भाग दोनों दशाओं में एक ही स्थिति धारण करता है। यह दोनों बिंदुओं A और B से होकर जाने वाली एक रेखा निरूपित करता है। अतः हम अपने प्रयोग से देखते हैं कि **दो बिंदुओं से होकर एक और केवल एक ही रेखा खींची जा सकती है।**



आकृति 10.5 (iii)

पुनः आइए एक कागज के पन्ने पर उसके तल में दो बिंदु निरूपित करने के लिए दो चिन्ह A और B अंकित करें। आइए अब कागज को इस प्रकार मोड़ें कि मोड़ का निशान A से होकर जाए। ऐसे हम कितने मोड़ के निशान बना सकते हैं? स्पष्ट है हम जितने चाहें उतने मोड़ के निशान बना सकते हैं। इसी प्रकार, हम B से होकर भी जितने चाहें उतने मोड़ के निशान बना सकते हैं। इनमें से कितने मोड़ के निशान A और B दोनों से होकर जाते हैं? स्पष्ट है, एक और केवल एक (देखिए आकृति 10.6)। चूंकि हमने कागज में जितने भी मोड़ के निशान बनाएँ हैं वे सभी A या B से होकर जानेवाली रेखाएँ निरूपित करते है, अतः हम **पुनः देखते हैं कि दो बिंदुओं A और B से होकर एक और केवल एक ही रेखा खींची जा सकती है।**



आकृति 10.6

उपर्युक्त दोनों ही प्रयोगों में हम देखते हैं कि दो बिंदुओं से होकर जाने वाली रेखा उस तल में पूर्णतया स्थित है जिसमें कि दोनों बिंदु स्थित है।

संक्षेप में, हमने देखा कि किसी तल में स्थित दो भिन्न बिंदुओं से होकर ठीक (*exactly*) एक रेखा खींची जा सकती है। यह रेखा पूर्णतया तल में स्थित होती है। हम इसे आपतन का पहला गुण (*First Incidence Property*) कहते हैं।

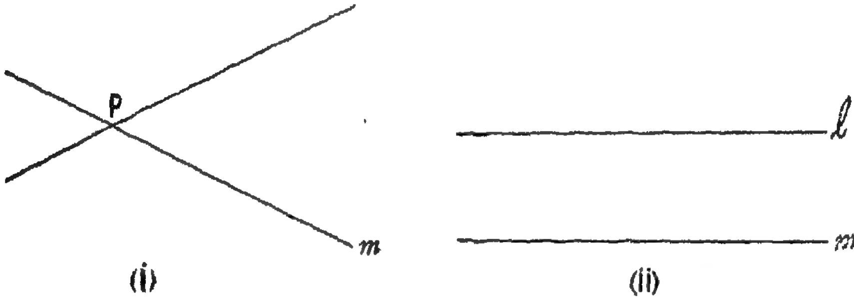
इसी गुण के कारण हम किसी रेखा को, जैसा कि हम अनुच्छेद 10.2.2 में पहले ही बता चुके हैं उस पर स्थित कोई भी दो बिंदु (उदाहरणार्थ, A और B) लेकर नामांकित कर सकते हैं।

व्यावहारिक रूप में हम दो बिंदुओं A और B से होकर रेखा निम्न प्रकार खींच सकते हैं; हम कागज़ पर दो बिंदु A और B अंकित कर लेते हैं और फिर कागज़ पर एक सीधे किनारे वाला रूलर (*straight edged ruler*) इस प्रकार रखते हैं कि दोनों चिन्ह उसके किनारे के अनुदिश स्थित हों। इसके बाद एक नुकीली पेंसिल को रूलर के किनारे के सहारे सहारे चलाकर हम कागज़ पर रेखा खींच लेते हैं। हम इस रेखा को AB से नामांकित कर सकते हैं।

10.4 दूसरा आपतन गुण

आइए, पुनः एक कागज का पन्ना लें और इसमें एक बार मोड़ का निशान बनाने के बाद कागज़ को खोलकर दूसरा मोड़ का निशान बनाएं और कागज को खोल लें। ये मोड़ के निशान कागज़ के तल में दो रेखाएं निरूपित करते हैं। मान लीजिए ये l और m हैं। आइए हम यह भी कल्पना करें कि इन मोड़ के निशानों का उनकी लम्बाइयों के अनुदिश दोनों दिशाओं में असीमित विस्तार हो सकता है। तब दो स्थितियां हो सकती हैं। या तो मोड़ के निशान

(i) परस्पर काटते हैं [देखिए आकृति 10.7 (i)] या
(ii) कभी नहीं काटते [देखिए आकृति 10.7 (ii)]



आकृति 10.7

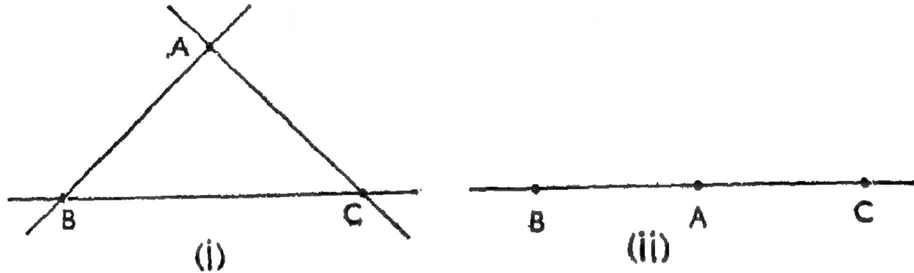
(i) में मान लीजिए कि मोड़ के निशान अर्थात् रेखाएं l और m बिंदु P पर काटते हैं। तब P, l और m दोनों में उभयनिष्ठ है। हम कहते हैं कि l और m एक बिंदु P पर **प्रतिच्छेद** (*intersect*) करती हैं। बिंदु P दोनों रेखाओं का **प्रतिच्छेद बिंदु** (*point of intersection*) कहलाता है।

(ii) में हम कहते हैं कि रेखाएँ **समांतर** (*parallel*) हैं। इस प्रकार, हम देखते हैं कि **किसी तल में दो रेखाएँ या तो ठीक एक बिंदु पर प्रतिच्छेद करती है या वे समांतर होती है।** हम इसे **आपतन का दूसरा गुण** (*Second Incidence property*) कहते हैं।

10.5 संरेखी बिंदु

हम यह देख चुके हैं कि यदि A किसी तल में स्थित कोई बिंदु हो तो तल में A से होकर जितनी चाहें उतनी रेखाएँ खींची जा सकती हैं। साथ ही, यदि A और B तल में स्थित कोई दो बिंदु हों तो (आपतन के पहले गुण द्वारा) A और B से होकर ठीक एक रेखा खींची जा सकती है और यह रेखा पूर्णतया तल में स्थित होती है।

अब यदि हमें तल में तीन बिंदु A, B और C दिए हों तो क्या होगा ? व्यापक रूप में अब तीन रेखाएँ होंगी। पहली, बिंदु युग्म B, C से होकर, दूसरी, बिंदु युग्म C, A से होकर और तीसरी, बिंदु युग्म A, B से होकर [देखिए आकृति 10.8(i)]। परन्तु यह भी हो सकता है कि बिंदु A, रेखा BC पर

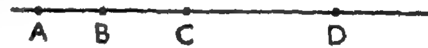


आकृति 10.8

स्थित हो जिससे कि तीनों रेखाओं की केवल एक ही रेखा रह जाए [देखिए आकृति 10.8 (ii)]। ऐसे तीनों बिंदु संरेखी (*collinear*) कहलाते हैं।

यदि एक ही तल में स्थित तीन या अधिक बिंदु एक ही रेखा पर स्थित हों तो वे संरेखी कहलाते हैं।

आकृति 10.9 में बिंदु A, B, C और D एक ही रेखा पर स्थित हैं। अतः ये संरेखी हैं।

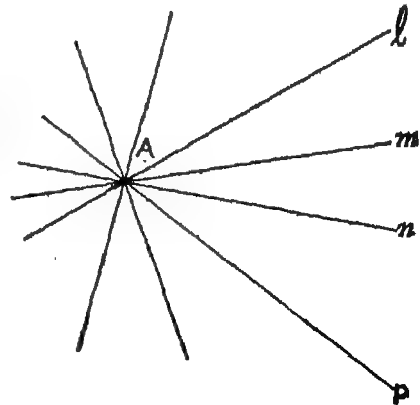


आकृति 10.9

10.6 संगामी रेखाएँ

आकृति 10.10 में रेखाएँ l, m, n, p,...एक ही बिंदु A से होकर जाती हैं। ऐसी रेखाएँ **संगामी रेखाएँ** (*concurrent lines*) कहलाती हैं और हम कहते हैं कि वे A **पर संगामी** (*concurrent at A*) हैं।

किसी तल में यदि तीन या अधिक रेखाएँ एक ही बिंदु से होकर जाएं तो वे संगामी कहलाती हैं।



आकृति 10.10

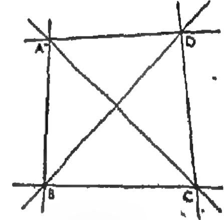
प्रश्नावली 10.1

1. एक दिए हुए बिंदु से होकर आप कितनी रेखाएँ खींच सकते हैं ?

2. दो दिए हुए बिंदुओं से होकर आप कितनी रेखाएँ खींच सकते हैं ?

3. आप तीन दिए हुए बिंदुओं में से दो दो बिंदु एक साथ लेकर कितनी रेखाएँ खींच सकते हो जब कि तीनों बिंदु (i) संगामी हैं, (ii) असंगामी (*non-collinear*) हैं।

4. किसी तल में A, B, C और D कोई चार बिंदु हैं। उनको दो दो के युग्मों में जोड़िए जैसा कि आकृति 10.11 में दिखाया गया है। ऐसी कितनी रेखाएँ हैं ? उनके नाम लिखिए।

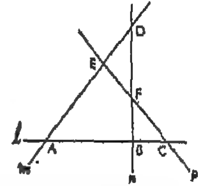


आकृति 10.11

5. आकृति 10.11 में रेखाएँ AB, AC तथा AD बिंदु A से होकर जाती हैं अर्थात् A पर संगामी हैं। उन रेखाओं के नाम बताइए जो (i) B, (ii) C, (iii) D पर संगामी हैं।

6. तीन रेखाओं के प्रतिच्छेद बिंदुओं की अधिकतम संख्या क्या है ?

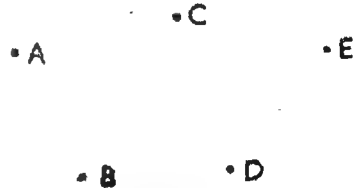
7. किसी तल में l, m n और p कोई चार रेखाएँ हैं। उनके प्रतिच्छेद बिंदु अंकित कीजिए जैसा कि आकृति 10.12 में दिखाया गया है। ऐसे कितने बिंदु हैं ? चार संख्याओं के प्रतिच्छेद बिंदुओं की अधिकतम संख्या क्या है ?



आकृति 10.12

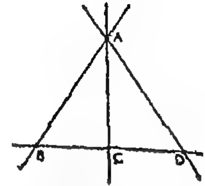
8. आकृति 10.12 में बिंदु A, B तथा C संरेखी है। अन्य संरेखी बिंदुओं के संग्रहों के नाम बताइए।

9. किसी तल में पाँच बिंदु A, B, C, D तथा E लीजिए जैसा कि आकृति 10.13 में दिखाया गया है। इनको दो दो के युग्मों में जोड़कर सभी रेखाएँ खींचिए। इन रेखाओं के नाम लिखिए और उनकी संख्या ज्ञात कीजिए।



आकृति 10.13

10. यदि प्रश्न 4 में चार बिंदुओं में से तीन बिंदु संरेखी हों तो उनको दो दो के युग्मों में जोड़ने से प्राप्त रेखाओं की संख्या चार रह जाएगी। (देखिए आकृति 10.14) यदि सभी चारों बिंदु संरेखी हों तो ऐसी रेखाओं की संख्या क्या होगी ?



आकृति 10.14

10.7 इतिहास सम्बन्धी एक टिप्पणी

10.7 तल में बिंदुओं और रेखाओं के उपर्युक्त आपतन गुणों एवं कुछ ऐसे ही और नियमों को लगभग 300 ई० पू० में महान यूनानी ज्यामितिविद **यूक्लिड** (*Euclid*) ने प्रतिपादित किया था। उन्होंने अपने समय से पूर्व विद्यमान सम्पूर्ण ज्यामितीय ज्ञान को संग्रहित किया। उनका यह कार्य **दी एलीमेंट्स** (*The Elements*) नामक तेरह ग्रन्थों में दिया हुआ है। यूक्लिड ने इन गुणों को **स्वयं सिद्ध प्रमाण** (*Postulates*) कहा अर्थात् इन नियमों की उसने प्रत्यक्ष सत्य के रूप में कल्पना की। इनका बाद का नाम **अभिग्रहीत** (*axioms*) है।

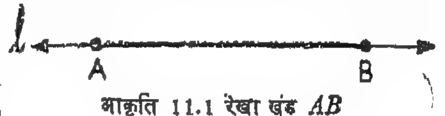
एकक XI

रेखाखंडों का मापन

11.1 रेखाखंड

हम देख चुके हैं कि रेखा को चूंकि वह दोनों दिशाओं में असीमित रूप से विस्तृत होती है, पूर्णतया आकृति में निरूपित नहीं किया जा सकता। अतः हम उसे उसके केवल एक भाग को ही खींचकर निरूपित करते हैं।

मान लीजिए l कोई रेखा है और A, B उस पर स्थित कोई दो बिंदु हैं। (देखिए आकृति 11.1) तब, A से लेकर B तक रेखा के भाग को रेखा l का खंड (*segment*) या केवल **रेखाखंड** (*line segment*) कहते हैं और इसे AB द्वारा व्यक्त किया जाता है। A और B इसके अंत बिंदु (*end points*) कहलाते हैं।



आकृति 11.1 रेखा खंड AB

मान लीजिए तल में कोई दो बिंदु P और Q हैं। (देखिए आकृति 11.2) तब P और Q से होकर केवल एक ही रेखा जा सकती है। (क्यों ?) P से लेकर Q तक इस रेखा का भाग P और Q को जोड़ने वाला रेखाखंड या केवल **रेखाखंड** PQ कहलाता है। चूंकि P और Q से होकर केवल एक ही रेखा जाती है अतः स्पष्ट है कि P और Q को जोड़ने वाला केवल एक ही रेखाखंड होगा। इस प्रकार,

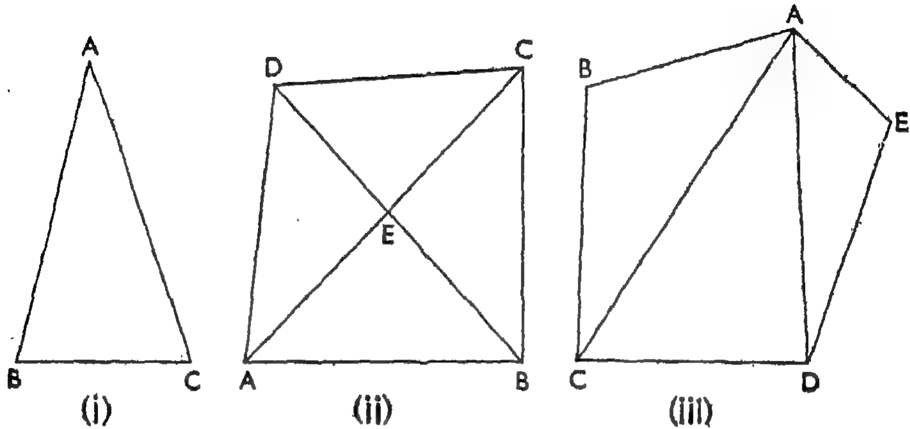


आकृति 11.2 रेखा खंड PQ

यदि किसी रेखाखंड के अंत बिंदु दिए हुए हों तो वह रेखाखंड पूर्णतया ज्ञात हो सकता है।

प्रश्नावली 11.1

1. निम्न आकृतियों में सभी रेखाखंडों के नाम लिखिए :



आकृति 11.3

प्रत्येक आकृति में कितने रेखाखंड हैं ?

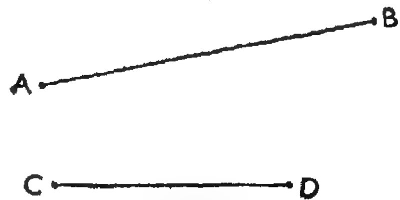
2. आकृति 11.4 में आप कितने रेखाखंड ज्ञात कर सकते हैं ?



आकृति 11.4

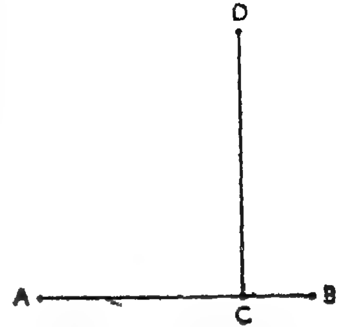
11.2 रेखाखंडों की तुलना

आकृति 11.5 में हमें दो रेखाखंड AB और CD दिए हुए हैं। इनमें कौनसा रेखाखंड अधिक लम्बा है ? आप तुरन्त कह सकते हैं कि AB, CD से लम्बा है।



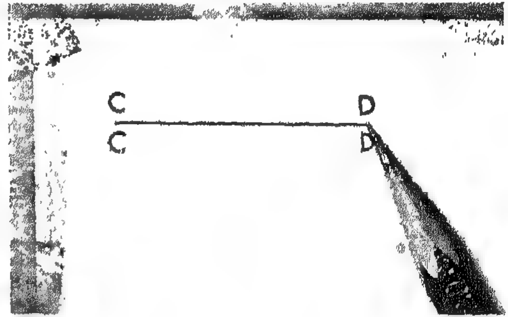
आकृति 11.5

आइए अब आकृति 11.6 को देखें। हमारे पास अब भी दो रेखाखंड AB और CD हैं। परन्तु ये कुछ भिन्न प्रकार से स्थित हैं। इनमें से कौनसा रेखाखंड लम्बा है ? आप शायद कहेंगे कि CD लम्बा रेखाखंड है चूँकि यह लम्बा **प्रतीत** होता है। परन्तु यह केवल हमारी दृष्टि का भ्रम ही है—सीधे खड़े रेखाखंड, क्षैतिज रेखाखंडों से लम्बे प्रतीत होते हैं। वास्तव में रेखाखंडों AB और CD में से कोई भी रेखाखंड एक दूसरे से लम्बा नहीं है। ये एक ही लम्बाई के हैं। फिर भी हमारी दृष्टि हमें धोखा दे सकती है। इसलिए हमें रेखाखंडों की तुलना करने के लिए कुछ अच्छी विधियों की आवश्यकता है। एक विधि इस प्रकार है :



आकृति 11.6

आइए एक अक्स खींचने का कागज (*tracing paper*) लें और उसे रेखाखंड CD के ऊपर रखें। आइए इस कागज पर रूलर और पेंसिल की सहायता से रेखाखंड CD का अक्स खींचे। अब हम रेखाखंड CD के इस अक्स को रेखाखंड AB पर इस प्रकार रखते हैं कि C, A पर गिरे और CD, AB के अनुदिश रहे। D को क्या होता है ? या तो D, A और B के बीच में गिरेगा [देखिए आकृति 11.7 (i)] या D, ठीक B पर गिरेगा [देखिए आकृति 11.7 (ii)] या D, B के आगे गिरेगा [देखिए आकृति 11.7 (iii)]। स्पष्ट है कि पहली स्थिति में हम कहेंगे कि AB, CD से **लम्बा** है या यह कि CD, AB से **छोटा** है। तीसरी स्थिति में हम कहते हैं कि AB, CD से छोटा है या यह कि CD, AB से लम्बा है। दूसरी स्थिति में हम कहते हैं कि AB और CD की **एक ही लम्बाई** है या यह कि रेखाखंड AB, रेखाखंड CD के **बराबर (सर्वांगसम)** है।





आकृति 11.7 (i)



आकृति 11.7 (ii)



आकृति 11.7 (iii)

संकेतन* में हम इन्हें निम्न प्रकार लिखते हैं:

स्थिति (i) में $AB > CD$ जिसे 'AB, CD से अधिक है' पढ़ा जाता है और इसका अर्थ है कि AB की लम्बाई, CD की लम्बाई से अधिक है।

स्थिति (ii) में $AB = CD$ जिसे 'AB, CD के बराबर है' पढ़ा जाता है और इसका अर्थ है कि AB की लम्बाई, CD की लम्बाई के बराबर है।

स्थिति (iii) में $AB < CD$ जिसे 'AB, CD से कम है' पढ़ा जाता है और इसका अर्थ है कि AB की लम्बाई, CD की लम्बाई से कम है।

रेखाखंडों के तुलना करने की एक अन्य विधि नीचे प्रश्नावली 11.2 के प्रश्न 1 में दी गई है।

---

*जब हम रेखाखंड और उसकी लम्बाई के लिए दो भिन्न संकेत प्रयोग करना चाहते हैं तो हम रेखाखंड AB को $\overline{AB}$ तथा उसकी लम्बाई को केवल AB लिखते हैं। परन्तु इस पुस्तक में हम रेखाखंड AB और साथ ही उसकी लम्बाई के लिए भी संकेत AB का ही प्रयोग करेंगे। यह संदर्भ से स्पष्ट हो जाएगा कि हमारा तात्पर्य रेखाखंड से है या उसकी लम्बाई से।

प्रश्नावली 11.2


आकृति 11.8
रेखाखंडों की तुलना

1. AB और CD दो रेखाखंड है। एक डिवाइडर (divider) लीजिए (आप परकार का भी प्रयोग कर सकते हैं)। डिवाइडर के एक पैर का नुकीला सिरा बिंदु C पर रखिए। अब डिवाइडर को सावधानीपूर्वक इस प्रकार खोलिए कि उसके दूसरे पैर का सिरा बिंदु D पर रहे। इस प्रकार दोनों पैरों के सिरे क्रमश: C और D पर रहेंगे। (देखिए आकृति 11.8) डिवाइडर को उठाइए और उसके फैलाव में बिना कुछ परिवर्तन किए उसे इस प्रकार रखिए कि एक पैर का सिरा A पर गिरे तथा दूसरे पैर को सिरा AB पर या AB को B के आगे बढ़ाने पर गिरे। आप रेखाखंडों AB और CD की लम्वाइयों के बारे में क्या कहसकते हैं जबकि दूसरे पैर का सिरा (i) A और B के बीच में हो ?, (ii) ठीक B पर हो ?, (iii) B के आगे हो ?



आकृति 11.9

2. एक कागज़ का पन्ना लीजिए और उसे मोड़िए। मान लीजिए मोड़ का निशान XY, रेखाखंड XY निरूपित करता है। कागज़ को अब इस प्रकार मोड़िए कि Y ठीक X पर पड़े। इससे बना दूसरा मोड़ का निशान RS मान लीजिए पहले मोड़ के निशान को Z पर काटता है। रेखाखंडों XZ और ZY की तुलना कीजिए। क्या ये बराबर हैं ?



आकृति 11.10

3. आकृति 11.10 में दो रेखाखंड a और b दिए हैं। क्या आप अनुमान लगा सकते हैं कि कौनसा रेखाखंड लम्बा है ? अपने डिवाइडर की सहायता से अपने उत्तर की जाँच कीजिए।

11.3 रेखाखंडों का मापन

हम रेखाखंडों की तुलना करने की दो विधियाँ देख चुके हैं। इससे भी एक अच्छी विधि यह होगी कि हम इन रेखाखंडों की एक निश्चित रेखाखंड की लम्बाई के पदों में लम्बाइयाँ मापें। इस निश्चित रेखाखंड की लम्बाई को हम मानक (*standard*) अर्थात् **मात्रक** (*unit*) मान लेते हैं। अभी लगभग पंद्रह वर्ष पहले तक हमारे देश में लम्बाई का मात्रक **फुट** (*foot*) था। यदि इसे 12 समान भागों में विभाजित कर दिया जाए तो प्रत्येक भाग एक **इंच** (*inch*) कहलाता है। परन्तु सन् 1962 में हमारे देश में यह मात्रक फुट से बदलकर **मीटर** (*metre*) कर दिया गया है। इस मात्रक को सबसे पहले सन् 1791 में फ्रांसीसियों ने फ्रांसीसी क्रान्ति (*French Revolution*) के बाद अपनाया था और इसे भूमध्यरेखा (*Equator*) और उत्तरी ध्रुव (*North Pole*) के बीच की दूरी के लगभग एक करोड़वें भाग के बराबर माना जाता था। आजकल, **पेरिस के निकट एक गुम्बज में रखी हुई प्लेटिनम** (*platinum*) **की छड़ पर लगे हुए दो निशानों के बीच की दूरी को मानक मीटर** (*standard metre*) **माना जाता है**। मीटर को 100 समान भागों में विभाजित करने पर प्रत्येक भाग एक **सेन्टीमीटर** (*centimetre*) कहलाता है। सेंटीमीटर को फिर और 10 भागों में विभाजित किया गया है और प्रत्येक भाग एक **मिलीमीटर** (*millimetre*) कहलाता है। इस प्रकार एक सेन्टीमीटर (सेमी), मीटर (मी) का सौवां भाग है तथा एक मिलीमीटर (मिमी), मीटर का हजारवाँ भाग है। दूसरे शब्दों में,

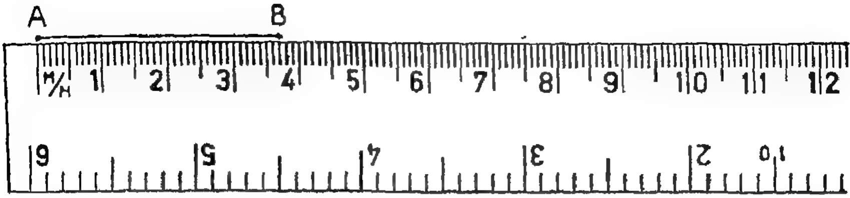
1 मी = 100 सेमी, 1 सेमी = 10 मिमी।

मीटर लम्बाई में तीन फुट से कुछ अधिक होता है तथा एक इंच, ढाई सेन्टीमीटर से कुछ अधिक होता है। हमारे ज्यामिति के कार्य के लिए मीटर काफी बड़ा है और इसलिए हम लम्बाई नापने के लिए सेन्टीमीटर और मिलीमीटर को ही प्राथमिकता देंगे। लम्बे रेखाखंड, उदाहरणार्थ, दो शहरों अथवा गाँवों के बीच की दूरी मापने के लिए हम किलोमीटर (संक्षेप में किमी) का प्रयोग करते हैं। यह 1000 मीटर के बराबर होता है।

कक्षा के अधिकांश कार्य के लिए सीधे किनारे वाले रूलर जिन पर सेन्टीमीटरों और इंचों दोनों के निशान लगे होते हैं उपलब्ध हैं। इन पर सेन्टीमीटर और इंच के उपभागों मिलीमीटर या इंच के दसवें भागों के भी निशान लगे होते हैं। ये प्रायः एक फुट या 6 इंच लम्बाई के होते हैं और इनके एक किनारे पर इंच के निशान तथा दूसरे किनारे पर सेन्टीमीटर के निशान बने होते हैं। वैज्ञानिक कार्य के लिए मीटर छड़ (*metre rods*) तथा अर्धमीटर छड़ (*half metre rods*) भी उपलब्ध हैं। रूलर पर लगे हुए निशान **अंशांकन** (*graduations*) कहलाते हैं तथा स्वयं रूलर, **अंशांकित रूलर** (*graduated ruler*) कहलाता है।

अब, मान लीजिए हमें एक दिए हुए रेखाखंड AB की लम्बाई नापनी है। आइए एक सेन्टीमीटर के निशान वाला रूलर लें और उसे रेखाखंड AB के अनुदिश इस प्रकार रखें कि उसका शून्य (0) का निशान A पर रहे जैसा कि आकृति 11.11 में दिखाया गया है।

इसके बाद हम रूलर पर B के तदनुरूपी निशान पढ़ते हैं। आकृति 11.11 में हम देखते हैं कि रूलर पर 3 के बाद सातवां लघु अंशांकन (*small graduation*), B के तदनुरूपी निशान है। दूसरे शब्दों में, रेखखंड AB में 3 पूर्ण सेन्टीमीटर तथा एक सेन्टीमीटर के सात दशांश (tenths) हैं।



आकृति 11.11: रेखाखंड का मापन

हम कह सकते हैं कि AB की लम्बाई 3 सेमी 7 मिमी है। इसे दशमलव संकेतन में हम 3.7 सेमी लिखते हैं।

प्राय: रूलर कुछ मिलीमीटर मोटा होता है। अत: हम देखेंगे कि रूलर पर लगे हुए निशान उस समतल में नहीं हैं जिसमें रेखाखंड AB है। इससे कभी कभी जब तक कि कोई बहुत अधिक साव-धान न रहे, A के सामने शून्य निशान रखने और B के तदनुरूपी निशान पढ़ने में कुछ त्रुटि हो जाती है। इस कठिनाई से बचने के लिए हम रेखाखंड मापने के लिए डिवाइडर (या परकार) का निम्न प्रकार प्रयोग कर सकते हैं।



(i)



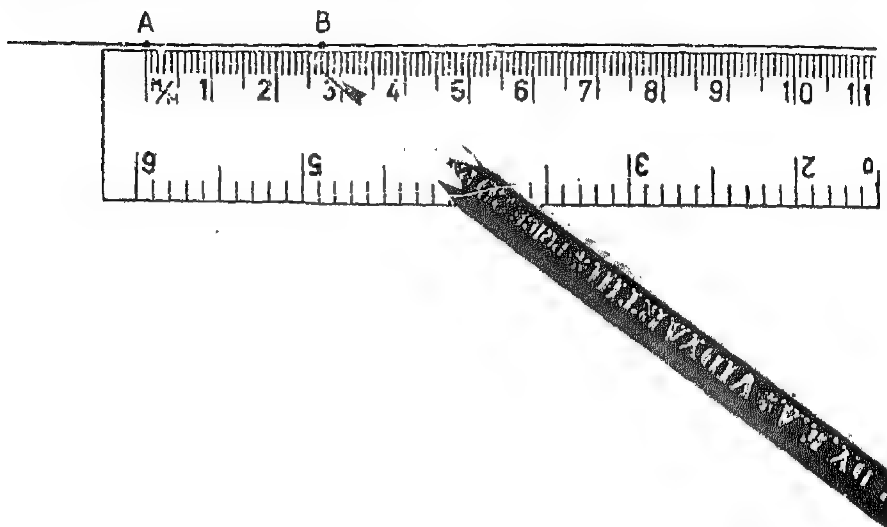
(ii)

आकृति 11.12: रेखाखंड का मापन

हम डिवाइडर को इतना खोलते हैं कि उसके एक पैर का सिरा A पर रहे तथा दूसरा ठीक B पर रहे। (देखिए आकृति 11.12) अब हम डिवाइडर को उठाते हैं और उसके फैलाव में बिना कोई परिवर्तन किए रूलर पर इस प्रकार रखते हैं कि एक पैर का सिरा शून्य निशान पर रहे। तब हम डिवाइडर के दूसरे पैर के सिरे के तदनुरूपी निशान पढ़ते हैं। आकृति 11.12 में दूसरा सिरा 3 सेमी के निशान के दाईं ओर 1 लघु निशान पर है। इस प्रकार दिए हुए रेखाखंड को लम्बाई 3 सेमी 1 मिमी अर्थात् 3.1 सेमी है।

11.4 दी हुई लम्बाई का रेखाखंड खींचना

मान लीजिए हमें 2.7 सेमी लम्बाई का रेखाखंड खींचना है। हम यह निम्न प्रकार करते हैं: हम एक रेखा खींचते हैं और उस पर कोई विंदु A ले लेते हैं। (देखिए आकृति 11.13) इसके बाद



आकृति 11.13

हम रेखा के अनुदिश एक रूलर इस प्रकार रखते हैं कि उसका शून्य का निशान A पर रहे। अब हम 2 सेमी के निशान के बाद सात छोटे भाग (*divisions*) गिन लेते हैं और रेखा पर इस निशान के तदनुरूपी बिंदु B अंकित कर लेते हैं। AB, 2.7 सेमी लम्बाई का वांछित रेखाखंड है।

यह सदैव आवश्यक नहीं है कि मापन शून्य चिन्ह से ही प्रारम्भ किया जाए। शून्य चिन्ह रूलर के एक सिरे पर होता है यदि यह सिरा टूट या घिस जाए तो हो सकता है कि इस चिन्ह का प्रयोग करना संभव न हो। तब हम अपना मापन किसी भी सेन्टीमीटर के निशान से प्रारम्भ कर सकते हैं। मान लीजिए हम रूलर इस प्रकार रखते हैं कि उसका 1 सेमी वाला निशान A के तदनुरूपी रहे। तब बिंदु B, 3 सेमी के निशान के बाद सातवें छोटे विभाग के तदनुरूपी लेना पड़ेगा। (क्यों?)

प्रश्नावली 11.3

1. निम्न को सेंटीमीटरों में बदलिए:
   (i) 3 मी, (ii) 2 मी 40 सेमी, (iii) 4.35 मी, (iv) 5.2 मी

2. निम्न को मिलीमीटरों में बदलिए :

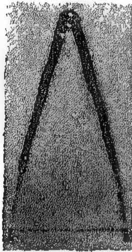
(i) 6 सेमी, (ii) 6.4 सेमी, (iii) 2 मी, (iv) 3 मी 40 सेमी, (v) 4.52 मी

3. निम्न लम्बाइयों के रेखाखंड खींचिए :

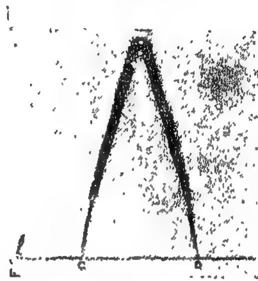
(i) 2 सेमी, (ii) 2 सेमी, 5 मिमी, (iii) 4.3 सेमी (iv) 3.4 सेमी, (v) 6.5 सेमी

4. रेखाखंड AB का एक अंत बिंदु A, रूलर के 1 सेंमी वाले निशान के साथ संपाती है तथा दूसरा अंत बिंदु B 4 सेमी वाले निशान के साथ। रेखाखंड AB की लम्बाई कितनी है ?

11.5 एक दी हुई रेखा से एक दिए हुए रेखाखंड की लम्बाई के बराबर रेखाखंड काटना :



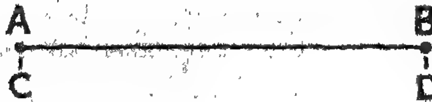
(i)



(ii)

आकृति 11.14

माना AB दिया हुआ रेखाखंड है और l दी हुई रेखा है। हम डिवाइडर को इतना खोलते हैं कि उसके दोनों पैरों के सिरे A और B पर रहें। [देखिए आकृति 11.14 (i)] तब हम डिवाइडर को उठाते हैं और उसके फैलाव में बिना कोई परिवर्तन किए उसे इस प्रकार रखते हैं कि दोनों पैरों के सिरे l पर रहें। हम इन दोनों के तदनुरूपी l पर बिंदु C और D अंकित कर लेते हैं। [देखिए आकृति 11.14 (ii)] तब रेखाखंड CD, रेखाखंड AB की लम्बाई के बराबर है। अर्थात् CD वांछित रेखाखंड है।



(i)

आकृति 11.15

हम इस कार्य के लिए कागज की एक पट्टी (*strip*) का भी प्रयोग कर सकते हैं। वास्तव में इसको कक्षा में गतिविधि हेतु एक सुझाव माना जा सकता है। आइए एक कागज को मोड़ें जिससे मोड़ का निशान एक सीधे किनारे (*straight edge*) का रूप धारण कर लेता है। अब इस किनारे को रेखाखंड AB के अनुदिश रखें। कागज के किनारे (*paper edge*) पर हम A और B के तदनुरूपी क्रमशः दो चिन्ह C और D अंकित करते हैं। [देखिए आकृति 11.15(i)] अब हम इस किनारे को रेखा l के अनुदिश रखते हैं। हम रेखा l पर चिन्ह C के तदनुरूपी बिंदु E तथा चिन्ह D के तदनुरूपी बिंदु F लेते हैं। [देखिए आकृति 11.15 (ii)] तब रेखाखंड EF की लम्बाई रेखाखंड AB की लम्बाई के बराबर है। अर्थात् EF वांछित रेखाखंड है।

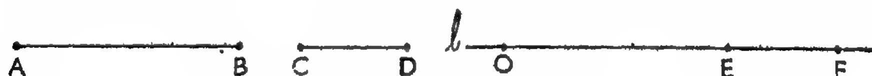


(ii)

आकृति 11.15

11.6 दो दिए हुए रेखाखंडों की लम्बाइयों के योग के बराबर लम्बाई का रेखाखंड खींचना।

मान लीजिए AB और CD दो रेखाखंड दिए हैं। आइए एक रेखा l लें और उस पर एक बिंदु O अंकित कर लें। हम l पर रेखाखंड AB की लम्बाई के बराबर एक रेखाखंड इस प्रकार खींचते हैं कि उसका एक अंत बिंदु O हो। मान लीजिए यह रेखाखंड OE है। अब हम l पर रेखाखंड CD



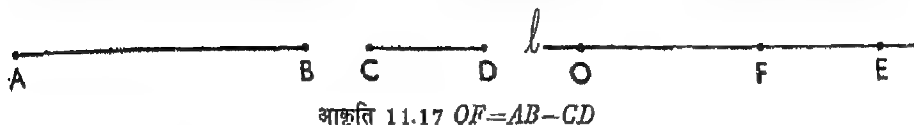
आकृति 11.16 $OF=AB+CD$

की लम्बाई के बराबर एक रेखाखंड EF इस प्रकार खींचते हैं कि E, O और F के मध्य स्थित हो। (देखिए आकृति 11.16) तब OF वांछित रेखाखंड है। (क्यों?) हम लिखते हैं कि $OF=AB+CD$।

11.7 दो दिए हुए रेखाखंडों की लम्बाइयों के अंतर के बराबर लम्बाई का रेखाखंड खींचना

मान लीजिए AB और CD दो दिए हुए रेखाखंड हैं। हम यह मान लेते हैं कि AB, CD से लम्बा है अर्थात् $AB > CD$। पहले ही की भांति हम एक रेखा l लेते हैं और उस पर एक बिंदु O अंकित कर लेते

हैं। हम l पर AB की लम्बाई के बराबर एक रेखाखंड OE खींचते हैं। इसके बाद हम l पर CD की लम्बाई के बराबर एक रेखाखंड EF इस प्रकार खींचते हैं कि F, O और E के मध्य स्थित हो।



आकृति 11.17 $OF=AB-CD$

(देखिए आकृति 11.17) OF वांछित रेखाखंड है। (क्यों ?) हम लिखते हैं कि $OF=AB-CD$।

यदि CD, AB से लम्बा हो तो हम $CD-AB$ की लम्बाई का रेखाखंड खींचते हैं।

प्रश्नावली 11.4

1. निम्न लम्बाई के रेखाखंड खींचिए :

   (i) 7.5 सेमी (ii) 3.4 सेमी (iii) 5.2 सेमी

2. अपनी कापी के एक पन्ने की सेन्टीमीटरों में लम्बाई व चौड़ाई नापिए।

3. 4.2 सेमी तथा 2.3 सेमी लम्बाइयों के दो रेखाखंड खींचिए। रूलर की सहायता से इन दोनों रेखाखंडों की लम्बाइयों के योग के बराबर लम्बाई का एक रेखाखंड खींचिए और उसकी लम्बाई मापिए।

4. AB एक रेखाखंड है। एक रेखाखंड खींचिए जिसकी लम्बाई AB की लम्बाई के दुगुने के बराबर हो। (यदि वांछित रेखाखंड CD हो तो हम लिखते हैं कि $CD=2AB$)

5. 12 सेमी लम्बाई का एक रेखाखंड लीजिए। इसमें से 4.5 सेमी लम्बाई का एक रेखाखंड इस प्रकार काटिए की इस रेखाखंड का एक अंत बिंदु वहीं रहे जो पहले रेखाखंड का है। बचे हुए रेखाखंड की लम्बाई मापिए।

6. अपनी कापी की लम्बाई और चौड़ाई मापिए। एक रेखाखंड खींचिए जिसकी लम्बाई इन लम्बाईयों के अंतर के बराबर हो।

7. दिया हुआ है कि $AB=3$ सेमी तथा $CD=2$ सेमी। निम्न रेखाखंड खींचिए :

   (i) $2AB$ (ii) $AB+CD$ (iii) $AB-CD$

   (iv) $2AB-CD$ (v) $3\,CD$

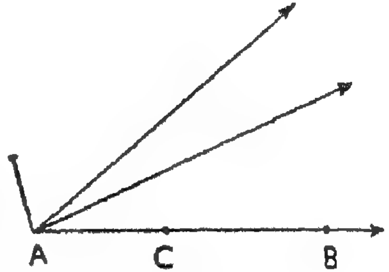
प्रत्येक स्थिति में आपने जो रेखाखंड खींचा है उसकी लम्बाई मापिए।

एकक XII

कोण

12.1 किरण

आइए, ड्राइंग बोर्ड के एक बिंदु A पर एक पिन लगाएँ। फिर एक डोरी लें और उसके एक सिरे को A पर लगी हुई पिन से बांध दें। डोरी को बोर्ड के जितना संभव हो सके समीप रखते हुए आइए अब उसके दूसरे सिरे B को खींचें जिससे कि डोरी तनी हुई रहे। डोरी इस स्थिति में एक रेखा का भाग AB निरूपित करती है। हम जानते हैं कि यदि हम यह कल्पना करें कि डोरी का दोनों दिशाओं अर्थात् A से B और B से A में असीमित विस्तार है तो यह एक रेखा निरूपित करती है। अब मान लीजिए कि डोरी केवल एक ही दिशा माना B से अर्थात् AB के अनुदिश विस्तृत है। तब हमें रेखा का केवल एक ऐसा भाग प्राप्त होता है जो A से प्रारम्भ होता है और जिसका AB दिशा में असीमित विस्तार है। हम इसे **किरण*** AB (*ray AB*) कहते हैं तथा A इसका **प्रारम्भिक बिंदु** (*initial point*) कहलाता है।



आकृति 12.1

मान लीजिए किरण AB पर C कोई अन्य बिंदु है। (देखिए आकृति 12.1) क्या किरण AC और किरण AB एक ही हैं? हां।

यदि हमें किरण का प्रारम्भिक बिंदु और उस पर स्थित कोई अन्य बिंदु ज्ञात हो तो किरण पूर्णतया ज्ञात हो जाती है।

उपर्युक्त प्रयोग में डोरी AB कितनी स्थितियाँ धारण कर सकती हैं? एक, दो या कितनी भी? **एक दिए हुए प्रारम्भिक बिंदु से चाहें जितनी किरणें खींची जा सकती हैं।**

---

*किरण AB को व्यक्त करने के लिए तथा इस बात पर जोर देने के लिए कि किरण का प्रारम्भिक बिंदु A है और उसकी दिशा A से B की ओर है संकेत $\overrightarrow{AB}$ का भी प्रयोग किया जाता है। इसी आधार पर रेखा AB के लिए संकेत $\overleftrightarrow{AB}$ का प्रयोग किया जाता है।

यद्यपि रेखा AB, किरण AB, रेखाखंड AB और रेखाखंड AB की लम्बाई व्यक्त करने के लिए क्रमश: चार भिन्न संकेत $\overleftrightarrow{AB}$, $\overrightarrow{AB}$, $\overline{AB}$ तथा AB प्रचलित है परन्तु हम वर्तमान स्तर पर इन्हें अनावश्यक अनुभव करते हैं। इसलिए इन चारों के लिए एक ही संकेत AB का प्रयोग करेंगे। यह संदर्भ से स्पष्ट हो जाएगा कि हमारा तात्पर्य चारों में से किससे है।

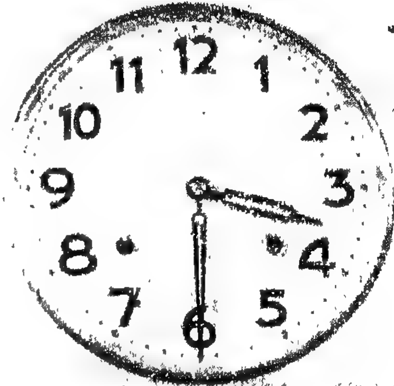
मान लीजिए l कोई रेखा है। आइए l पर तीन बिंदु O, A और B इस प्रकार लें कि A और B, O की विपरीत दिशाओं में रहें। (देखिए आकृति 12.2) हमें एक ही प्रारम्भिक बिंदु O वाली दो किरणें OA और OB प्राप्त होती हैं। ऐसी किरणें **विपरीत किरणें** (*opposite rays*) कहलाती हैं तथा दिशाएँ OA और OB **विपरीत दिशाएँ** (*opposite direction*) कहलाती हैं।



आकृति 12.2

12.2 कोण

आइए आकृति 12.3 में दी हुई घड़ी को देखें। घंटे की सुई 3 और 4 के बीच में है तथा मिनट की सुई 6 पर है। हम कहते हैं कि सुइयाँ एक दूसरे पर झुकी हुई हैं या यह कि सुइयाँ परस्पर एक कोण बनाती हैं।



आकृति 12.3

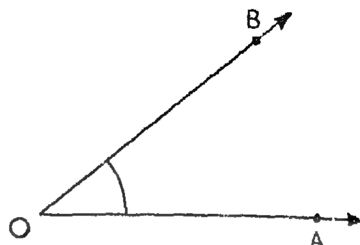
पुनः एक डिवाइडर लें और उसे कागज़ के तल में इस प्रकार रखें कि उसके दोनों पैर सटे हुए रहें। उसके एक पैर को OA पर स्थिर किए हुए, आइए अब डिवाइडर को खोलें जिससे कि दूसरा पैर जोड़ O पर लगे हुए कब्जे (*hinge*) के चारों ओर घूमने (*rotate*) लगता है। (देखिए आकृति 12.4) मान लीजिए अब दूसरा पैर OB पर है। हम कह सकते हैं कि OA और OB एक कोण बनाते हैं। यदि पैर OB को और अधिक घुमाएँ तो कोण का क्या होता है?



आकृति 12.4

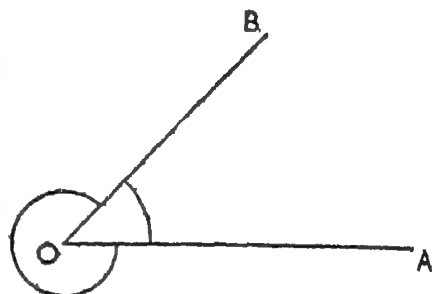
यदि घड़ी की सुइयों या डिवाइडर के पैरों को किरणों OA और OB का निरूपण माने तो हम कहते हैं कि दोनों किरणें एक कोण बनाती हैं। साथ ही, चूँकि दोनों किरणों के बीच का झुकाव बढ़ाया या घटाया जा सकता है अतः हम देखते हैं कि कोण का एक परिमाण (*magnitude*) होता है। किरण OB को उसकी प्रारम्भिक स्थिति OA से अंतिम स्थिति तक लाने में आवश्यक कुल घूर्णन (*rotation*) की मात्रा से इस परिमाण को मापा जा सकता है।

एक ही प्रारम्भिक बिंदु O से निकलने वाली किरणों OA और OB के लिए यह कहा जाता है कि वे एक कोण बनाती हैं। (देखिए आकृति 12.5) बिंदु O इसका शीर्ष (*vertex*) तथा किरणें OA और OB भुजाएँ (*arms*) कहलाती हैं। एक भुजा को शीर्ष के चारों ओर घुमाकर दूसरी भुजा की स्थिति में लाने के लिए आवश्यक घूर्णन की मात्रा से इस कोण का **परिमाण** मापा जाता है। प्रायः कोण व्यक्त करने के लिए दोनों भुजाओं को एक वृतीय चाप से जोड़ देते हैं जैसा कि आकृति 12.5 में दिखाया गया है।



आकृति 12.5

दो किरणें OA और OB वास्तव में दो कोण बनाती हैं जैसा कि आकृति 12.6 में दर्शाया गया है। इनमें से किसी को भी कोण AOB से व्यक्त किया जा सकता है। फिर भी, इनमें से एक कोण दूसरे से बड़ा है। संदिग्ध स्थिति से बचने के लिए हम यह मान लेते हैं कि जब तक कि कुछ कहा न जाए, किरणों OA और OB से बनने वाले दोनों कोणों में से छोटा कोण, कोण AOB होगा।

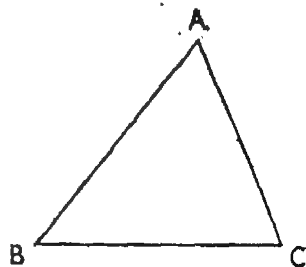


आकृति 12.6

हम कोण व्यक्त करने के लिए संकेत '$\angle$' का प्रयोग करते हैं और 'कोण AOB' या 'कोण BOA' के लिए क्रमशः $\angle AOB$ या $\angle BOA$ लिखते हैं। कोण का नाम लिखते समय शीर्ष सदैव मध्य में लिखा जाता है। कभी कभी हम कोण को केवल उसके शीर्ष से ही व्यक्त करते हैं जैसे कि $\angle O$ या कोण O।

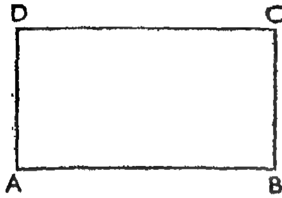
प्रश्नावली 12.1

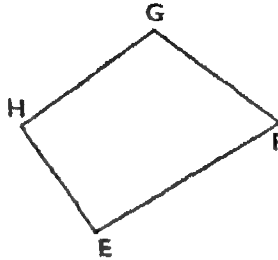
1. आकृति 12.7 में तीन कोण दिए हैं। इनमें से एक $\angle BAC$ या केवल $\angle A$ है। क्या आप अन्य दोनों कोणों के नाम बता सकते हैं?

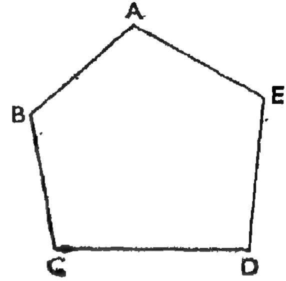


आकृति 12.7

2. आकृति 12.8 में दिए हुए प्रत्येक बहुभुज के कोणों के नाम बताइए।

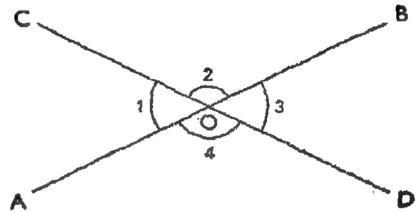






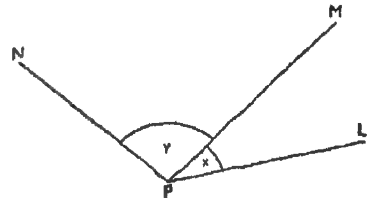
आकृति 12.8

3. कभी कभी कोणों को व्यक्त करने के लिए संख्यांक या अक्षरों का प्रयोग सुविधाजनक रहता है। आकृति 12.9 में $\angle AOC$ को $\angle 1$ नाम दिया गया है। क्या आप 2, 3 तथा 4 द्वारा अंकित कोणों के पूरे नाम बता सकते हैं ?



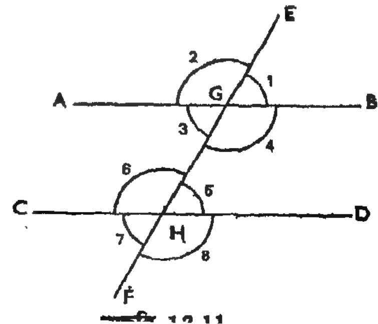
आकृति 12.9

4. आकृति 12.10 में $\angle x$, $\angle LPM$ है। y द्वारा अंकित कोण का पूरा नाम बताइए।



आकृति 12.10

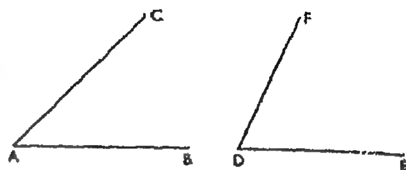
5. आकृति 12. 11 में 1, 2, 3,...8 द्वारा अंकित कोणों के पूरे नाम बताइए।



आकृति 12.11

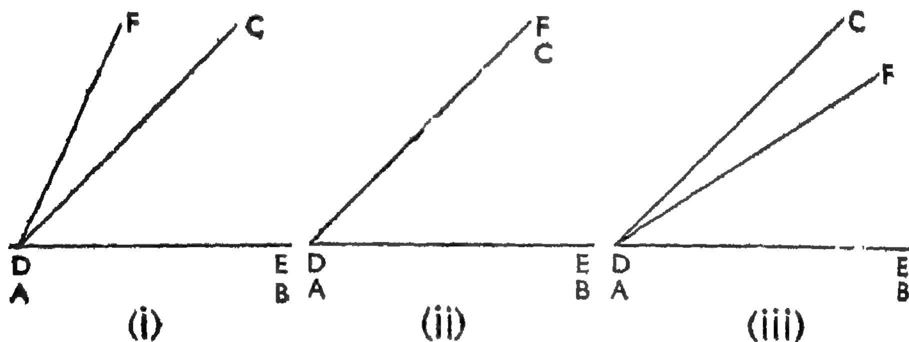
12.3 कोणों की तुलना

आइए आकृति 12.12 में दिए हुए कोणों BAC तथा EDF को देखें। इनमें कौन बड़ा है? केवल देखकर ही हम कह सकते हैं कि $\angle EDF$, $\angle BAC$ से बड़ा है। परन्तु हम अपनी दृष्टि पर सदैव भरोसा नहीं रख सकते। अतः हम दो कोणों के परिमाणों की तुलना के लिए निम्न विधि का प्रयोग करते हैं।



आकृति 12.12

हम एक अक्स करने के कागज़ को एक कोण माना $\angle BAC$ पर रखते हैं और इस कागज़ पर उसका अक्स उतार लेते हैं। अब हम इस अक्स को उठाकर $\angle EDF$ पर इस प्रकार रखते हैं कि शीर्ष A शीर्ष D पर पड़े तथा भुजा AB, भुजा DE के अनुदिश रहे। (देखिए आकृति 12.13)



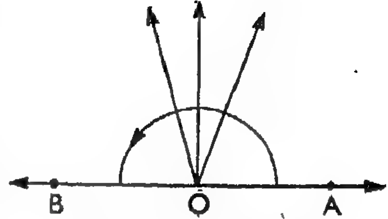
आकृति 12.13

भुजा AC की क्या संभव स्थितियाँ हो सकती हैं? यह DE और DF के बीच में पड़ सकती है [देखिए आकृति 12.13(i)] या यह ठीक DF पर पड़ सकती है (देखिए आकृति 12.13 (ii)] या यह DF के आगे पड़ सकती है [देखिए आकृति 12.13(iii)]। पहली स्थिति में हम कहते हैं कि $\angle BAC < \angle EDF$, दूसरी स्थिति में यह कि $\angle BAC = \angle EDF$ तथा तीसरी स्थिति में यह कि $\angle BAC > \angle EDF$। (हम कोण तथा उसके परिमाण दोनों के लिए ही एक ही संकेतन का प्रयोग करेंगे) **स्थिति (ii) में दोनों कोण बराबर हैं।**

कोणों के परिमाणों के तुलना करने की यह विधि बहुत संतोषजनक नहीं है। एक अच्छी विधि यह होगी कि हम कोणों को एक मानक कोण (*standard angle*), जिसे हम मापन का मात्रक मान लेते हैं, के पदों में मापें।

12.4 कोण की अंशीय माप

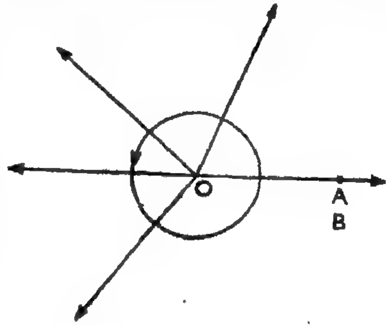
आइए एक किरण OA लें और उसे बिंदु O के चारों ओर घुमाएं। (देखिए आकृति 12.14) जैसे जैसे यह किरण घूमती है यह अपनी प्रारम्भिक स्थिति से बढ़ते हुए परिमाण के कोण बनाती है। मान लीजिए यह तब तक घूमती है जब तक कि यह स्थिति OB पर न आ जाए जो कि प्रारम्भिक स्थिति OA की विपरीत दिशा में है। इस प्रकार बना हुआ कोण AOB **ऋजु कोण** (*straight angle*) कहलाता है। ऋजु कोण की दोनों भुजाएँ विपरीत किरणें हैं।



आकृति 12.14 : ऋजुकोण

यदि हम किरण OA को और घूमने दें तो वह O के चारों ओर एक चक्कर (*revolution*) पूरा करने के बाद OA के संपाती (*coincident*) हो जाती है (अर्थात् OB और OA एक ही किरण है)। इस प्रकार बना कोण AOB **संपूर्ण कोण** (*complete angle*) कहलाता है। (देखिए आकृति 12.15)

यदि किरण OA बिल्कुल भी न घुमाई जाए तो हम कहते हैं कि हमें **शून्य कोण** (*zero angle*) प्राप्त हो गया है। दूसरे शब्दों में यदि कोण की दोनों भुजाएँ संपाती हों तो हम कहते हैं कि कोण का परिमाण शून्य है। हम देखते हैं कि संपूर्ण कोण में भी दोनों भुजाएँ संपाती होती हैं। परन्तु ये भुजाएँ केवल एक पूरे चक्कर के बाद संपाती हुई हैं। निस्संदेह, संपूर्ण कोण का परिमाण शून्य नहीं है।



आकृति 12.15 : संपूर्ण कोण

आइए एक कागज का पन्ना लें और उसे मोड़ें ताकि हमें एक मोड़ का निशान अर्थात् रेखा AB प्राप्त हो जाए। अब यदि हम कागज को खोल लें और उसे दुबारा इस प्रकार मोड़ें कि A, B पर पड़े तथा मोड़ का निशान AB स्वयं पर संपाती हो, तो हमें एक अन्य मोड़ का निशान CD प्राप्त हो जाता है। रेखाएँ AB और CD माना परस्पर O पर काटती हैं। (देखिए आकृति 12.16) अब हमें O पर चार बराबर कोण प्राप्त हो जाते हैं। (क्यों?) इनमें से प्रत्येक कोण **समकोण** (*right angle*) कहलाता है। इस प्रकार $\angle AOC$, $\angle COB$, $\angle BOD$ और $\angle DOA$ में से प्रत्येक एक समकोण है।



आकृति 12.16

अतः एक ऋजु कोण, जैसे कि $\angle AOB$ में दो समकोण होते हैं। इसी प्रकार एक संपूर्ण कोण में चार समकोण होते हैं।

हम कोण मापने के लिए समकोण को अपना मात्रक मान सकते हैं। परन्तु हमारे कार्य के लिए यह बहुत बड़ा मात्रक है क्योंकि तब हमें बहुत से कोणों को एक समकोण के भागों में मापना पड़ेगा।

इसलिए हम समकोण को नब्बे बराबर भागों में जो कि प्रत्येक एक **अंश** (*degree*) कहलाता है, विभाजित करते हैं और अंश को कोण मापन का मात्रक मान लेते हैं। 'अंश' को संख्या के ऊपर एक छोटा वृत्त '*o*' लिखकर व्यक्त किया जाता है। इस प्रकार एक अंश को 1° लिखा जाता है।

पुनः अंश को 60 मिनटों (*minutes*) तथा प्रत्येक मिनट को 60 सैकण्डों (*seconds*) में विभाजित किया गया है। 'मिनट' को संख्या के ऊपर एक तिरछी छोटी लकीर खींच कर तथा 'सैकण्ड' को दो तिरछी छोटी लकीरें खींच कर व्यक्त किया जाता है। इस प्रकार एक मिनट को $1'$ तथा एक सैकण्ड को $1''$ लिखा जाता है। मिनट और सैकण्ड का प्रयोग केवल तब ही किया जाता है जबकि बहुत ही शुद्ध मापनों (*accurate measurements*) की आवश्यकता हो उदाहरणार्थ जैसे खगोल विद्या संबंधी (*astromonical*) कार्य में।

संक्षेप में,

1 संपूर्ण कोण $= 360^\circ$

1 ऋजु कोण $= 180^\circ$

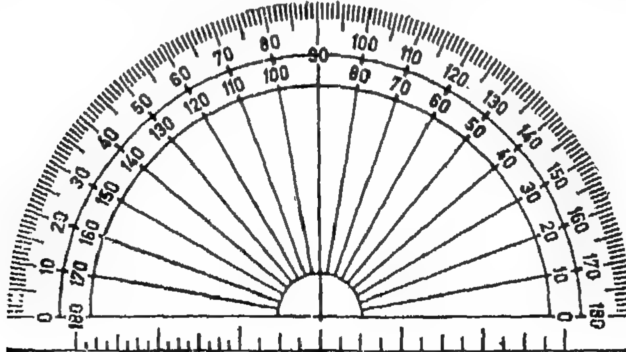
1 समकोण $= 90^\circ$

$1^\circ = 60'$

$1' = 60''$

12.5 चाँदा और उसके उपयोग

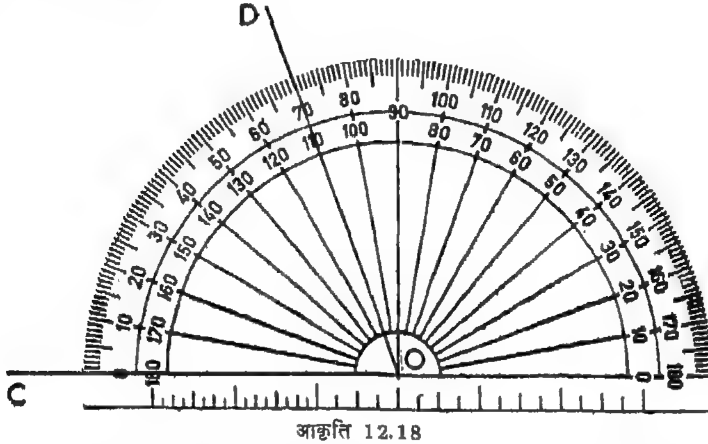
चाँदा (*protractor*) धातु या प्लास्टिक (*plastic*) का एक अर्धवृत्ताकार खंड होता है। इसके अर्धवृत्तीय किनारे पर अंश के निशान लगे होते हैं तथा सीधे किनारे के अनुदिश या अधिकतर सीधे



आकृति 12.17

किनारे के समांतर एक 0-180 रेखा बनी होती है। (देखिए आकृति 12.17) इस 0-180 रेखा का मध्य-बिंदु चाँदे का केन्द्र कहलाता है। अर्धवृत्ताकार किनारे पर अंशों के चिन्ह हैं। अंशों 0, 10,

20, 30, ..., 170, 180 के चिन्ह विशिष्ट रूप से दर्शाए जाते हैं तथा ये किनारे के अनुदिश लिखे होते हैं। यही चिन्ह उल्टे क्रम में भी लिखे होते हैं जिससे कि आकृति 12.18 में दिए हुए कोण COD जैसे कोणों को सुविधाजनक रूप से मापा जा सकता है। इसी कारण 0-180 रेखा को 0-0 (शून्य-शून्य) रेखा भी कहा जा सकता है।

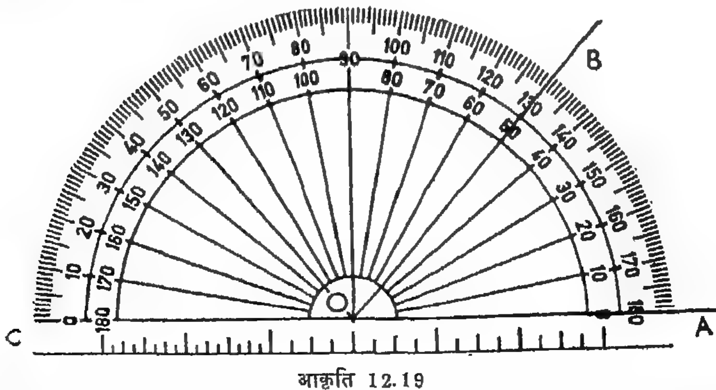


आकृति 12.18

अपने चाँदे को देखिए। किसी चिन्ह विशेष पर लिखी हुई दोनों संख्याओं का योग क्या है?

चाँदे का, कोण मापने तथा साथ ही दिए हुए परिमाण का कोण खींचने में उपयोग किया जाता है।

(क) दिया हुआ कोण मापना



आकृति 12.19

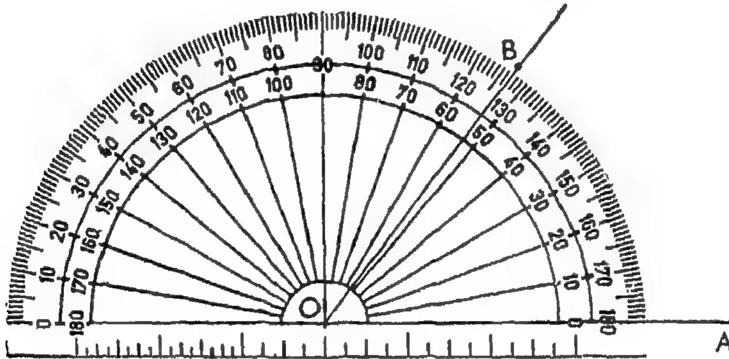
माना दिया हुआ कोण AOB है। हम चाँदे को इस प्रकार रखते हैं कि उसका केन्द्र कोण के शीर्ष पर पड़े तथा 0-0 रेखा, भुजा OA के अनदिश रहे। (देखिए आकृति 12.19) तब हम A पर

0° से प्रारम्भ करके अंश चिन्हों (*degree marks*) को बढ़ते हुए क्रम वाली दिशा में पढ़ते हुए वह चिन्ह पढ़ते हैं जिससे होकर भुजा OB जाती है।

आकृति 12.19 में, OB के तदनुरूपी चिन्ह 50 है। इस प्रकार $\angle AOB$ का परिमाण 50° है। हम लिखते हैं कि $\angle AOB = 50°$। यह देखना शिक्षाप्रद और रुचिकर होगा कि $\angle COB = 130°$। (क्यों?)

(ख) दिए हुए परिमाण का कोण बनाना

मान लीजिए हमें 53° का कोण बनाना है। हम एक किरण OA खींचते हैं और उस पर चाँदे को इस प्रकार रखते हैं कि उसका केन्द्र O पर पड़े तथा 0-0 रेखा OA के अनुदिश रहे। तब हम चाँदे पर 53° वाला चिन्ह ढूँढ़ते हैं तथा एक नुकीली पेंसिल से इस चिन्ह के तदनुरूपी एक बिंदु मान लीजिए B अंकित कर लेते हैं। अब हम O और B को जोड़ते हैं। (देखिए आकृति 12.20) तब AOB वांछित कोण है।

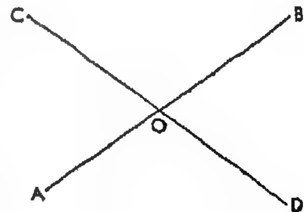


आकृति 12.20

हम देखते हैं कि चाँदे की सहायता से हम 180° तक के कोण माप या बना सकते हैं।

प्रश्नावली 12.2

1. AB और CD दो रेखाएँ हैं जो कि बिंदु O पर काटती हैं। (देखिए आकृति 12.21) $\angle AOC$ तथा $\angle BOD$ को मापिए। क्या ये बराबर हैं? कोण AOC और BOD **शीर्षाभिमुख कोण** (*vertically opposite angles*) कहलाते हैं। शीर्षाभिमुख कोणों के अन्य युग्म $\angle BOC$ तथा $\angle AOD$ को मापिए। आप इनके परिमाण के बारे में क्या कह सकते हैं? उपर्युक्त प्रक्रिया को किसी अन्य प्रतिच्छेदी रेखाओं के युग्म के साथ दोहराइए। आप क्या निष्कर्ष निकाल सकते हैं?



आकृति 12.21

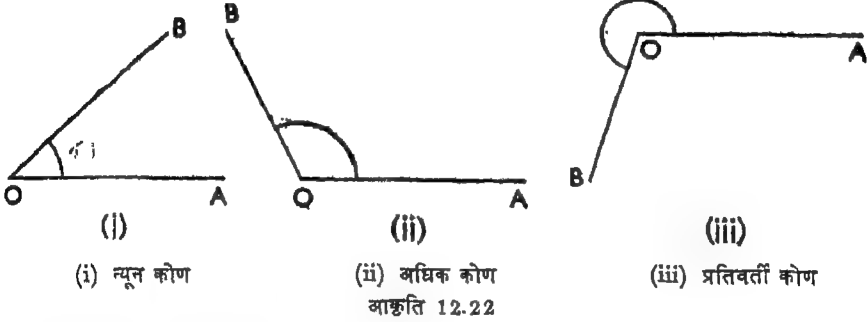
(हम देखते हैं कि शीर्षाभिमुख कोण बराबर होते हैं)

2. अपने चाँदे की सहायता से निम्न कोण बनाइए :

(i) 15°	(ii) 27°	(iii) 30°	(iv) 36°
(v) 45°	(vi) 54°	(vii) 60°	(viii) 75°
(ix) 120°	(x) 135°		

12.6 कोणों के प्रकार

हम पहले ही देख चुके हैं कि समकोण में 90°, ऋजुकोण में 180° तथा संपूर्ण कोण में 360° होते हैं।



(i) न्यून कोण (ii) अधिक कोण (iii) प्रतिवर्ती कोण

आकृति 12.22

0° वाले कोण को छोड़कर समकोण से छोटा कोई भी कोण **न्यून** (*acute*) कोण कहलाता है [देखिए आकृति 12.22(i)]

समकोण से बड़ा परन्तु ऋजुकोण से छोटा कोण **अधिक** (*obtuse*) कोण कहलाता है [देखिए आकृति 12.22(ii)]

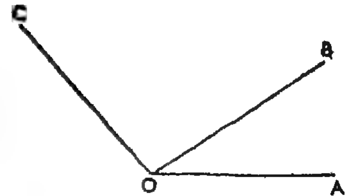
ऋजुकोण से बड़ा परन्तु संपूर्ण कोण से छोटा कोण **प्रतिवर्ती** (*reflex*) कोण कहलाता है [देखिए आकृति 12.22(iii)]

इस प्रकार, न्यूनकोण 0° और 90° के बीच, अधिक कोण 90° और 180° के बीच तथा प्रतिवर्ती कोण 180° और 360° के बीच होता है।

12.7 कोणों के युग्म

12.7.1 आसन्न कोण

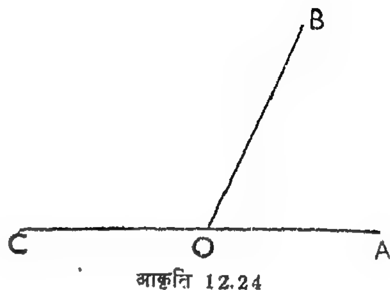
आइए आकृति 12.23 को देखें। हमारे पास उभयनिष्ठ भुजा OB वाले दो कोण AOB तथा BOC हैं। साथ ही, इनकी अन्य भुजाएँ OA तथा OC, OB की विपरीत दिशाओं में हैं। ऐसे दो कोण, **आसन्न कोण** (*adjacent angle*) कहलाते हैं।



आकृति 12.23

12.7.2 रैखिक युग्म

आइए आकृति 12.24 को देखें। AOC एक रेखा है। AOB तथा BOC आसन्न कोण हैं जिनकी बाहरी भुजाएँ एक रेखा में हैं। ऐसे आसन्न कोणों का युग्म **रैखिक युग्म** (*linear pair*) कहलाता है।



आकृति 12.24

12.7.3 पूरक कोण

यदि दो कोणों का योग $90°$ हो तो वे **पूरक कोण** (*complementary angles*) कहे जाते हैं तथा प्रत्येक कोण एक दूसरे का **पूरक** (*complement*) कहलाता है। उदाहरणार्थ $40°$ और $50°$ के कोण पूरक हैं। $40°$ का कोण $50°$ के कोण का पूरक है तथा $50°$ का कोण $40°$ के कोण का पूरक है।

12.7.4 संपूरक कोण

यदि दो कोणों का योग $180°$ हो तो वे **संपूरक कोण** (*supplementary angles*) कहे जाते हैं तथा प्रत्येक कोण एक **दूसरे का संपूरक** (*supplement*) कहलाता है। उदाहरणार्थ $70°$ और $110°$ के कोण संपूरक है तथा प्रत्येक एक दूसरे का संपूरक है।

हम देखते हैं कि **कोणों का रैखिक युग्म संपूरक है।**

प्रश्नावली 12.3

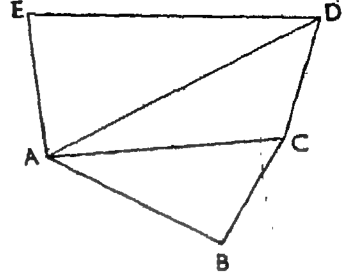
1. अपने आस पास देखिए और समकोण, न्यून कोण एवं अधिक कोणों के **कुछ** उदाहरण दीजिए।

2. अपनी भुजाओं का प्रयोग करके दिखाइए कि आप न्यून कोण, अधिक कोण तथा समकोण किस प्रकार बनाते हैं।

3. दत्त पश्चिम की ओर मुँह करके खड़ा है। वह दाईं ओर एक समकोण पर घूम जाता है। अब वह किस दिशा में देख रहा है?

4. शीला उत्तर की दिशा में एक नाव खे रही है। वह उसे एक ऋजु कोण पर घुमा देती है। अब वह किस दिशा में नाव खे रही है?

5. **कुछ** न्यून और अधिक कोण खींचिए और उन्हें मापिए।



आकृति 12.25

6. आकृति 12.25 में तीन आसन्न कोणों के युग्मों के नाम लिखिए।

7. निम्न कोणों के पूरक ज्ञात कीजिए :

   50°, 70°, 80°, 30°, 45°

8. निम्न कोणों के संपूरक ज्ञात कीजिए :

   150°, 50°, 70°, 80°

9. निम्न कोण युग्मों में जाँच कीजिए कि कौन से कोण पूरक हैं और कौन से संपूरक :

   (i) 20°, 70° (ii) 30°, 60° (iii) 44°, 46°

   (iv) 40°, 140° (v) 75°, 105° (vi) 42°, 138°

   (vii) 45°, 45° (viii) 15°, 75° (ix) 60°, 120°

10. ऐसा कोण खींचिए जोकि 35° के कोण का संपूरक हो।

11. एक कोण अपने पूरक के बराबर है। कोण का परिमाण क्या है?

12. एक कोण अपने संपूरक के बराबर है। कोण का परिमाण क्या है?

13. क्या निम्न कथन सत्य हैं?

   (क) यदि दो कोण रैखिक युग्म बनाते हैं तो उनका योग 180° है।

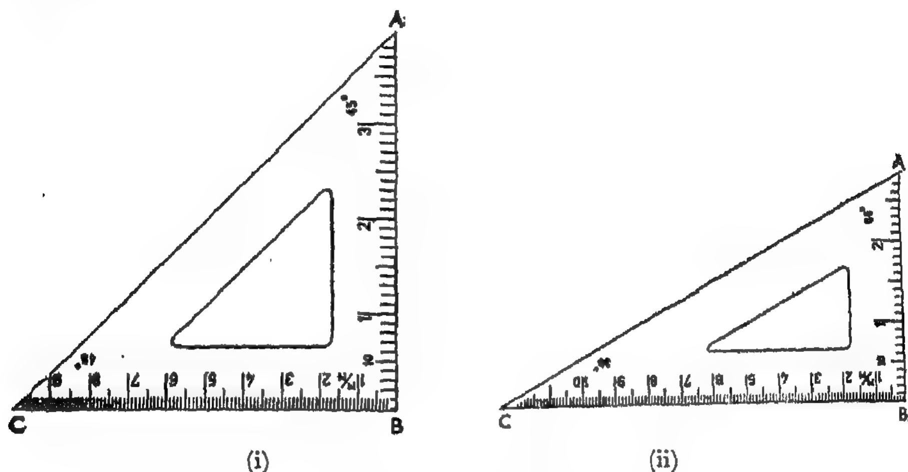
   (ख) यदि दो कोणों का योग 180° हो तो वे एक रैखिक युग्म बनाते हैं।

14. रैखिक युग्म का एक कोण अधिक कोण है। आप दूसरे कोण के बारे में क्या कह सकते हैं?

15. रैखिक युग्म का एक कोण 48° का है। दूसरा कोण ज्ञात कीजिए।

12.8 सेट स्क्वायर

आपने अपने ज्यामिति बक्स में दो त्रिभुजाकार उपकरण (*tools*) देखे होंगे। इनके चित्र नीचे आकृति 12.26 में दिए हैं।



आकृति 12.26: सेट स्क्वायर

ये **सेट स्क्वायर** (*set squares*) कहलाते हैं। एक सेट स्क्वायर के कोण 45°, 90° और 45° हैं तथा दूसरे के 60°, 90°, और 30°। अपने चाँदे से इसकी जाँच कीजिए। कुछ अन्य कोणों तथा साथ ही लम्ब (*perpendicular*) और समांतर रेखाएँ खींचने में सेट स्क्वायर का प्रयोग किया जा सकता है। ये धातु या लकड़ी या पारदर्शक प्लास्टिक के बने होते हैं तथा प्रायः कुछ मिलीमीटर मोटाई के होते हैं। कभी कभी समकोण वाले दोनों किनारे अंशांकित भी होते हैं। एक किनारा सेंटीमीटरों में तथा दूसरा किनारा इंचों में अंशांकित होता है। ऐसे सेट स्क्वायर का रेखाखंडों की लम्बाइयाँ मापने में भी प्रयोग किया जा सकता है।

हम आकृति 12.26 (i) के सेट स्क्वायर को **45° सेट स्क्वायर** तथा आकृति 12.26 (ii) के सेट स्क्वायर को **30° सेट स्क्वायर** कह सकते हैं।

प्रश्नावली 12.4

1. अपने सेट स्क्वायर के किनारे मापिए। आकृति 12.26 के अनुसार उनके नाम लिखकर जाँच कीजिए कि 45° सेट स्क्वायर में $AB=BC$ है तथा 30° सेट स्क्वायर में $AC=2\,AB$ है।

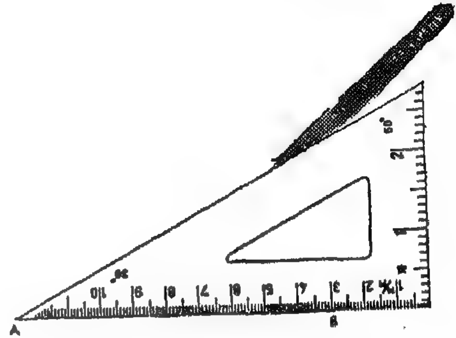
12.9 विभिन्न रचनाओं में सेट स्क्वायर का उपयोग

30°, 45°, 60° तथा 90° के कोण दोनों सेट स्क्वायर में से एक या दूसरे का प्रयोग करके तुरन्त ही खींचे जा सकते हैं। उदाहरणार्थ 30° का कोण हम निम्न चरणों में खींचते हैं :

चरण 1 : हम 30° सेट स्क्वायर को कागज़ पर एक उपयुक्त स्थिति में रखते हैं जैसा कि आकृति 12.27 में दिखाया गया है।

चरण 2 : हम एक हाथ से सेट स्क्वायर को कस कर पकड़े रहते हैं तथा दूसरे हाथ से एक नुकीली पेंसिल की सहायता से 30° कोण वाले दोनों किनारों के अनुदिश दो किरणें AB और AC खींचते हैं।

तब, $\angle BAC$ वांछित कोण है।

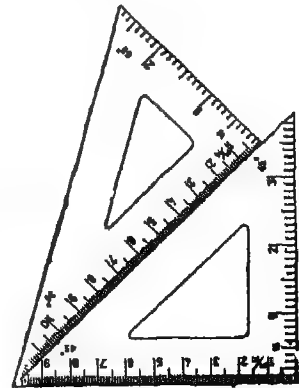


आकृति 12.27

प्रश्नावली 12.5

1. सेट स्क्वायर की सहायता से 45°, 60° तथा 90° के कोण खींचिए।

2. 75° के कोण की रचना कीजिए।
[संकेत : $75^\circ = 45^\circ + 30^\circ$, आकृति 12.28 भी देखिए।]



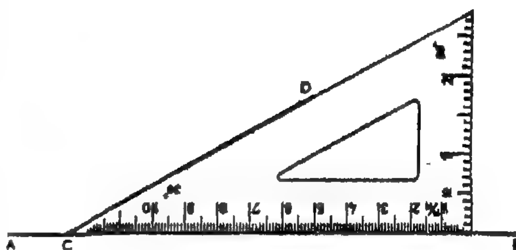
आकृति 12.28

3. 105° के कोण की रचना कीजिए।

4. 15° के कोण की रचना कीजिए। [संकेतः $15^\circ = 45^\circ - 30^\circ$]

हम सेट स्क्वायर की सहायता से, एक दी हुई रेखा से, उस पर एक दिए हुए बिंदु पर भी कोई कोण, मान लीजिए 30°, बना सकते हैं। माना AB दी हुई रेखा है और C उस पर स्थित कोई बिंदु दिया है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : हम 30° सेट स्क्वायर को इस प्रकार रखते हैं कि 30° कोण की एक भुजा AB के अनुदिश रहे तथा उसका शीर्ष C पर रहे।



आकृति 12.29

चरण 2 : हम एक हाथ से सेट स्क्वायर को कस कर पकड़े रहते हैं तथा दूसरे हाथ से एक नुकीली पेंसिल की सहायता से 30° कोण की दूसरी भुजा के अनुदिश किरण CD खींचते हैं। (देखिए आकृति 12.29) तब, $\angle BCD$ वांछित कोण है ?

यह देखा जा सकता है कि यह विधि अधिक उपयुक्त नहीं है क्योंकि सेट स्क्वायर के कोने प्रायः टूटे और घिसे हुए होते हैं। ऐसी दशा में न तो सेट स्क्वायर के 30° कोने को ठीक प्रकार से C पर रखना संभव है और न ही सही तरीके से किरण CD खींचना इतना सरल है।

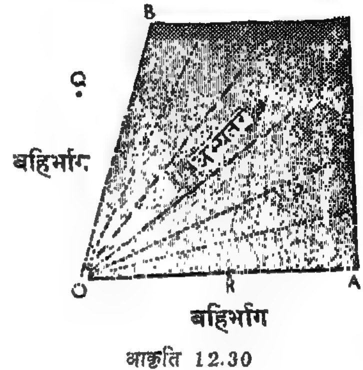
प्रश्नावली 12.6

1. सेट स्क्वायर की सहायता से एक दी हुई रेखा के दिए हुए बिंदु पर निम्न कोण खींचिए :
   (i) 45° (ii) 60° (iii) 90°

हम लम्ब और समांतर रेखाएँ खींचने में सेट स्क्वायर के उपयोग के बारे में अगले एकक में बताएंगे।

12.10 कोण का अभ्यंतर और बहिर्भाग

मान लीजिए AOB कोई कोण है। किरणें OA और OB तल के (सभी) बिंदुओं को तीन भागों में विभाजित करती हैं। **पहला** ऐसे बिंदु P जो कि O के चारों ओर किरण OA द्वारा स्थिति OA से OB तक घूमने में बनी किसी किरण पर स्थित होते हैं जैसा कि आकृति 12.30 में दिखाया गया है। **दूसरा**, Q की तरह के बिंदु जो कि किसी ऐसी किरण पर स्थित नहीं होते तथा **तीसरा** R की तरह के बिंदु जो कि कोण की किसी एक भुजा अर्थात् OA या OB पर स्थित होते हैं।



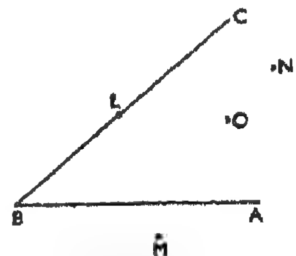
आकृति 12.30

तल का वह भाग या क्षेत्र जिसमें ऐसे सभी बिंदु P स्थित हैं, $\angle AOB$ का **अभ्यंतर** (*interior*) कहलाता है। वह क्षेत्र जिसमें ऐसे सभी बिंदु Q स्थित हैं, $\angle AOB$ का **बहिर्भाग** (*exterior*) कहलाता है। भुजाओं OA और OB से दोनों क्षेत्रों की **उभयनिष्ठ सीमा** (*common boundary*) बनती है।

हम देखते हैं कि यदि हमें P से Q या Q से P तक जाना हो तो हमें OA या OB में से एक भुजा को अवश्य पार करना पड़ेगा। दूसरे शब्दों में, कोण की भुजाएँ उसके अभ्यंतर को उसके बहिर्भाग से पृथक करती हैं।

प्रश्नावली 12.7

1. आकृति 12.31 में बिंदुओं L, M, O तथा N में से कौन से बिंदु $\angle ABC$ के अभ्यंतर में स्थित हैं? कौन से बहिर्भाग में?



आकृति 12.31

2. क्या कोण का शीर्ष अभ्यंतर में स्थित होता है? क्या बहिर्भाग में स्थित होता है?

3. किसी कोण को दो बराबर आसन्न कोणों में विभाजित करने वाली किरण को कोण का **समद्विभाजक** (*bisector*) कहते हैं। क्या किसी कोण का समद्विभाजक पूरी तरह से उसके अभ्यंतर में स्थित होता है? क्या यह बहिर्भाग में स्थित है?

[संकेत : उपर्युक्त प्रश्न 2 का अध्ययन कीजिए।]

एकक XIII

समांतर रेखाएँ

13.1 समांतर रेखाएँ

आपको याद होगा कि तल में जो रेखाएँ कभी नहीं मिलतीं, चाहे उन्हे किसी भी दिशा में कितना ही बढ़ाया जाएं, **समांतर रेखाएँ** (*parallel lines*) कहलाती हैं। ब्लैकबोर्ड के सम्मुख किनारे, आयताकार मेज की ऊपरी सतह के सम्मुख किनारे, आपके कक्ष के सम्मुख किनारे, आपकी पुस्तक के सम्मुख किनारे, आपके रूलर के सम्मुख किनारे इत्यादि सब समांतर रेखाओं के युग्मों के उदाहरण हैं।

मान लीजिए AB और CD दो समांतर रेखाएँ हैं। (देखिए आकृति 13.1) हम AB, CD के समांतर है, व्यक्त करने के लिए $AB \parallel CD$ लिखते हैं। हम इसे $CD \parallel AB$ भी लिख सकते हैं।

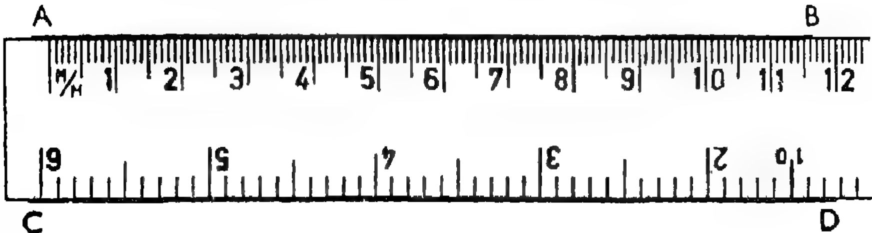


आकृति 13.1

आइए अब समांतर रेखाओं के एक महत्वपूर्ण गुण को देखें। हम निम्न प्रयोगों पर विचार करते हैं।

प्रयोग 1

आइए कागज के पन्ने पर अपना रूलर रखें और एक नुकीली पेंसिल से रूलर के दोनों सम्मुख किनारों के अनुदिश दो रेखाएँ AB और CD खींचे। अब रूलर की सहायता से दोनों रेखाओं को दाईं ओर की तरफ जितना हो सके उतना बढ़ाएं। क्या ये रेखाएँ कहीं मिलती हैं?



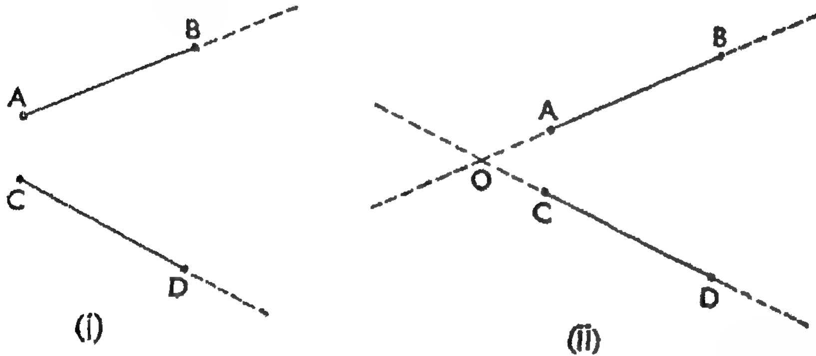
आकृति 13.2

पुनः अब रेखाओं को बाईं ओर की तरफ जितना हो सके उतना बढ़ाएँ। क्या ये रेखाएं कहीं मिलती हैं?

अतः हम देखते हैं कि दोनों रेखाओं के बीच की दूरी प्रत्येक जगह रूलर की चौड़ाई के बराबर है।

प्रयोग 2

आइए दो रेखाएँ AB और CD लें तथा उन्हें दाईं ओर की तरफ बढ़ाएँ। [देखिए आकृति 13.3 (i)] क्या दोनों रेखाएँ अधिकाधिक अलग होती जाती हैं?



आकृति 13.3

अब इन्हें बाईं ओर की तरफ बढ़ाएँ। [देखिए आकृति 13.3 (ii)] क्या दोनों रेखाएँ परस्पर समीप आती जाती हैं?

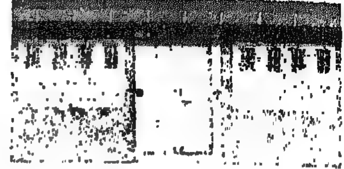
हम देखते हैं कि ये एक बिंदु O पर मिलती हैं। यदि इन्हें और आगे बढ़ाएँ तो क्या होता है? क्या ये पुनः अलग होना प्रारम्भ कर देती हैं?

हम उपर्युक्त दो प्रयोगों से देखते हैं कि

यदि दो रेखाएँ समांतर हों तो प्रत्येक जगह ये एक दूसरे से समान 'दूरी' पर होती हैं। दूसरे शब्दों में हम दो समांतर रेखाओं के बीच की 'दूरी' का अस्तित्व मान सकते हैं।

यदि रेखाएँ समांतर न हों तो उनके बीच की कोई स्थिर दूरी नहीं होती। ये ऐसी दो सड़कों की तरह हैं जो कि चौराहे पर मिलने के पश्चात् पुनः अलग होना शुरू हो जाती हैं।

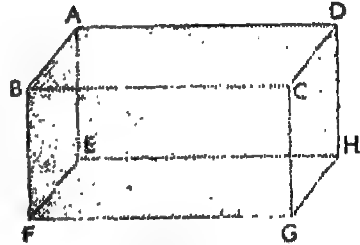
प्रश्नावली 13.1



आकृति 13.4

1. आकृति 13.4 में हम एक दरवाजे और दो खिड़कियों वाली दीवार का चित्र देखते हैं। किन्हीं पाँच समांतर रेखाओं के युग्मों के नाम लिखिए।

2. अपने आस पास की वस्तुओं में समांतर रेखाओं के युग्मों के पाँच उदाहरण दीजिए।



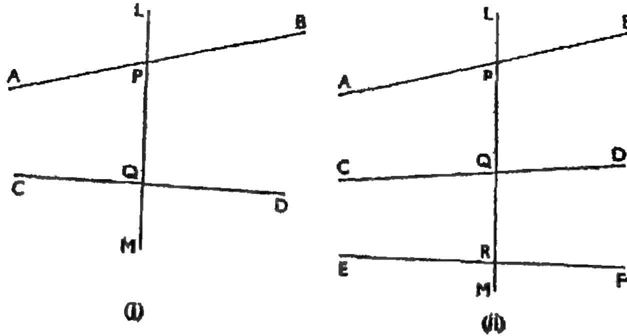
आकृति 13.5

3. आकृति 13.5 में आप लकड़ी का एक आयताकार ब्लाक देख रहे हैं। किनारों BC और FG को बढ़ाइए। चाहे इन्हें कितना भी बढ़ाया जाए ये किनारे कभी नहीं मिलते।
क्या ये समांतर हैं? कुछ अन्य समांतर किनारों के युग्म बताइए।

13.2 तिर्यक रेखा

आकृति 13.6(i) में AB और CD दो रेखाएं हैं तथा एक अन्य रेखा LM उन्हें दो विभिन्न बिंदुओं P और Q पर काटती है। आकृति 13.6(ii) में AB, CD और EF तीन रेखाएँ हैं तथा एक अन्य रेखा LM उन्हें तीन विभिन्न बिंदुओं P, Q और R पर काटती है। प्रत्येक स्थिति में LM अन्य रेखाओं की **तिर्यक रेखा** (*transversal*) कहलाती है।

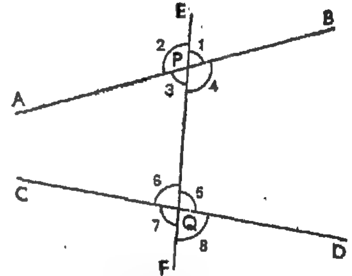
वह रेखा जो दो या दो से अधिक दी हुई रेखाओं को विभिन्न बिंदुओं पर काटती है तिर्यक रेखा कहलाती है। दी हुई रेखाएँ समांतर भी हो सकती हैं और असमांतर भी। परन्तु इन सभी रेखाओं के तिर्यक रेखा के साथ प्रतिच्छेद बिंदु अवश्य ही भिन्न भिन्न होने चाहिए।



आकृति 13.6

13.3 दो रेखाओं से तिर्यक रेखा द्वारा बनाए गए कोण

आकृति 13.7 में AB और CD दो रेखाएँ हैं और तिर्यक रेखा EF इन्हें क्रमशः बिंदुओं P और Q पर काटती है। रेखाओं AB और CD से तिर्यक रेखा आठ कोण बनाती है। इन कोणों को आकृति में 1, 2, 3, 4, 5, 6, 7, 8 लिखा गया है। क्या आप इन कोणों के पूरे नाम बता सकते हैं?



आकृति 13.7

कोण 1, 2, 7 और 8 तिर्यक रेखा द्वारा दोनों रेखाओं से बनाए गए **बाह्य कोण** (*exterior angles*) कहलाते हैं। कोण 3, 4, 5 और 6 **अंतः कोण** (*interior angles*) कहलाते हैं।

कोणों 1 और 5 से **संगत कोणों** (*corresponding angles*) का एक युग्म बनता है। इसी प्रकार 2 और 6, 3 और 7, 4 और 8 भी संगत कोणों के युग्म हैं।

कोणों 3 और 5 से **अंतः एकांतर कोणों** (*alternate interior angles*) का एक युग्म बनता है। इन्हें केवल **एकांतर कोण** (*alternate angles*) भी कहते हैं। इसी प्रकार 4 और 6 भी एकांतर कोणों का युग्म है।

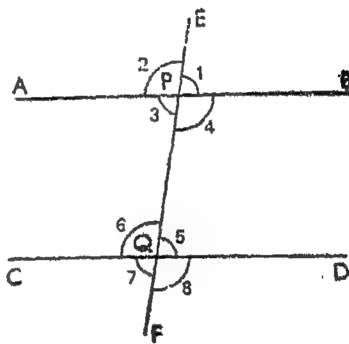
प्रश्नावली 13.2

1. आकृति 13.7 में निम्न कोणों के संगत कोण लिखिए :
   (i) $\angle APE$ (ii) $\angle QPB$
2. आकृति 13.7 में $\angle APQ$ के एकांतर कोण का नाम बताइए।

13.4 दो समांतर रेखाओं से तिर्यक रेखा द्वारा बनाए गए कोण

आइए निम्न प्रयोग को देखें :

चरण 1 : अपने रूलर को कागज के पन्ने पर रखें और उसके सम्मुख किनारों के अनुदिश दो रेखाएँ AB और CD खींचें। रूलर को हटाने पर हमें दो समांतर रेखाएं AB और CD प्राप्त हो जाती हैं।



आकृति 13.8

चरण 2 : अब एक तिर्यक रेखा EF खींचें जो कि AB और CD को क्रमश: P और Q पर काटे। साथ ही, तिर्यक रेखा द्वारा दोनों रेखाओं से बनाए गए आठों कोणों को 1, 2, 3, 4, 5, 6, 7 और 8 से निरूपित करें जैसा कि आकृति 13.8 में दिखाया गया है।

चरण 3 : अब चाँदे से एकांतर कोणों 3 और 5 को मापें। क्या ये बराबर हैं? एकांतर कोण 4 और 6 भी मापें। हमें क्या ज्ञात होता है?

चरण 4 : पुन: आइए संगत कोणों 1 और 5 को मापें। क्या ये बराबर हैं? संगत कोण 4 और 8 भी मापें। हमें क्या ज्ञात होता है? इसी प्रकार संगत कणों के अन्य दो युग्म भी मापें। हमें क्या ज्ञात होता है?

चरण 5 : अब अंत: कोणों 4 और 5 जोकि तिर्यक रेखा के एक ही ओर हैं, को मापें। क्या इनका योग $180°$ है? पुन: अंत: कोण 3 और 6 मापें और उनका योग ज्ञात करें। हम क्या देखते हैं?

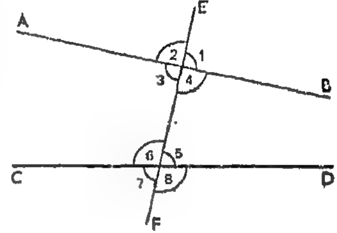
आइए इस प्रयोग को समांतर रेखाओं के एक अन्य युग्म और उनकी तिर्यक रेखा के साथ दोहराएं।

हमें दो समांतर रेखाओं से उनकी तिर्यक रेखा द्वारा बनाए गए कोणों के निम्न गुण प्राप्त होंगे।

यदि एक तिर्यक रेखा दो समांतर रेखाओं को काटे तो (i) एकांतर कोणों के युग्म बराबर होते हैं (देखिए चरण 3), (ii) संगत कोणों के युग्म बराबर होते हैं (देखिए चरण 4), (iii) तिर्यक रेखा

के एक ही ओर के अंतः कोणों का योग 180° होता है (देखिए चरण 5)। दूसरे शब्दों में, तिर्यक रेखा के एक ही ओर के अंतः कोण संपूरक होते हैं।

अब इस प्रयोग को दो असमांतर रेखाओं AB तथा CD और उनकी तिर्यक रेखा EF के साथ दोहराएँ। (देखिए आकृति 13.9) आइए कोणों 1, 2, 3, 4, 5, 6, 7 और 8 को मापें। हम देखते हैं कि संगत कोणों के युग्म 1, 5; 2, 6; 4, 8 या 3, 7 में से कोई भी युग्म बराबर नहीं है। एकांतर कोणों के युग्म 3, 5 या 4, 6 में से कोई भी युग्म बराबर नहीं है। अंत में, कोणों 4 और 5 या 3 और 6 का योग 180° नहीं है। इस प्रकार हम देखते हैं कि यदि रेखाएँ समांतर न हों तो (i), (ii) तथा (iii) में से कोई भी गुण सत्य नहीं होगा। दूसरे शब्दों में, यदि एक तिर्यक रेखा दो रेखाओं को काटे तथा (i), (ii) या (iii) में से कोई एक भी गुण सत्य हो तो दोनों रेखाएँ समांतर होती हैं।

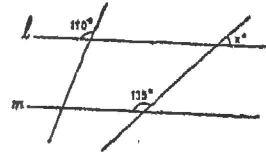


आकृति 13.9

हम इनमें से कुछ गुणों का बिना इनका नाम विशेष बताए लम्ब और समांतर रेखाएँ खींचने में उपयोग करेंगे।

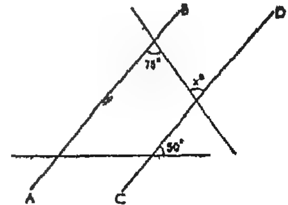
प्रश्नावली 13.3

1. आकृति 13.10 में, l और m दो समांतर रेखाएँ हैं। x का मान ज्ञात कीजिए।

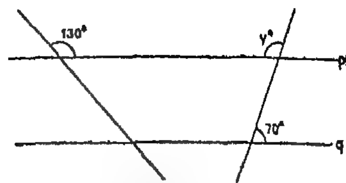


आकृति 13.10

2. आकृति 13.11 में, AB और CD समांतर हैं। x का मान ज्ञात कीजिए।



आकृति 13.11



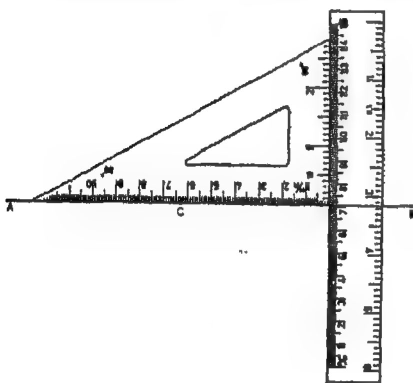
आकृति 13.12

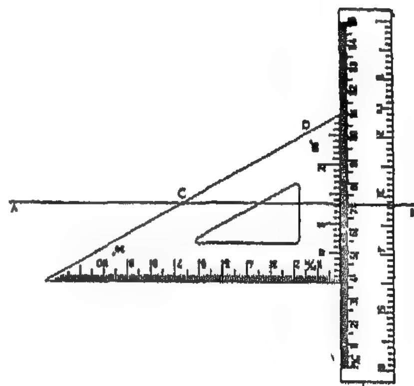
3. आकृति 13.12 में p और q समांतर हैं। y का मान ज्ञात कीजिए।

13.5 सेट स्क्वायर से कुछ और रचनाएँ

13.5.1 एक दी हुई रेखा से उसके एक दिए हुए बिंदु पर एक कोण, माना 30°, बनाना

माना दी हुई रेखा AB है तथा उस पर दिया हुआ बिंदु C है।





(i) आकृति 13.13 (ii)

हम यह निम्न चरणों में करते हैं :

चरण 1 : आइए 30° सेट स्क्वायर को आकृति 13.13 (i) के अनुसार रखें जिससे 30° कोण की एक भुजा रेखा AB के अनुदिश रहे। तब बिंदु C भी इसी भुजा पर स्थित होगा।

चरण 2 : अब सेट स्क्वायर को कसकर पकड़े रहें तथा 30° के सामने वाले किनारे के अनुदिश एक रूलर रखें। (रूलर के स्थान पर हम दूसरे सेट स्क्वायर का प्रयोग भी कर सकते हैं।)

चरण 3 : अब रूलर को स्थिर रखते हुए सेट स्क्वायर को रूलर के अनुदिश तब तक सरकाएँ जब तक कि यह ऐसी स्थिति में न आ जाए कि C, 30° कोण की दूसरी भुजा पर स्थित हो। [देखिए आकृति 13.13 (ii)]

चरण 4 : अंत में सेट स्क्वायर को इस स्थिति में स्थिर रखते हुए उसके किनारे के अनुदिश किरण CD खींचें।

तब $\angle BCD$ वांछित कोण है।

हम देखते हैं कि इस रचना में हमें सेट स्क्वायर के कोनों का प्रयोग नहीं करना पड़ा। इस प्रकार यदि सेट स्क्वायर के कोने टूटे हुए हों तो भी कोण की रचना की जा सकती है।

प्रश्नावली 13.4

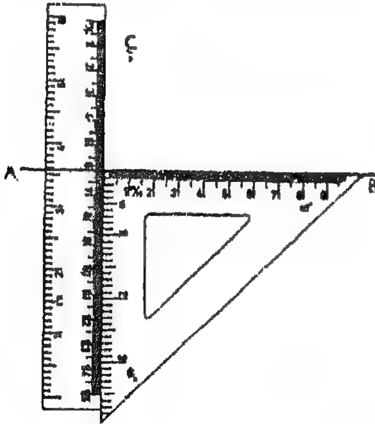
1. अनुच्छेद 13.5.1 की विधि का प्रयोग करते हुए एक दी हुई रेखा के दिए हुए बिंदु पर निम्न कोणों की रचना कीजिए :

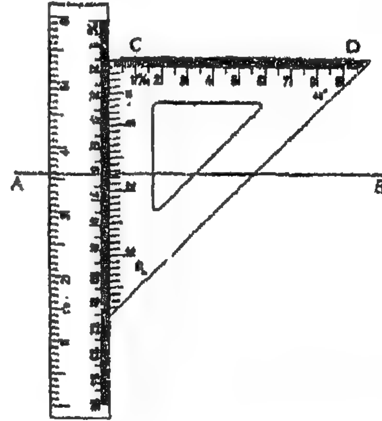
(i) 45° (ii) 60° (iii) 90°

13.5.2 एक दी हुई रेखा के समांतर उस पर न स्थित एक दिए हुए बिंदु से होकर एक रेखा खींचना

माना दी हुई रेखा AB है तथा C उसके बाहर कोई बिंदु है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : आइए दोनों में से किसी भी सेट स्क्वायर को इस प्रकार रखें कि समकोण की एक भुजा AB के अनुदिश रहे। [देखिए आकृति 13.14 (i)]





(i) आकृति 13.14 (ii)

चरण 2 : अब सेट स्क्वायर को पकड़े रहें तथा रूलर (या दूसरे सेट स्क्वायर) को समकोण की दूसरी भुजा के अनुदिश रखें।

चरण 3 : अब रूलर को कसकर पकड़े हुए, सेट स्क्वायर को रूलर के अनुदिश तब तक सरकाएं जब तक कि वह ऐसी स्थिति में न आ जाए कि बिंदु C समकोण की पहली भुजा पर स्थित हो। [देखिए आकृति 13.14 (ii)]

चरण 4 : अंत में सेट स्क्वायर को इस स्थिति में स्थिर रखते हुए C से होकर इस किनारे के अनुदिश रेखा CD खींचें।

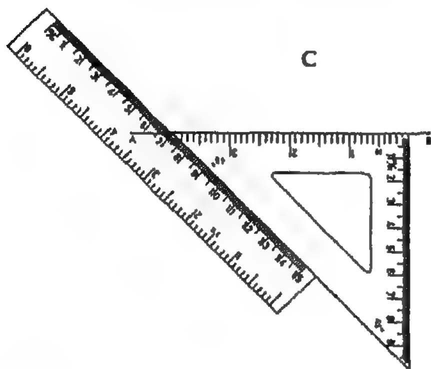
तब CD ही, C से होकर जाने वाली तथा AB के समांतर वांछित रेखा है।

13.5.3 एक दी हुई रेखा पर उस पर न स्थित एक दिए हुए बिंदु से लम्ब खींचना

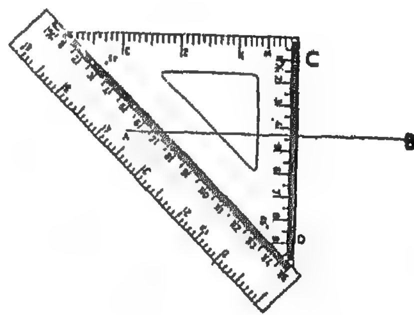
जब दो रेखाएँ एक दूसरे से $90°$ का कोण बनाती हैं तो वे **परस्पर लम्ब** (*perpendicular*) या **समकोण पर** (*at right angles*) कहलाती हैं। हम लम्ब को संकेत '$\perp$' से व्यक्त करते हैं।

माना AB दी हुई रेखा है और C उसके बाहर कोई बिंदु है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : आइए दोनों में से किसी भी सेट स्क्वायर को इस प्रकार रखें कि समकोण की एक भुजा AB के अनुदिश रहे। [देखिए आकृति 13.15 (i)]



(i)



(ii)

आकृति 13.15

चरण 2 : अब सेट स्क्वायर को कसकर पकड़े रहें तथा एक रूलर (या दूसरे सेट स्क्वायर के सबसे लम्बे किनारे) को इस सेट स्क्वायर के समकोण के सामने वाले किनारे के अनुदिश रखें।

चरण 3 : अब रूलर को कसकर पकड़े हुए, सेट स्क्वायर को रूलर के अनुदिश तब तक सरकाएँ जब तक कि वह ऐसी स्थिति में न आ जाए कि C समकोण की दूसरी भुजा पर स्थित हो। [देखिए आकृति 13.15 (ii)]

चरण 4 : अंत में सेट स्क्वायर को इस स्थिति में स्थिर रखते हुए C से होकर इस किनारे के अनुदिश रेखा CD खींचें।

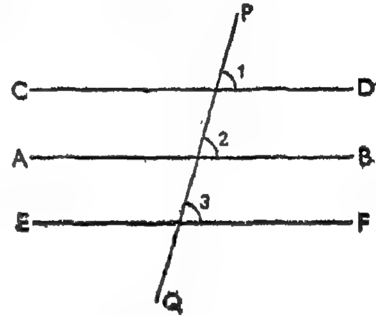
तब CD ही AB पर C से होकर वांछित लम्ब रेखा है।

प्रश्नावली 13.5

1. एक रेखाखंड AB खींचिए तथा इसके विपरीत ओर बिंदु C और E लीजिए। C से होकर $CD \parallel AB$ खींचिए तथा E से होकर $EF \parallel AB$ खींचिए। सेट स्क्वायर की सहायता से जाँच कीजिए कि $CD \parallel EF$ है।

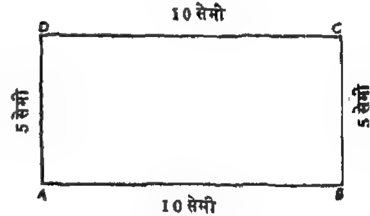
आप क्या देखते हैं ? **एक ही रेखा के समांतर रेखाएँ परस्पर समांतर** होती हैं।

2. समांतर रेखाओं CD, AB और EF की एक तिर्यक रेखा खींचिए। (देखिए आकृति 13.16) कोणों को 1, 2 तथा 3 लिखिए और उन्हें मापिए। क्या ये बराबर हैं ?



आकृति 13.16

3. 10 सेमी का एक रेखाखंड AB लीजिए। A से होकर $AD \perp AB$ इस प्रकार खींचिए कि $AD = 5$ सेमी हो। D से होकर $DC \parallel AB$ इस प्रकार खींचिये कि $DC = 10$ सेमी हो। C और B को जोड़िए। (देखिए आकृति 13.17) तब $ABCD$ एक **आयत** होगा जिसकी भुजाएँ 10 सेमी और 5 सेमी हैं।



आकृति 13.17

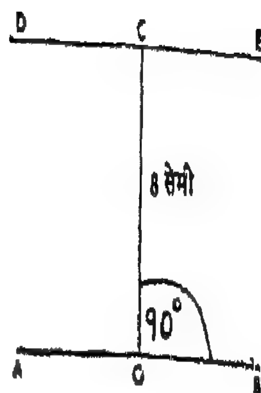
4. 8.5 सेमी तथा 5.6 सेमी भुजाओं वाला एक आयत खींचिए।

5. 10 सेमी की भुजा का एक वर्ग खींचिए।

6. स्केल 1 मीटर = 1 सेमी का प्रयोग करते हुए 5 मीटर चौड़ी एक सीधी सड़क का चित्र बनाइए।

[प्रश्न में तात्पर्य यह है कि सड़क के किनारे समांतर हैं तथा प्रत्येक जगह उनकी परस्पर दूरी 5 सेमी है। चूंकि स्केल 1 मीटर = 1 सेमी है अतः हमें 5 सेमी की दूरी पर दो समांतर रेखाएँ खींचनी हैं।

हम कोई रेखा AB और उस पर एक बिंदु O लेते हैं। अब हम AB पर लम्ब OC इस प्रकार खींचते हैं कि $OC = 5$ सेमी हो। C से होकर हम $DE \perp AB$ खींचते हैं। (देखिए आकृति 13.18)



आकृति 13.18

दोनों रेखाएँ AB और DE ही सड़क का चित्र है।]

7. स्केल 1 मीटर = 1 सेमी का प्रयोग करते हुए 6 मीटर चौड़ी सीधी सड़क का चित्र खींचिए।

8. एक आयताकार कमरे के फर्श का चित्र खींचिए जिसकी लम्बाई और चौड़ाई क्रमशः 6 मीटर और 4 मीटर हैं। स्केल 1 मीटर = 1 सेमी का प्रयोग कीजिए।

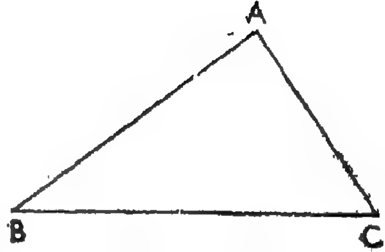
9. स्केल 10 मीटर = 1 सेमी का प्रयोग करते हुए 100 मीटर लम्बे और 75 मीटर चौड़े एक आयताकार खेत का चित्र बनाइए।

एकक XIV

त्रिभुज

14.1 त्रिभुज

आइए आकृति 14.1 को देखें। क्या आप इस आकृति का नाम बता सकते हैं? यह एक त्रिभुज (*triangle*) है। आइए हम त्रिभुजों का पुनरावलोकन करें तथा उनके कुछ और तथ्यों का अध्ययन करें।



आकृति 14.1 : त्रिभुज

त्रिभुज तीन असंरेखी बिंदुओं जैसे, B, C और A को जोड़ने से प्राप्त रेखाखंडों BC, CA और AB से बनता है। (देखिए आकृति 14.1) इस प्रकार हमें त्रिभुज ABC प्राप्त होता है। त्रिभुज शब्द को व्यक्त करने के लिए संकेत '$\triangle$' का प्रयोग किया जाता है। हम देखते हैं कि $\triangle ABC$ को $\triangle BCA$, $\triangle CAB$, $\triangle BAC$ इत्यादि भी कहा जा सकता है।

रेखाखंड BC, CA और AB इस त्रिभुज की **भुजाएँ** (*sides*) कहलाती हैं तथा इन रेखाखंडों की लम्बाइयाँ **भुजाओं की लम्बाइयाँ** कहलाती हैं। बिंदु A, B और C **शीर्ष** (*vertices*) कहलाते हैं। शीर्ष A भुजा BC के **सम्मुख** (*opposite*), शीर्ष B भुजा CA के **सम्मुख** तथा शीर्ष C भुजा AB के **सम्मुख** है।

तीनों रेखाखंडों BC, CA और AB से कितने (अंतः) कोण बनते हैं? स्पष्ट है, तीन कोण। ये कोण BAC, CBA तथा ACB हैं। इन्हें केवल $\angle A$, $\angle B$ तथा $\angle C$ से भी व्यक्त किया जा सकता है। $\angle A$ भुजा BC के **सम्मुख**, $\angle B$ भुजा CA के **सम्मुख** तथा $\angle C$ भुजा AB के **सम्मुख** है।

आकृति 14.1 में हम कह सकते हैं कि भुजा BC त्रिभुज का **आधार** (*base*) तथा सम्मुख कोण A **शीर्ष कोण** (*vertical angle*) है। परन्तु, वास्तव में त्रिभुज की किसी भी भुजा को आधार तथा उसके सम्मुख कोण को शीर्ष कोण माना जा सकता है।

भुजाओं की लम्बाइयों का योग अर्थात् $BC+CA+AB$, $\triangle ABC$ का **परिमाप** (*perimeter*) कहलाता है।

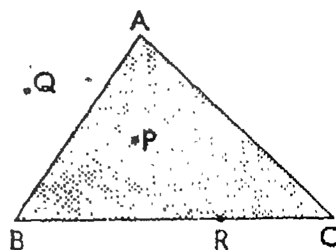
14.2 त्रिभुज का अभ्यंतर और बहिर्भाग

हम किसी घर या खेत के अभ्यंतर और बहिर्भाग के बारे बात किया करते हैं। हम यह भी कहा करते हैं कि फुटबाल का खिलाड़ी D के अंदर अर्थात् अभ्यंतर [प्रत्येक गोल (goal) के पास का अर्धवृत्ताकार क्षेत्र] में है।

आइए एक भूखंड और उसकी सीमा (boundary) पर विचार करें। सीमा के अंदर कोई भी वस्तु उसके अभ्यंतर में है तथा सीमा के बाहर कोई भी वस्तु उसके बहिर्भाग में है। यदि कोई व्यक्ति अभ्यंतर से बहिर्भाग या विलोमतः बहिर्भाग से अभ्यंतर में जाना चाहे तो उसे सीमा को अवश्य पार करना पड़ेगा।

हम कोण के अभ्यंतर और बहिर्भाग की चर्चा करते समय इन कल्पनाओं का पहले ही प्रयोग कर चुके हैं। अब हम त्रिभुज के लिए भी ऐसा ही करेंगे।

आइए कागज़ के पन्ने पर एक त्रिभुज ABC खींचें। त्रिभुज, तल के (सभी) बिंदुओं को तीन भागों में विभाजित करता है। **पहला** ऐसे बिंदु P, जिन्हें हम कहते हैं कि ये त्रिभुज के अंदर स्थित हैं। **दूसरा** ऐसे बिंदु Q, जिन्हें हम कहते हैं कि ये त्रिभुज के बाहर स्थित हैं। तथा **तीसरा** ऐसे बिंदु R, जो कि त्रिभुज की किसी एक भुजा पर स्थित हैं। (देखिए आकृति 14.2) तल का वह भाग जिसमें ऐसे सभी बिंदु P स्थित हैं **त्रिभुज का अभ्यंतर** (*interior of the triangle*) कहलाता है। R जैसे सभी बिंदु **अभ्यंतर की सीमा** (*boundary of the interior*) बनाते हैं। $\triangle ABC$ के अभ्यंतर और उस की सीमा को मिलाकर **त्रिभुजाकार क्षेत्र** (*triangular region*) कहा जाता है।



आकृति 14.2

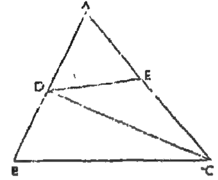
तल का वह भाग जिसमें ऐसे सभी बिंदु Q स्थित हैं, **त्रिभुज का बहिर्भाग** (*exterior of the triangle*) कहलाता है। अब आकृति 14.2 में बिंदुओं P और A को देखें। ये BC के एक ही ओर स्थित हैं। P और B के बारे में आप क्या देखते हैं? P और C के बारे में आप क्या देखते हैं? हम देखेंगे कि अभ्यंतर के सभी बिंदु जैसे कि P त्रिभुज की प्रत्येक भुजा के उसी ओर स्थित हैं जिस ओर कि सम्मुख शीर्ष।

आइए अब बिंदुओं Q और A को देखें। ये BC के एक ही ओर स्थित हैं। इसी प्रकार बिंदु Q और B, CA के एक ही ओर स्थित हैं। परन्तु बिंदुओं Q और C के बारे में आप क्या देखते हैं? ये AB के विपरीत ओर स्थित हैं। इस प्रकार त्रिभुज के बहिर्भाग के बिंदु उसके अभ्यंतर के बिंदुओं से कुछ भिन्न प्रकार के हैं। क्या अब हम त्रिभुज के अभ्यंतर और बहिर्भाग को पहिचानने का कोई नियम बता सकते हैं? नियम है कि

त्रिभुज के अभ्यंतर के सभी बिंदु प्रत्येक भुजा के उसी ओर स्थित होते हैं जिस ओर कि सम्मुख शीर्ष।

हम यह भी देखते हैं कि हम त्रिभुज की एक भुजा को बिना पार किए (काटे) P से Q या Q से P तक नहीं पहुँच सकते।

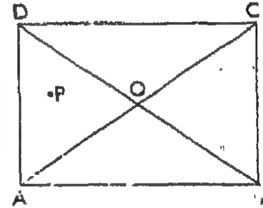
प्रश्नावली 14.1



आकृति 14.3

1. आकृति 14.3 में कितने विभिन्न त्रिभुज हैं? प्रत्येक का नाम बताइए।

2. आकृति 14.3 के कौन कौन से त्रिभुजों के बहिर्भाग में B स्थित हैं? कौन कौन से त्रिभुजों की कम से कम एक भुजा पर D स्थित है?



आकृति 14.4

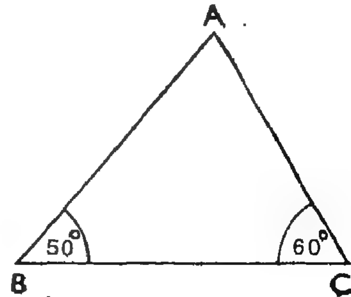
3. आकृति 14.4 के सभी त्रिभुजों के नाम बताइए। इनमें से कौन से त्रिभुजों के अभ्यंतर में P स्थित है? कौनसे त्रिभुजों के बहिर्भाग में A स्थित है? कितने त्रिभुजों की कम से कम एक भुजा पर A स्थित है? कितने त्रिभुजों की कम से कम एक भुजा पर B स्थित है?

14.3 त्रिभुज के कोणों का योग

14.3.1 एक नुकीली पेंसिल की सहायता से आइए अपने सेट स्कावायर की बाहरी रूप रेखा (*out lines*) खींचें। प्रत्येक एक त्रिभुज है। अब प्रत्येक त्रिभुज के कोणों को मापें। 30° सेट स्क्वायर से बने त्रिभुज के कोण 30°, 90°, और 60° होंगे। इन कोणों का योग क्या है? 45° सेट स्क्वायर से बने त्रिभुज में कोणों का योग क्या है? हम देखेंगे कि प्रत्येक त्रिभुज में कोणों का योग 180° है।

14.3.2 आइए एक त्रिभुज ABC बनाएं और उसके कोणों को चाँदे से मापें। इसके कोणों का योग क्या है?

14.3.3 आइए एक रेखाखंड BC लें और बिंदु B पर 50° का एक कोण बनाएँ तथा C पर 60° का। (देखिए आकृति 14.5) हमें एक $\triangle ABC$ प्राप्त होता है। अब $\angle A$ को मापें। $\angle A + \angle B + \angle C$ कितना है?



आकृति 14.5

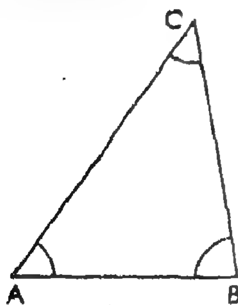
B और C पर विभिन्न कोण बनाकर, आइए इस प्रयोग को दोहराएँ तथा प्रत्येक स्थिति में तीनों कोणों का योग ज्ञात करें। हम देखेंगे कि

त्रिभुज के कोणों का योग 180° होता है।

14.4 न्यून कोण, समकोण और अधिक कोण त्रिभुज

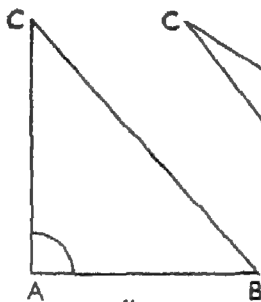
हम त्रिभुजों का उनके कोणों के अनुसार निम्न प्रकार वर्गीकरण करते हैं :

वह त्रिभुज जिसके सभी कोण न्यून कोण हों **न्यून कोण त्रिभुज** (*acute triangle*) कहलाता है। [देखिए आकृति 14.6 (i)]



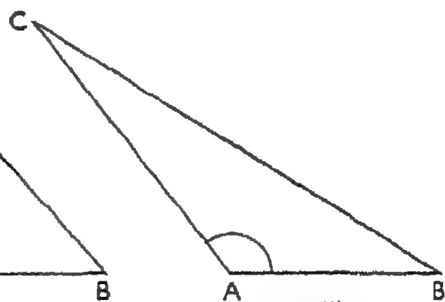
(i)

न्यूनकोण त्रिभुज



(ii)

समकोण त्रिभुज



(iii)

अधिककोण त्रिभुज

आकृति 14.6

वह त्रिभुज जिसका एक कोण समकोण हो **समकोण त्रिभुज** (*right triangle*) कहलाता है। [देखिए आकृति 14.6 (ii)] दोनों सेट स्क्वायर समकोण त्रिभुजों के उदाहरण हैं।

अंत में, वह त्रिभुज जिसका एक कोण अधिक कोण हो **अधिक कोण त्रिभुज** (*obtuse triangle*) कहलाता है। [देखिए आकृति 14.6 (iii)]

प्रश्नावली 14.2

1. क्या आप एक से अधिक समकोण वाला एक त्रिभुज खींच सकते हैं ?
2. क्या आप एक से अधिक, अधिक कोण वाला एक त्रिभुज खींच सकते हैं ?

3. $\triangle ABC$ में

(i) $\angle A = 40°$, $\angle B = 60°$। $\angle C$ ज्ञात कीजिए।
(ii) $\angle A = \angle B = 60°$। $\angle C$ ज्ञात कीजिए।
(iii) $\angle A = 110°$, $\angle B = 40°$। $\angle C$ ज्ञात कीजिए।
(iv) $\angle B = 90°$, $\angle A = \angle C$। $\angle A$ और $\angle C$ ज्ञात कीजिए।

4. प्रश्न 3 में कौनसे त्रिभुज

(क) न्यून कोण त्रिभुज
(ख) समकोण त्रिभुज
(ग) अधिक कोण त्रिभुज
हैं ?

5. क्या निम्न कथन सत्य हैं ?

(i) किसी त्रिभुज के सभी कोण न्यून कोण हो सकते हैं।
(ii) किसी त्रिभुज का एक कोण समकोण तथा शेष कोण न्यून कोण हो सकते हैं।
(iii) किसी त्रिभुज के दो कोण समकोण और तीसरा कोण न्यून कोण हो सकता है।
(iv) किसी त्रिभुज में दो अधिक कोण हो सकते हैं।

6. एक त्रिभुज ABC खींचिए। AB को एक बिंदु D तक इस प्रकार बढ़ाइए कि B, A और D के मध्य स्थित हो। $\angle CBD$, $\angle ACB$ तथा $\angle CAB$ को मापिए तथा जाँच कीजिए कि $\angle CBD = \angle CAB + \angle ACB$

7. एक त्रिभुज ABC में $AB = 2.4$ सेमी, $AC = 1.8$ सेमी तथा $BC = 2.4$ सेमी है। उसका परिमाप ज्ञात कीजिए।

8. $\triangle ABC$ का परिमाप $\triangle DEF$ के परिमाप से दुगुना है। क्या $\angle A + \angle B + \angle C = \angle D + \angle E + \angle F$ है ?

9. किसी त्रिभुज के कोण $1:2:3$ के अनुपात में हैं। उसके कोण ज्ञात कीजिए। यह किस प्रकार का त्रिभुज है ?

10. किसी त्रिभुज के कोण $1 : 1 : 2$ के अनुपात में हैं। उसके कोण ज्ञात कीजिए

11. कोई त्रिभुज ABC खींचिए और AB का मध्य-बिंदु X मान लीजिए। $XY \parallel BC$ इस प्रकार खींचिए कि Y भुजा AC पर स्थित हो।

(i) AY और YC को मापिए और जांच कीजिए कि ये बराबर हैं।
(ii) XY और BC को मापिए और जाँच कीजिए कि $XY = \frac{1}{2} BC$

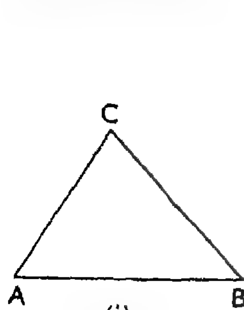
12. कोई त्रिभुज ABC खींचिए। BC को एक बिंदु E तक इस प्रकार बढ़ाइए कि C बिंदुओं B और E के मध्य स्थित हो। सेट स्क्वायर का प्रयोग करते हुए $CF \parallel BA$ खींचिए।

(i) $\angle BAC$ और $\angle ACF$ को मापिए तथा जाँच कीजिए कि ये बराबर हैं।
(ii) $\angle ABC$ और $\angle FCE$ को मापिए तथा जाँच कीजिए कि ये बराबर हैं।

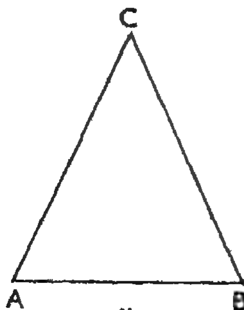
14.5 विषमबाहु, समद्विबाहु और समबाहु त्रिभुज

हम त्रिभुजों का उनकी भुजाओं के अनुसार निम्न प्रकार वर्गीकरण करते हैं :

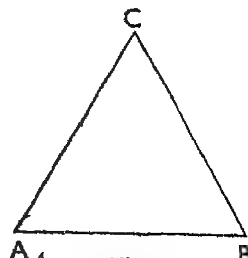
यदि त्रिभुज की सभी भुजाएँ लम्बाई में असमान हो तो वह **विषमबाहु त्रिभुज** (*scalene triangle*) कहलाता है। [देखिए आकृति 14.7 (i)] हम प्रायः इस नाम का प्रयोग नहीं करते क्योंकि अधिकतर जो त्रिभुज हम खींचते हैं या जो हमें देखने को मिलते हैं वे विषमबाहु ही होते हैं।



(i)
विषमबाहु त्रिभुज



(ii)
समद्विबाहु त्रिभुज



(iii)
समबाहु त्रिभुज

आकृति 14.7

यदि त्रिभुज की दो भुजाएँ बराबर (समान) हों तो वह **समद्विबाहु त्रिभुज** (*isosceles triangle*) कहलाता है। [देखिए आकृति 14.7 (ii)]

यदि त्रिभुज की सभी भुजाएँ बराबर हों तो वह **समबाहु त्रिभुज** (*equilateral triangle*) कहलाता है। [देखिए आकृति 14.7 (iii)]

हम एकक *XVI* में देखेंगे कि विषमबाहु त्रिभुज के सभी कोण असमान होते हैं, समद्विबाहु त्रिभुज में बराबर भुजाओं के सम्मुख कोण बराबर होते हैं तथा यह कि समबाहु त्रिभुज के सभी कोण बराबर होते हैं।

14.6 त्रिभुज की किन्हीं दो भुजाओं का योग

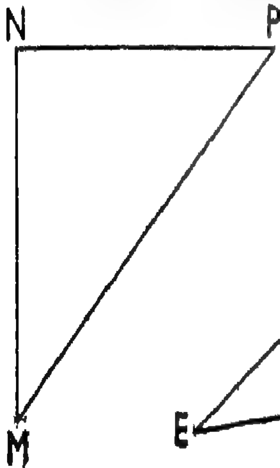
आइए एक त्रिभुज ABC खींचें और उसकी भुजाएँ मापें।

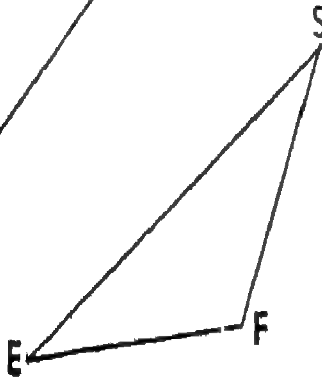
क्या $BC+CA>AB$ है ? क्या $BC+AB>CA$ है ? क्या $CA+AB>BC$ है ?

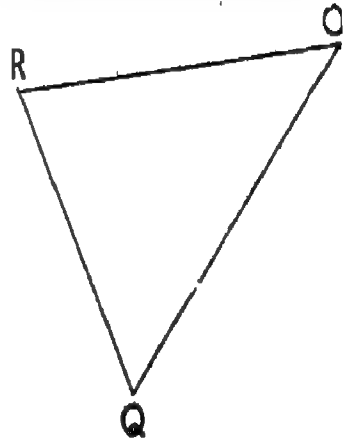
अब इस प्रयोग को कुछ और त्रिभुजों के साथ दोहराएँ। आप क्या देखते हैं ? हम देखेंगे कि **त्रिभुज की किन्हीं दो भुजाओं का योग तीसरी भुजा से बड़ा होता है।**

प्रश्नावली 14.3

1. आकृति 14.8 में तीन समद्विबाहु त्रिभुज दिए हैं। उनकी बराबर भुजाओं के नाम बताइए।







(i) (ii) (iii)

आकृति 14.8

2. एक त्रिभुज की प्रत्येक भुजा, परिमाप की एक तिहाई है। यह त्रिभुज किस प्रकार का है ?

3. एक समद्विबाहु त्रिभुज का एक कोण $120°$ है। अन्य दोनों कोण ज्ञात कीजिए।

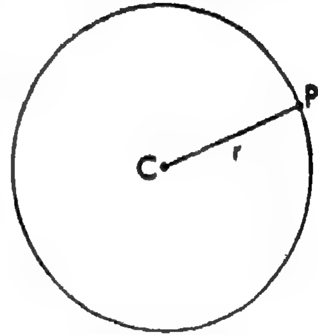
4. एक समद्विबाहु त्रिभुज का एक कोण $90°$ है। अन्य दोनों कोण ज्ञात कीजिए।

एकक XV

वृत्त

15.1 वृत्त

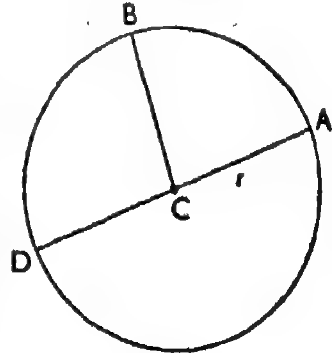
हम पिछली कक्षाओं में वृत्त से संबंधित अनेक संकल्पनाओं जैसे कि केन्द्र (*centre*), त्रिज्या (*radius*), वृत्तखंड (*segment of a circle*), त्रिज्यखंड (*sector*), उसका अभ्यंतर (*interior*) और बहिर्भाग (*exterior*) के बारे में पहले ही पढ़ चुके हैं। हम इस एकक में इनका संक्षेप में पुनरावलोकन करेंगे तथा कुछ और संकल्पनाओं का अध्ययन करेंगे। **वृत्त,** तल के ऐसे सभी बिंदुओं से बना होता है जो कि तल में एक दिए हुए बिंदु, मान लीजिए C, से समान दूरी, मान लीजिए r, पर हों। (देखिए आकृति 15.1) बिंदु C वृत्त का **केन्द्र** तथा दूरी r **त्रिज्या** कहलाती है। कभी कभी 'वृत्त' शब्द को संकेत '$\odot$' से भी व्यक्त किया जाता है।



आकृति 15.1 : वृत्त

मान लीजिए बिंदु A केन्द्र C और त्रिज्या r वाले किसी वृत्त पर स्थित है। C और A को जोड़िए। (देखिए आकृति 15.2) रेखाखंड CA जिसकी लम्बाई r है, भी वृत्त की त्रिज्या कहलाती है। यह बेहतर होगा कि इस रेखाखंड को कोई दूसरा नाम दिया जाए।

त्रिज्य रेखाखंड (*radial segment*) एक अच्छा नाम होगा। परन्तु त्रिज्या शब्द का दो अर्थों में प्रयोग चिर प्रचलित है और हम इन्हीं प्रयोगों का अनुकरण करेंगे। हम देखते हैं कि दूरी r के अर्थ में वृत्त की केवल एक ही त्रिज्या है जबकि केन्द्र को वृत्त के किसी बिंदु से जोड़ने वाले रेखाखंड के अर्थ में चाहें जितनी त्रिज्याएँ हो सकती हैं।



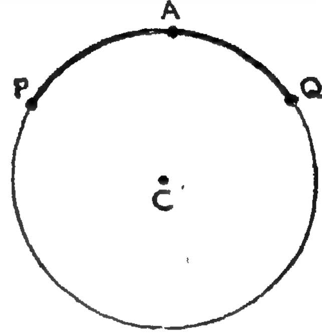
आकृति 15.2

यदि वृत्त पर कोई अन्य बिंदु B स्थित हो तो रेखाखंड CB की वही लम्बाई होगी जो कि रेखाखंड CA की है। इस प्रकार CB, CA के बराबर है। हम यह निष्कर्ष निकालते हैं कि **वृत्त की सभी त्रिज्याएँ बराबर होती हैं।**

आइए, आकृति 15.2 में त्रिज्या AC को बढ़ाएं जिससे वह वृत्त से पुनः D पर मिले। रेखाखंड AD, वृत्त का **व्यास** (*diameter*) कहलाता है। स्पष्ट है कि $AC=CD$। इस प्रकार व्यास $AD=2AC$।

दूसरे शब्दों में, **व्यास की लम्बाई $2r$ अर्थात् त्रिज्या की दुगुनी है।** हम केवल यह भी कह सकते हैं कि वृत्त का व्यास $2r$ है। वृत्त की परिमाप उसकी **परिधि** (*circumference*) कहलाती है।

15.2 चाप और जीवाएँ

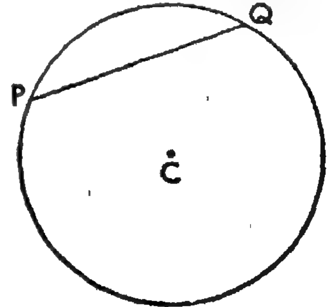


माना केन्द्र C के किसी वृत्त पर P और Q दो बिंदु हैं। (देखिए आकृति 15.3) बिंदु P और Q वृत्त को दो भागों में विभाजित करते हैं। प्रत्येक भाग **चाप** (*arc*) कहलाता है।

आकृति 15.3

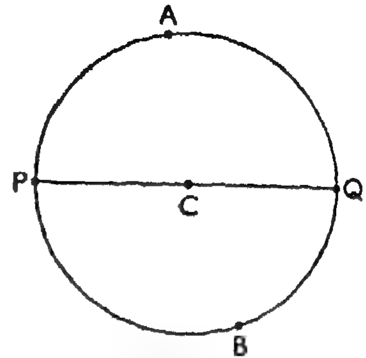
उपर्युक्त आकृति में दोनों में से किसी भी भाग को चाप PQ कह सकते हैं। हम चाप PQ को व्यक्त करने के लिए संकेत $\widehat{PQ}$ का प्रयोग करते हैं। चूँकि यहाँ दो चाप PQ हैं, इसलिए हमारे कार्य में इससे कुछ भ्रम उत्पन्न हो सकता है। इस भ्रम से बचने के लिए हम वृत्त पर एक तीसरा बिंदु, मान लीजिए A, जैसा कि आकृति में दिखाया गया है अंकित कर लेते हैं और इस भाग को चाप PAQ अर्थात् $\widehat{PAQ}$ कहते हैं।

सामान्यतः दोनों चापों PQ में से एक, दूसरे चाप से बड़ा होता है। बड़ा चाप **दीर्घ चाप** (*major arc*) तथा छोटा चाप **लघु चाप** (*minor arc*) कहलाता है। इसका दोनों चापों की पहिचान करने में भी उपयोग किया जाता है।



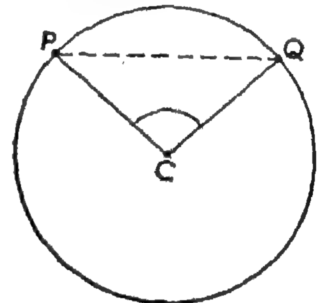
यदि हम वृत्त पर स्थित किन्हीं दो बिंदुओं P और Q को मिलाएं तो रेखाखंड PQ वृत्त की **जीवा** (*chord*) कहलाती है। (देखिए आकृति 15.4) निस्संदेह, जब जीवा केन्द्र C से होकर जाए तो वह व्यास बन जाती है। (देखिए आकृति 15.5) दूसरे शब्दों में, वृत्त का व्यास वह जीवा है जो केन्द्र से होकर जाए।

आकृति 15.4



आकृति 15.5

वृत्त का व्यास जैसे कि PCQ वृत्त को दो बराबर चापों, मान लीजिए, PAQ और PBQ में विभाजित करता है। आकृति को व्यास PQ के अनुदिश मोड़ कर यह जाँच की जा सकती है कि दोनों चाप बराबर हैं। मोड़ने पर चाप PAQ ठीक चाप PBQ पर पड़ेगा। इनमें से प्रत्येक चाप **अर्धवृत्तीय चाप** (*semicircular arc*) या केवल **अर्धवृत्त** (*semicircle*) कहलाता है।



आकृति 15.6

माना C केन्द्र वाले वृत्त पर P और Q कोई दो बिंदु हैं। आइए P और C तथा Q और C को जोड़ें। (देखिए आकृति 15.6) हम कहते हैं कि **चाप PQ केन्द्र पर कोण PCQ बनाता है।** हम यह भी कह सकते हैं कि **जीवा PQ केन्द्र पर कोण PCQ बनाती है।** वृत्त का व्यास केन्द्र पर कैसा कोण बनाता है? स्पष्ट है, एक ऋजु कोण।

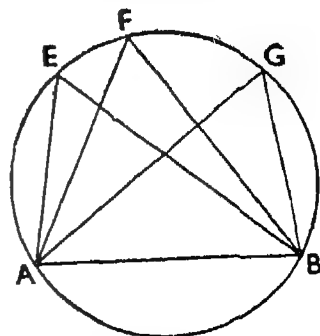
प्रश्नावली 15.1

1. निम्न त्रिज्याओं के वृत्त खींचिए :
   (क) 4 सेमी (ख) 2.6 सेमी (ग) 3.5 सेमी
2. निम्न व्यास वाले वृत्त खींचिए :
   (क) 6 सेमी (ख) 8.4 सेमी (ग) 5 सेमी
3. बताइए कि निम्न में से कौन से कथन सत्य हैं और कौन से असत्य।
   (क) वृत्त की सभी त्रिज्याएँ बराबर होती हैं।
   (ख) त्रिज्या, वृत्त की एक जीवा है।
   (ग) वृत्त का व्यास उसकी त्रिज्या का दुगुना होता है।
   (घ) एक वृत्त के दो केन्द्र हो सकते हैं।

4. केन्द्र O और 3.5 सेमी त्रिज्या लेकर एक वृत्त खींचिए। दो त्रिज्याएँ OA और OB इस-प्रकार खींचिए कि $\angle AOB = 60°$ हो। (चाँदे का प्रयोग कीजिए) A और B को जोड़िए और जीवा AB मापिए। क्या $\triangle OAB$ समबाहु त्रिभुज है?

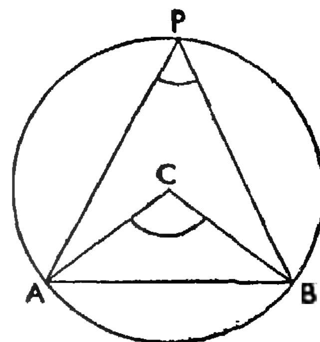
5. C को केन्द्र मानकर किसी भी त्रिज्या का एक वृत्त खींचिए। कोई तीन जीवाएँ खींचिए जो केन्द्र से होकर न जाएँ। साथ ही वृत्त का एक व्यास भी खींचिए। व्यास और प्रत्येक जीवा की लम्बाई मापिए। कौन सबसे लम्बा है? हम देखेंगे कि **व्यास, वृत्त की सबसे लम्बी जीवा है।**

6. किसी भी त्रिज्या का एक वृत्त और उसकी एक जीवा AB खींचिए जैसा कि आकृति 15.7 में दिखाया गया है। माना जीवा के एक ही ओर वृत्त पर तीन बिंदु E, F और G हैं। इन बिंदुओं को A और B से मिलाइए। $\angle AEB$, $\angle AFB$ और $\angle AGB$ मापिए। क्या ये बराबर हैं?



आकृति 15.7

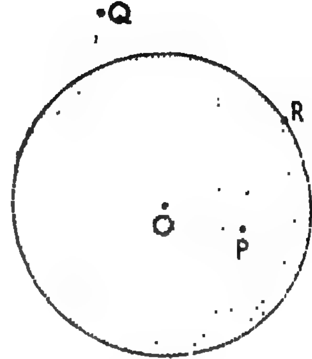
7. C केन्द्र मानकर किसी भी त्रिज्या का एक वृत्त खींचिए। मान लीजिए AB कोई जीवा है और P वृत्त पर स्थित कोई बिंदु है जैसा कि आकृति 15.8 में दिखाया गया है। AP, BP, AC और BC खींचिए। $\angle APB$ तथा $\angle ACB$ को मापिए तथा जाँच कीजिए कि $\angle ACB = 2\angle APB$।



आकृति 15.8

8. वृत्त का एक चाप केन्द्र पर $100°$ का कोण बनाता है। तदनुरूपी दीर्घ चाप द्वारा केन्द्र पर बनाया गया कोण ज्ञात कीजिए।

15.3 वृत्त का अभ्यंतर और बहिर्भाग



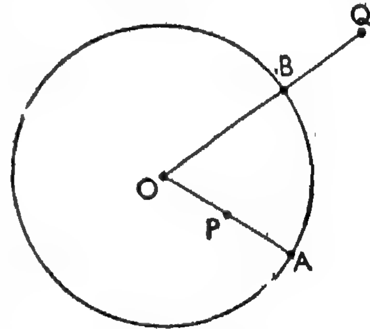
आकृति 15.9

माना तल में कोई बिंदु O है। आइए O को केन्द्र मानकर और कोई त्रिज्या r लेकर एक वृत्त खींचें। तब, वृत्त तल के (सभी) बिंदुओं को तीन भागों में विभाजित करता है। पहला ऐसे बिंदु P, जो वृत्त के अन्दर स्थित हैं। दूसरा ऐसे बिंदु Q, जो वृत्त के बाहर स्थित हैं तथा तीसरा ऐसे बिंदु R, जो कि वृत्त पर स्थित हैं। (देखिए आकृति 15.9) तल का वह भाग जिसमें ऐसे सभी बिंदु P स्थित हैं वृत्त का **अभ्यंतर** कहलाता है। R जैसे सभी बिंदुओं से **अभ्यंतर की सीमा** बनती है। वृत्त के अभ्यंतर और उसकी सीमा को मिलाकर **वृत्तीय क्षेत्र** (*circular region*) कहते हैं।

तल का वह भाग जिसमें ऐसे सभी बिंदु Q स्थित हों **वृत्त का बहिर्भाग** कहलाता है।

माना O केन्द्र और त्रिज्या r वाले वृत्त के अभ्यंतर में P कोई बिंदु है। आइए O और P को जोड़ें और OP को बढ़ाएँ ताकि वह वृत्त को A पर काटे। (देखिए आकृति 15.10) स्पष्ट है कि OP वृत्त की त्रिज्या OA से छोटा है। दूसरे शब्दों में, वृत्त के अभ्यंतर के किसी भी बिंदु P के लिए $OP<r$ होता है।

पुनः माना Q वृत्त के बहिर्भाग में कोई बिंदु है। अब O और Q को जोड़ें। रेखाखंड OQ वृत्त को माना बिंदु B पर काटता है। (देखिए आकृति 15.10) स्पष्ट है $OQ>OB$। परन्तु OB वृत्त की एक त्रिज्या है। इस प्रकार, वृत्त के बहिर्भाग के किसी भी बिंदु Q के लिए $OQ>r$ होता है।



आकृति 15.10

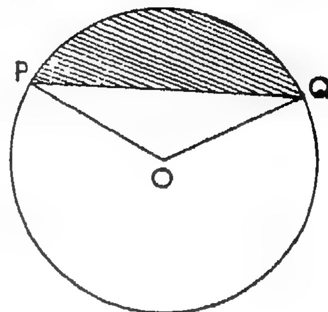
क्या अब हम यह निर्धारित करने का कोई नियम बता सकते हैं कि एक बिंदु वृत्त के अभ्यंतर में है या बहिर्भाग में है या कि वृत्त पर है? नियम है कि :

बिंदु P केन्द्र O और त्रिज्या r वाले वृत्त के अभ्यंतर में, वृत्त पर या वृत्त के बहिर्भाग में स्थित होगा जबकि क्रमश : $OP<r$, $OP=r$ या $OP>r$ हो।

हम यह भी देखते हैं कि यदि हमें अभ्यंतर के किसी बिंदु से बहिर्भाग के किसी बिंदु तक या विलोमतः बहिर्भाग के किसी बिंदु से अभ्यंतर के किसी बिंदु तक जाना हो तो हमें वृत्त को अवश्य ही पार करना (काटना) पड़ेगा।

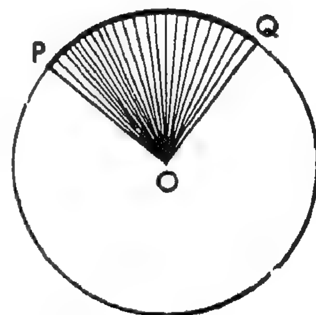
15.4 वृत्त-खंड और त्रिज्यखंड

माना O केन्द्र वाले वृत्त पर P और Q कोई दो बिंदु हैं। आइए P और Q को जोड़ें। **जीवा PQ** वृत्तीय क्षेत्र को दो भागों में विभाजित करता है। इनमें से प्रत्येक एक **वृत्त-खंड** (*segment of a circle*) कहलाता है। हम इनमें से प्रत्येक को **वृत्त-खंड** PQ कहते हैं। स्पष्ट है जिस वृत्त-खंड PQ में केन्द्र स्थित है वह बड़ा वृत्त-खंड है। **परन्तु यहाँ हम यह मान लेते हैं कि जब तक कि अन्यथा कहा न जाए वृत्त-खंड PQ से हमारा तात्पर्य दोनों वृत्त-खंडों में से छोटे वाले वृत्त-खंड से होगा। आकृति 15.11 में वृत्त-खंड PQ को छायामय** (*shaded*) **दिखाया गया है।**



आकृति 15.11

पुनः माना O केन्द्र वाले वृत्त पर P और Q कोई दो बिंदु हैं। आइए O और P तथा O और Q को जोड़ें। दोनों त्रिज्याएँ OP और OQ वृत्तीय क्षेत्र को दो भागों में विभाजित करती हैं। इनमें से प्रत्येक वृत्त का **त्रिज्यखंड** (*sector*) कहलाता है। हम इनमें से प्रत्येक को त्रिज्यखंड OPQ कहते हैं। स्पष्ट है जिस त्रिज्यखंड OPQ में दीर्घ चाप सम्मिलित है वही दोनों त्रिज्यखंडों में बड़ा है। परन्तु **हम यह मान लेते हैं कि जब तक कि अन्यथा कहा न जाए त्रिज्यखंड OPQ से हमारा तात्पर्य छोटे वाले त्रिज्यखंड से होगा अर्थात् उस त्रिज्यखंड से होगा जिसमें लघुचाप सम्मिलित है।**



आकृति 15.12

आकृति 15.12 में त्रिज्यखंड OPQ को छायामय दिखाया गया है। $\angle POQ$ **त्रिज्यखंड का कोण** (*angle of the sector*) कहलाता है।

प्रश्नावली 15.2

1. दो बिंदु O और P दिए हैं। O को केन्द्र मानकर एक वृत्त खींचिए जो कि बिंदु P से होकर जाए।

2. दो बिंदु C और Q दिए हैं। C को केन्द्र मानकर एक ऐसा वृत्त खींचिए कि Q उसके अभ्यंतर में रहे।

3. दो बिंदु C और R दिए हैं। C को केन्द्र मानकर एक ऐसा वृत्त खींचिए कि R उसके बहिर्भाग में रहे।

4. एक वृत्त और उसके बहिर्भाग में एक रेखा खींचिए। क्या आप एक ऐसी रेखा खींच सकते हैं जो कि पूर्णतया वृत्त के अभ्यंतर में हो?

5. 3 सेमी त्रिज्या का एक वृत्त खींचिए और इसका एक वृत्त-खंड PQ ऐसा बनाइए कि जीवा PQ की लम्बाई (i) 3 सेमी, (ii) 4 सेमी, (iii) 5 सेमी हो।

6. 3.5 सेमी त्रिज्या का एक वृत्त खींचिए तथा इसमें वृत्त-खंड PQ इस प्रकार छायांकित (*shade*) कीजिए कि जीवा PQ द्वारा केन्द्र पर बनाया गया कोण
(i) $30°$, (ii) $45°$, (iii) $60°$ हो।
[इन कोणों को बनाने के लिए चाँदे का प्रयोग कीजिए।]

7. 4 सेमी त्रिज्या का एक वृत्त खींचिए और उसमें निम्न कोणों के त्रिज्यखंड खींचिए:
(i) $35°$ (ii) $75°$ (iii) $120°$ (iv) $240°$
प्रत्येक के लिए अलग अलग चित्र बनाइए।

एकक XVI

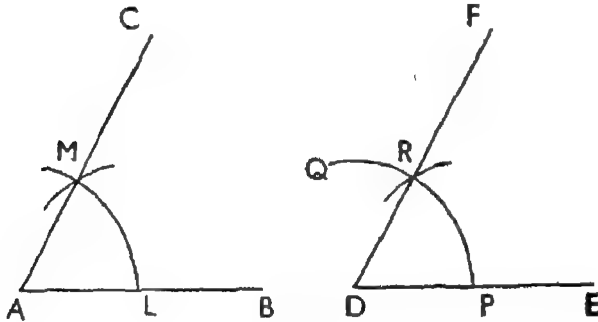
परकार से कुछ रचनाएँ

16.1 एक दिए हुए कोण के बराबर कोण बनाना

माना BAC दिया हुआ कोण है। इस कोण के बराबर कोण हम निम्न चरणों में बनाते हैं :

चरण 1 : हम एक किरण DE खींचते हैं।

चरण 2 : हम परकार (*compasses*) को थोड़ा खोलकर और उसके नुकीले सिरे को A पर रखकर एक चाप LM खींचते हैं जो कि AB को L पर तथा AC को M पर काटता है।



आकृति 16.1

चरण 3 : परकार के फैलाव में बिना कोई परिवर्तन किए अब हम उसके नुकीले सिरे को D पर रखकर DE को P पर काटता हुआ एक चाप PQ खींचते हैं।

चरण 4 : इसके बाद हम परकार को उठाकर उसके नुकीले सिरे को L पर रखते हैं तथा उसके फैलाव को इस प्रकार व्यवस्थित करते हैं कि पेंसिल वाला सिरा M पर रहे।

चरण 5 : पुन: परकार के फैलाव में बिना कोई परिवर्तन किए अब हम नुकीले सिरे को P पर रखकर चाप PQ को R पर काटता हुआ एक चाप खींचते हैं।

चरण 6 : अंत में हम D और R को जोड़कर किरण DF खींचते हैं।

तब, $\angle EDF$ **वांछित कोण है।** (देखिए आकृति 16.1)

प्रश्नावली 16.1

1. अपने चाँदे की सहायता से जाँच कीजिए कि आकृति 16.1 में दिए कोण BAC और EDF बराबर हैं।

2. पुन: आकृति 16.1 लीजिए। एक अक्स करने वाले कागज़ पर $\angle BAC$ का अक्स खींचिए। क्या आप इस अक्स को $\angle EDF$ पर इस प्रकार रख सकते हैं कि यह ठीक $\angle EDF$ पर पड़े ?

[क्या इससे एक अक्स करने के कागज़ और नुकीली पिन की सहायता से दिए हुए कोण के बराबर कोण बनाने की एक अन्य विधि का संकेत मिलता है ?]

3. एक न्यून कोण BAC दिया हुआ है। परकार का प्रयोग करते हुए निम्न कोणों के बराबर कोण बनाइए :

(क) $2\angle BAC$ (ख) $3\angle BAC$

4. चाँदे की सहायता से 50° का एक कोण बनाइए तथा परकार की सहायता से इस कोण के बराबर कोण बनाइए।

5. कोई दो न्यून कोण BAC और EDF खींचिए। परकार की सहायता से इन कोणों के योग के बराबर एक कोण बनाइए।

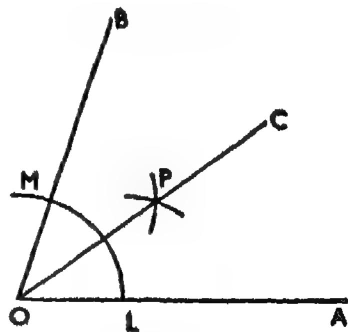
6. कोई दो कोण POQ और BAC खींचिए। परकार की सहायता से इन कोणों के अंतर के बराबर कोण बनाइए।

16.2 दिए हुए कोण को समद्विभाजित करना

आपको याद होगा कि किसी **कोण को समद्विभाजित करने का अर्थ होता है उसको दो बराबर कोणों में विभाजित करना।**

माना AOB दिया हुआ कोण है। हम इसे निम्न चरणों में समद्विभाजित करते हैं :

चरण 1 : O को केन्द्र मानकर तथा एक उपयुक्त त्रिज्या लेकर आइए OA को L तथा OB को M पर काटता हुआ एक चाप खींचें।



आकृति 16.2

चरण 2 : फिर हम L को केन्द्र मानकर तथा $\frac{1}{2}$ LM से अधिक (क्यों?) त्रिज्या लेकर एक चाप खींचते हैं।

चरण 3 : अब हम M को केन्द्र मानकर तथा चरण 2 वाली त्रिज्या लेकर एक चाप खींचते हैं जो कि माना चरण 2 में खींचे गए चाप को P पर काटता है।

चरण 4 : अब हम किरण OC खींचने के लिए O और P को जोड़ते हैं।

तब OC वांछित किरण है जो कि $\angle AOB$ को समद्विभाजित करती है। (देखिए आकृति 16.2)

क्या आप कागज़ को OC पर मोड़कर यह जाँच कर सकते हैं कि $\angle AOC = \angle COB$?

प्रश्नावली 16.2

1. चाँदे की सहायता से आकृति 16.2 में दिए कोणों AOB, AOC तथा COB को मापिए। निम्न की जाँच कीजिए :

(क) $\angle AOC = \angle COB$

(ख) $\angle AOC + \angle COB = \angle AOB$

(ग) $2\angle AOC = 2\angle COB = \angle AOB$

2. आकृति 16.2 लीजिए। $\angle AOC$ का एक अक्स करने वाले कागज़ पर अक्स खींचिए और इस अक्स को $\angle COB$ पर रखिए। क्या $\angle AOC$ और $\angle COB$ बराबर हैं?

3. चाँदे की सहायता से एक $70°$ का कोण बनाइए तथा परकार की सहायता से इसे समद्विभाजित कीजिए। प्रत्येक आधे को मापिए।

4. चाँदे की सहायता से एक $132°$ का कोण बनाइए तथा परकार से इसे समद्विभाजित कीजिए। चाँदे से प्रत्येक आधे को मापिए।

*5. $120°$ का एक कोण बनाइए तथा परकार से इसे चार बराबर भागों में विभाजित कीजिए। चाँदे की सहायता से जाँच कीजिए कि प्रत्येक भाग $30°$ के बराबर है।

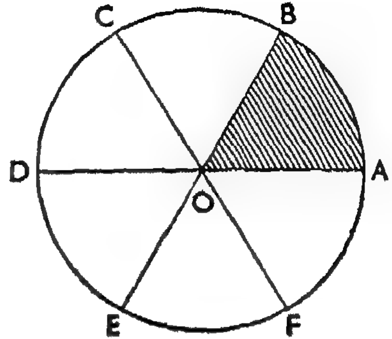
*6. कोई त्रिभुज ABC खींचिए। परकार की सहायता से कोणों A, B और C के समद्विभाजक खींचिए। आप क्या देखते हैं?

उपर्युक्त रचना को कोई और त्रिभुज लेकर दोहराइए। पुनः आप क्या देखते हैं? हम देखेंगे कि **त्रिभुज के तीनों कोणों के समद्विभाजक एक ही बिंदु पर मिलते हैं।**

16.3 एक वृत्तीय क्षेत्र को छः बराबर त्रिज्यखंडों में विभाजित करना

माना दिए हुए वृत्त का केन्द्र O है तथा त्रिज्या r है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : आइए केन्द्र O को वृत्त पर स्थित किसी बिंदु A से जोड़ें।



आकृति 16.3

चरण 2 : अब हम A को केन्द्र मानकर तथा दी हुई त्रिज्या r लेकर वृत्त को B पर काटता हुआ एक चाप खींचते हैं।

चरण 3 : फिर B को केन्द्र मानकर तथा वही त्रिज्या r लेकर हम वृत्त को C पर काटता हुआ एक अन्य चाप खींचते हैं। इसी प्रक्रिया को जारी रखते हुए हम बिंदु D, E और F प्राप्त करते हैं।

चरण 4 : OB, OC, OD, OE और OF खींचिए।

त्रिज्यखंड AOB, BOC, COD, DOE, EOF और FOA ही वृत्त के छः बराबर वांछित त्रिज्यखंड हैं। (देखिए आकृति 16.3)

कोणों AOB, BOC, COD, DOE, EOF तथा FOA को मापिए। क्या ये बराबर हैं? हम देखेंगे कि प्रत्येक कोण 60° का है।

16.4 60° का कोण बनाना

अनुच्छेद 16.3 में प्रत्येक त्रिज्यखंड का कोण 60° है। अतः इस रचना से हमें 60° का कोण बनाने के लिए निम्न चरणों का संकेत मिलता है :

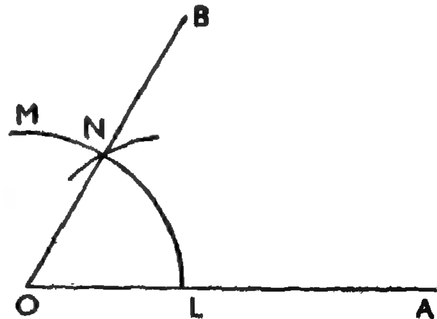
चरण 1 : हम एक किरण OA खींचते हैं।

चरण 2 : O को केन्द्र मानकर और एक उपयुक्त त्रिज्या लेकर आइए OA को L पर काटता हुआ एक चाप LM खींचें।

चरण 3 : अब L को केन्द्र मानकर तथा वही त्रिज्या (OL के बराबर) लेकर, जो कि चरण 2 में ली थी, हम चाप LM को N पर काटता हुआ एक अन्य चाप खींचते हैं।

चरण 4 : अब हम किरण OB खींचने के लिए O और N को जोड़ते हैं।

तब $\angle AOB$, 60° का वांछित कोण है। (देखिए आकृति 16.4)



आकृति 16.4

प्रश्नावली 16.3

1. 30° का एक कोण बनाइए।
   [संकेत : $30^\circ = \frac{1}{2}(60^\circ)$]
2. 120° का एक कोण बनाइए।
   [संकेत : $120^\circ = 2(60^\circ)$]
3. 15°, 75° और 240° के कोण बनाइए।

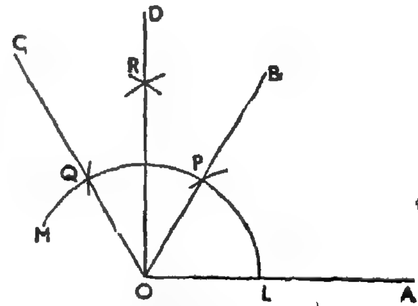
*4. 90° का एक कोण बनाइए।
   [संकेत : 60° का एक कोण बनाइए और उसे समद्विभाजित कीजिए। अब 30° में 60° या 60° में 30° जोड़िए।]

16.5 90° का कोण बनाना

प्रश्नावली 16.3 के प्रश्न 4 में हम 90° के कोण बनाने की एक विधि का संकेत दे चुके हैं। अब हम नीचे एक दूसरी विधि दे रहे हैं। निम्न चरण आवश्यक हैं :

चरण 1 : हम एक किरण OA खींचते हैं।

चरण 2 : O को केन्द्र मानकर और एक उपयुक्त त्रिज्या लेकर हम OA को L पर काटता एक चाप LM खींचते हैं।



आकृति 16.5

चरण 3 : फिर L को केन्द्र मानकर तथा वही त्रिज्या लेकर (OL के बराबर) जो कि चरण 2 में थी हम LM को P पर काटता हुआ एक चाप खींचते हैं।

चरण 4 : तब P को केन्द्र मानकर तथा वही त्रिज्या लेकर (OL के बराबर) चाप LM को एक अन्य बिंदु Q पर काटता हुआ एक चाप खींचते हैं।

चरण 5 : आइए अब किरण OB खींचने के लिए O और P को तथा किरण OC खींचने के लिए O और Q को जोड़ें।

तब, AOB और BOC में से प्रत्येक कोण 60° का है।

चरण 6 : अब हम $\angle BOC$ को किरण OD से समद्विभाजित करते हैं।
तब, $\angle AOD$, 90° का वांछित कोण हैं। (देखिए आकृति 16.5) हम यह भी कहते हैं कि OD, बिंदु O पर OA पर लम्ब है या यह कि DO, D से OA पर लम्ब है। संकेतन में हम इसे $OD \perp OA$ या $DO \perp OA$ लिखते हैं।

यह देखा जा सकता है कि 90° के कोण की रचना एक ऋजु कोण को समद्विभाजित करके भी की जा सकती है। (देखिए प्रश्नावली 16.4 का प्रश्न 3)

प्रश्नावली 16.4

1. 45° का एक कोण बनाइए।
   [**संकेत :** $45^\circ = \frac{1}{2}(90^\circ)$]
2. 135° का एक कोण बनाइए।
   [**संकेत :** $135^\circ = 90^\circ + 45^\circ$]
3. एक ऋजु कोण को समद्विभाजित करके 90° का एक कोण बनाइए।
4. निम्न कोणों की रचना कीजिए :
   $22\frac{1}{2}^\circ$, 150°

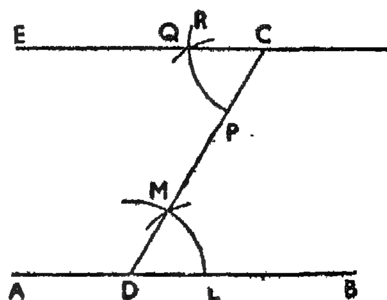
16.6 एक दी हुई रेखा के बाहर एक दिए हुए बिंदु से होकर उस रेखा के समांतर एक रेखा खींचना

माना दी हुई रेखा AB है और C उसके बाहर कोई बिंदु है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : आइए AB पर कोई बिंदु D लें और उसे C से मिला दें।

चरण 2 : फिर हम D को केन्द्र मानकर तथा एक उपयुक्त त्रिज्या लेकर AB को L तथा DC को M पर काटता हुआ एक चाप खींचते हैं।

चरण 3 : अब C को केन्द्र मानकर और वही त्रिज्या (DL के बराबर) लेकर जो कि चरण 2 में ली थी हम CD को P पर काटता हुआ एक चाप PR खींचते हैं।



आकृति 16.6

चरण 4 : अब हम परकार के नुकीले सिरे को L पर रखकर उसके फैलाव को इस प्रकार व्यवस्थित करते हैं कि पेंसिल वाला सिरा M पर रहे। तब हम उसके फैलाव में बिना कोई परिवर्तन किए तथा P को केन्द्र मानकर चाप PR को Q पर काटता हुआ एक चाप खींचते हैं।

चरण 5 : अब हम रेखा CE खींचने के लिए C और Q को जोड़ते हैं।
तब CE ही दिए हुए बिंदु C से होकर जाने वाली तथा AB के समांतर वांछित रेखा है। (देखिए आकृति 16.6)

यह देखा जा सकता है कि हमने समांतर रेखाएँ खींचने के लिए बराबर एकांतर कोणों के एक युग्म की रचना की है।

प्रश्नावली 16.5

1. कोई त्रिभुज ABC खींचिए और AB का मध्य-बिंदु D मान लीजिए। D से होकर BC के समांतर एक रेखा खींचिए जो कि AC को, मान लीजिए, E पर काटती है। AE और EC मापिए। क्या E, AC का मध्य-बिंदु है?

2. 8 सेमी लम्बाई का एक रेखाखंड AB खींचिए। इसके बाहर कोई बिंदु C लीजिए। C से होकर AB के समांतर एक रेखा खींचिए।

16.7 त्रिभुज की रचना करना जब कि उसकी तीनों भुजाएँ दी हुई हैं

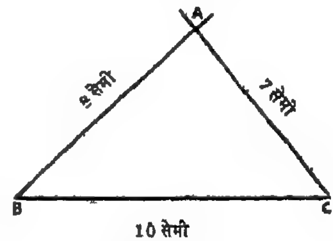
मान लीजिए हमें उस त्रिभुज की रचना करनी है जिसकी भुजाएँ क्रमशः 10 सेमी, 8 सेमी तथा 7 सेमी हैं। हम यह निम्न चरणों में करते हैं :

चरण 1 : हम 10 सेमी लम्बाई का एक रेखाखंड BC खींचते हैं।

चरण 2 : फिर B को केन्द्र मानकर और 8 सेमी त्रिज्या लेकर हम एक चाप खींचते हैं।

चरण 3 : तब, C को केन्द्र मानकर और 7 सेमी त्रिज्या लेकर हम एक दूसरा चाप खींचते हैं जो कि पहले चाप को माना A पर काटता है।

चरण 4 : अब हम A और B तथा A और C को जोड़ते हैं। ABC वांछित त्रिभुज है। (देखिए आकृति 16.7)



आकृति 16.7

प्रश्नावली 16.6

1. निम्न भुजाओं के त्रिभुज की रचना कीजिए :
   (i) 4 सेमी, 4.5 सेमी, 2.9 सेमी
   (ii) 6.2 सेमी, 6 सेमी, 9 सेमी
   (iii) 4 सेमी, 4.3 सेमी, 5 सेमी
   (iv) 5 सेमी, 4 सेमी, 3 सेमी
   (v) 13 सेमी, 12 सेमी, 5 सेमी

2. क्या आप ऐसा त्रिभुज खींच सकते हैं जिसकी भुजाएं 8 सेमी, 4 सेमी और 3 सेमी हों ?

[पहले 8 सेमी लम्बाई का एक रेखाखंड, मान लीजिए, BC खींचिए। तब B को केन्द्र मानकर तथा 4 सेमी त्रिज्या लेकर एक चाप खींचिए। पुन: C को केन्द्र मानकर और 3 सेमी त्रिज्या लेकर एक दूसरा चाप खींचिए। (देखिए आकृति 16.8) क्या दोनों चाप परस्पर काटते हैं ? हम देखते हैं कि यदि हम चापों की जगह पूरे वृत्त भी खींच लें तो भी दोनों चाप परस्पर नहीं काटते। अत: हमारी **रचना असफल रहती है**। हम 8 सेमी, 4 सेमी और 3 सेमी भुजाओं का त्रिभुज नहीं खींच सकते। क्या आप इसका कारण बता सकते हैं ? क्या आपको याद है कि **त्रिभुज में किन्हीं भी दो भुजाओं का योग तीसरी भुजा से अवश्य ही बड़ा होना चाहिए।**



आकृति 16.8

3. एक समबाहु त्रिभुज की रचना कीजिए जिसकी प्रत्येक भुजा 6 सेमी हो। इसके कोण मापिए। आप क्या देखते हैं ?

प्रत्येक भुजा 5 सेमी लेकर इस रचना को दोहराइए। पुन: आप क्या देखते हैं ?
हम देखेंगे कि **समबाहु त्रिभुज के सभी कोण बराबर होते हैं। साथ ही प्रत्येक कोण 60° का है।**

4. एक त्रिभुज ABC खींचिए जिसमें $BC=6$ सेमी तथा $AB=AC=5$ सेमी हो। यह त्रिभुज किस प्रकार का है ? कोणों B और C को मापिए। आप क्या देखते हैं ?

5. एक त्रिभुज ABC खींचिए जिसमें $BC=7$ सेमी, $AB=6$ सेमी तथा $CA=7$ सेमी हो। यह त्रिभुज किस प्रकार का है ? कोणों A और B को मापिए। क्या ये बराबर हैं ?

प्रश्न 4 और 5 से हम देखेंगे कि **समद्विबाहु त्रिभुज में बराबर (समान) भुजाओं के सम्मुख कोण बराबर होते हैं।**

6. 5 सेमी, 6 सेमी तथा 7 सेमी भुजाओं वाला एक त्रिभुज खींचिए और उसके कोण मापिए।

7 4.5 सेमी, 5.6 सेमी तथा 7 सेमी भुजाओं वाला एक त्रिभुज खींचिए और उसके कोण मापिए।

प्रश्न 6 और 7 से हम देखेंगे कि **विषमबाहु त्रिभुज में सभी कोण असमान** (*unequal*) **होते हैं।**

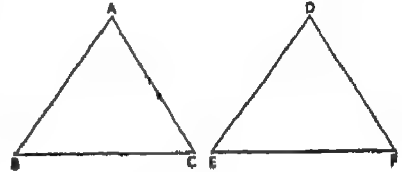
एकक XVII

बहुफलकों की पहिचान

17.1 इस एकक में हम कुछ सरल ठोसों के आकारों का अध्ययन करेंगे। परन्तु पहले हम कुछ ऐसी आधारभूत संकल्पनाओं की व्याख्या करेंगे जो कि हमारे लिए बाद में उपयोगी होंगी।

17.2 समतल आकृतियों की सर्वांगसमता

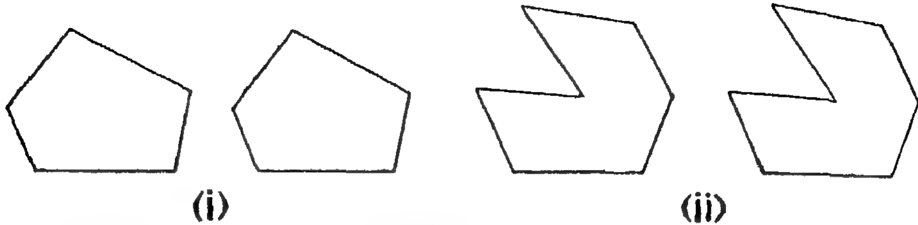
आइए एक गत्ते का त्रिभुजाकार टुकड़ा लें। हम इसे एक कागज के पन्ने पर रखते हैं और इसकी रूप रेखा (*outline*) खींचते हैं। हमें एक त्रिभुज, मान लीजिए, ABC प्राप्त होता है। अब हम गत्ते को दूसरे स्थान पर रखते हैं और पुनः उसकी रूप रेखा खींचते हैं। हमें फिर एक त्रिभुज, मान लीजिए, DEF प्राप्त हो जाता है। हम इन दोनों त्रिभुजों की भुजाओं के बारे में क्या कह सकते हैं? आइए कल्पना करें कि दोनों त्रिभुज स्वयं गत्ते के बने हुए हैं। क्या हम एक त्रिभुज को उठाकर दूसरे पर इस प्रकार रख सकते हैं कि वह दूसरे को पूर्णतया ढक ले या दूसरे शब्दों में दूसरे के साथ संपाती हो जाए? स्पष्ट है, हाँ! ऐसे दो त्रिभुज **सर्वांगसम** (*congruent*) कहलाते हैं। दूसरे शब्दों में, **दो त्रिभुज सर्वांगसम होते हैं यदि उनकी तदनुरूपी भुजाएँ बराबर हों।**



आकृति 17.1

सर्वांगसमता की संकल्पना किन्हीं भी दो समतल आकृतियों के लिए लागू की जा सकती है। इस प्रकार, **दो समतल आकृतियाँ सर्वांगसम होती हैं यदि, यह कल्पना करते हुए कि वे गत्ते के टुकड़े हैं, एक को उठाकर दूसरे पर इस प्रकार रखा जा सके कि वह दूसरी को पूर्णतया ढक ले अर्थात् दूसरे के साथ संपाती हो जाए। ये सर्वांगसम आकृतियाँ सभी प्रकार से बराबर होती हैं।**

क्या अब आप कह सकते हैं कि दो आयत सर्वांगसम होंगे यदि उनकी लम्बाइयाँ बराबर हों तथा उनकी चौड़ाइयाँ बराबर हों? दो वर्ग कब सर्वांगसम होंगे?

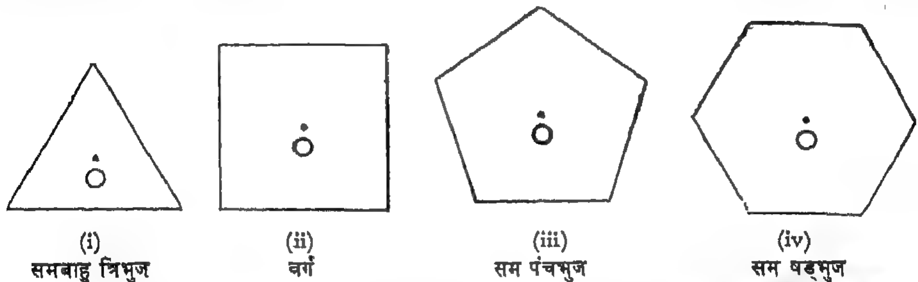


आकृति 17.2

आकृतियों 17.2 (i) और (ii) में सर्वांगसम बहुभुजों (*polygons*) के दो युग्म दिखाए गए हैं।

17.3 सम बहुभुज

यदि किसी बहुभुज की सभी भुजाएँ बराबर हों तथा सभी कोण बराबर हों तो वह सम बहुभुज (*regular polygon*) **कहलाता है।** आकृति 17.3 में क्रमशः 3, 4, 5 और 6 भुजाओं के सम बहुभुज दिखाए गए हैं।



आकृति 17.3

प्रत्येक सम बहुभुज के अंदर एक ऐसा बिंदु, मान लीजिए, *O* है जोकि प्रत्येक शीर्ष से बराबर दूरी पर है। यह बिंदु सम बहुभुज का **केन्द्र** (*centre*) कहलाता है।

17.4 ऊर्ध्वाधर और क्षैतिज दिशाएँ

आइए एक डोरी का एक सिरा पकड़ें और उसके दूसरे सिरे पर एक पत्थर बांध कर लटकाएं। डोरी तनी हुई लटकी रहती है और एक रेखा निरूपित करती है। (देखिए आकृति 17.4) इस रेखा की दिशा एक **ऊर्ध्वाधर दिशा** (*vertical direction*) तथा यह रेखा एक **ऊर्ध्वाधर रेखा** (*vertical line*) कहलाती है। यदि हम एक दूसरी डोरी इसी प्रकार लटकाएँ तो वह भी ऊर्ध्वाधर दिशा में लटकेगी। दोनों डोरियों की दिशाएँ एक ही हैं तथा इन दिशाओं के अनुदिश रेखाएँ समांतर हैं। वास्तव में, **सभी ऊर्ध्वाधर रेखाएँ समांतर होती हैं।** इस प्रकार, हम केवल **एक ऊर्ध्वाधर दिशा** की बात कर सकते हैं।

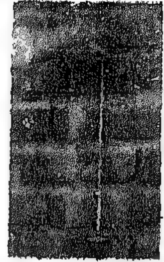


आकृति 17.4: डोरी से लटका हुआ पत्थर

ऊर्ध्वाधर रेखा का एक अन्य उदाहरण साहुल (plumb-line) है। राज (mason) इसका प्रयोग यह जाँच करने के लिए करता है कि वह जो दीवार बना रहा है वह सीधी है या नहीं। **आइए एक आयताकार कमरे के फर्श, छत और**

दीवारों को देखें। फर्श चौरस है। हम कहते हैं कि यह एक क्षैतिज तल (*horizontal plane*) में है। छत का तल भी एक क्षैतिज तल है। दीवारें फर्श पर सीधी खड़ी हैं और प्रत्येक जगह साहुल के समांतर हैं। दीवारों के फलक (faces) ऊर्ध्वाधर तलों (*vertical planes*) के उदाहरण हैं।

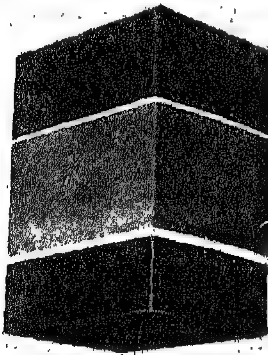
क्षैतिज तल में स्थित प्रत्येक रेखा एक क्षैतिज रेखा (*horizontal line*) कहलाती है। आप फर्श या छत के किनारों के बारे में क्या कह सकते हैं? उन किनारों के बारे में क्या कह सकते हैं जिनमें दिवारों के युग्म परस्पर मिलते हैं?



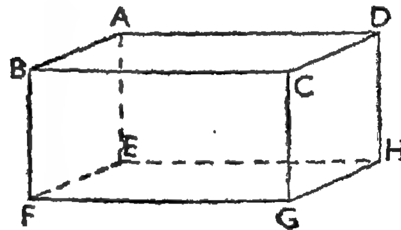
आकृति 17.5 : साहुल

17.5 घनाभ

आइए आकृति 17.6 (i) में दिए एक बक्स के चित्र को देखें। हम इसे आयताकार समांतर षड्फलक (*rectangular parallelopiped*) या केवल घनाभ (*cuboid*) कहते हैं। घनाभ के अन्य उदाहरण हैं : ईंट, आयताकार कमरा या अलमारी।



(i) बक्स का चित्र



(ii) घनाभ

आकृति 17.6

हम देखते हैं कि घनाभ के आठ कोने अर्थात् शीर्ष होते हैं। मान लीजिए ये A, B, C, D, E, F, G और H हैं। [देखिए आकृति 17.6 (ii)]

इसके बारह (सीधे) किनारे होते हैं। ये हैं : AB, EF, DC, HG; AD, BC, EH, FG; AE, BF, DH, CG। साथ ही, इसके छः समतल फलक होते हैं। ये हैं :

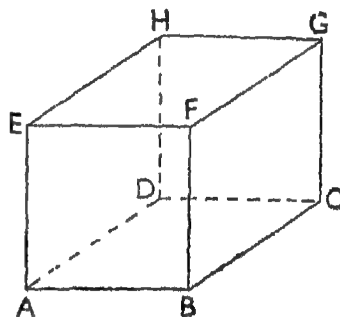
$ABCD$, $EFGH$; $ABFE$, $DCGH$; $ADHE$, $BCGF$

आइए फलक $ABCD$ का आकार देखें। हम देखेंगे कि भुजाएँ AB और CD बराबर हैं। साथ ही भुजाएँ AD और BC बराबर हैं। इसके साथ ही, सभी कोण A, B, C और D समकोण हैं। इस प्रकार $ABCD$ एक आयत है। इसी प्रकार, शेष पाँचों फलक भी आयत हैं।

घनाभ के बारह किनारों को ऊपर चार चार के तीन समूहों में लिखा गया है। हम देखते हैं कि पहले समूह के चारों किनारे AB, EF, DC और HG बराबर हैं और समांतर हैं। यही अन्य समूहों के किनारों के लिए सत्य है। इस प्रकार हम देखते हैं कि बारह किनारों की केवल तीन विभिन्न लम्बाइयाँ हैं। प्राय: इनमें सबसे बड़ी को घनाभ की **लम्बाई** (*length*) कहते हैं। शेष दो में से एक, **चौड़ाई** (*breadth* या *width*) तथा दूसरी **मोटाई** (*thickness*) या **गहराई** (*depth*) या **ऊँचाई** (*height*) कहलाती हैं। ये तीनों लम्बाइयाँ घनाभ की तीन **विमाएँ** (*dimensions*) कहलाती हैं।

17.6 घन

एक घनाभ जिसकी लम्बाई, चौड़ाई और ऊँचाई बराबर हों, **घन** (*cube*) कहलाता है। घन में सभी छ: फलक सर्वांगसम वर्ग होते हैं तथा सभी बारह किनारे बराबर होते हैं। [देखिए आकृति 17.7]



आकृति 17.7 : घन

प्रश्नावली 17.1

1. आकृति 17.6 (ii) में फलक $ABCD$ और $EFGH$ देखिए। हम देखते हैं कि ये सर्वांगसम आयत हैं और समांतर हैं। फलकों के अन्य दो युग्मों के नाम बताइए जो कि सर्वांगसम और समांतर हैं।

2. यदि आकृति 17.6 (ii) में $AB=AD$ हो तो उसके फलकों $ABCD$ और $EFGH$ के क्या आकार होंगे?

3. यदि आकृति 17.6 (ii) में $AD=AE$ हो तो उसके फलकों $ADHE$ और $BCGF$ के क्या आकार होंगे?

4. आकृति 17.6 (ii) में यदि तीन किनारे AB, AD और AE बराबर हों तो घनाभ के फलकों का क्या आकार होगा?

5. आकृति 17.6 (ii) में देखिए कि फलक $ABCD$ तथा $ABFE$ में एक उभयनिष्ठ किनारा

(common edge) AB है। हम यह भी कहते हैं कि फलक $ABCD$ और $ABFE$ किनारे AB में **प्रतिच्छेद करते** या **मिलते** हैं। इसी प्रकार प्रत्येक किनारा दो फलकों में उभयनिष्ठ है। उन फलको के युग्मों के नाम बताइए जिनके उभयनिष्ठ किनारे निम्न हैं :

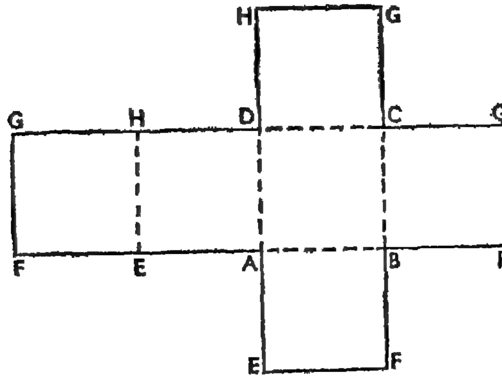
(i) BF (ii) CD (iii) HE

6. किनारों AB, AD और AE में शीर्ष A सर्वनिष्ठ (common) है। [देखिए आकृति 17.6 (ii)] हम यह भी कहते हैं कि ये किनारे A पर **मिलते** या **प्रतिच्छेद** करते हैं। इसी प्रकार प्रत्येक शीर्ष पर तीन किनारे मिलते हैं। उन तीन किनारों के नाम लिखिए जो

(i) B, (ii) D, (iii) G पर मिलते हैं।

7. घनाभ के छः फलक होते हैं। तीन फलकों $ABCD$, $ABFE$ और $ADHE$ में शीर्ष A सर्वनिष्ठ है अर्थात् ये फलक एक बिंदु A पर **मिलते** हैं। अन्य शेष तीन फलक $EFGH$, $DCGH$ और $BCGF$, G पर मिलते हैं। [देखिए आकृति 17.6 (ii)] A और G को घनाभ के शीर्षों का **सम्मुख युग्म** (*opposite pairs*) कहते हैं तथा AG घनाभ का **विकर्ण** (*diagonal*) कहलाता है। अन्य तीन सम्मुख शीर्षों तथा तदनुरूपी विकर्णों के नाम बताइए।

8. एक गत्ता लीजिए। इस पर 5 सेमी भुजा के छः सर्वांगसम वर्ग बनाइए जैसा कि आकृति 17.8 में दिखाया गया है।



आकृति 17.8 : घन का जाल

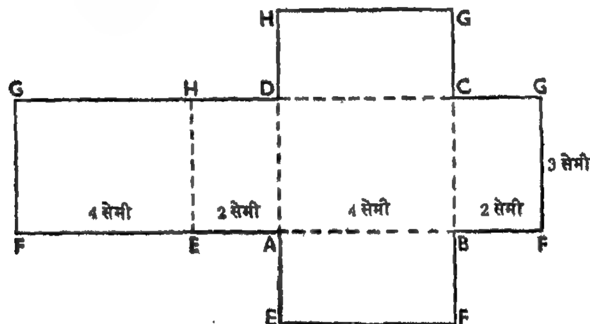
इन वर्गों में से प्रत्येक, घन का एक फलक निरूपित करता है। अब इस आकृति को गत्ते में से काट लीजिए और इसे बिंदुंकित रेखाओं (*dotted lines*) के अनुदिश इस प्रकार मोड़िए कि एक से अंकित शीर्ष पास पास आ जाएँ। हमें घन का एक प्रतिरूप (*model*) प्राप्त हो जाता है। [इस प्रतिरूप

को सेलोटेप (*cellotape*) या गोंद लगे हुए कागज़ के टेप से जोड़ा जा सकता है] क्या अब आप देख सकते हैं कि वर्गों को आकृति 17.8 में दिखाए गए एक विशेष प्रकार से ही क्यों खींचा गया था ?

इस आकृति में दिए गए आकार को घन का **जाल** (*net*) कहते हैं।

9. एक घनाभ, जिसकी लम्बाई, चौड़ाई और ऊँचाई क्रमशः 4 सेमी, 3 सेमी और 2 सेमी है, का जाल खींचिए। इस जाल से घनाभ का प्रतिरूप बनाइए।

[**संकेत** : घनाभ का जाल आकृति 17.9 में दिखाया गया है। इसे बिंदुकित रेखाओं के अनुदिश मोड़िए ताकि एक से अंकित बिंदु पास पास आ जाएँ। इससे हमें घनाभ का प्रतिरूप प्राप्त हो जाता है जिसकी लम्बाई, चौड़ाई और ऊँचाई क्रमशः 4 सेमी, 3 सेमी और 2 सेमी हैं।



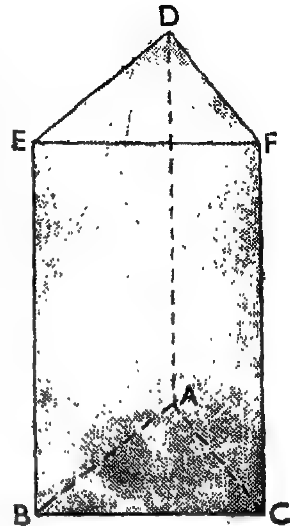
आकृति 17.9 : घनाभ का जाल

यदि आप इसमें से आयत *EFGH* को छोड़ दें तो आपको एक बिना ढक्कन के बक्स का जाल प्राप्त हो जाएगा। वास्तव में गत्ते आदि के डिब्बे या बक्से ऐसे ही जालों से बनाए जाते हैं।]

10 एक घनाभ का प्रतिरूप बनाइए जिसकी लम्बाई, चौड़ाई और ऊँचाई क्रमशः 5 सेमी, 4 सेमी और 3 सेमी हैं।

टिप्पणी जब कोई घनाभ (या घन) किसी मेज़ पर रखा जाता है तो उसके ऊपरी और निचले फलक क्षैतिज होते हैं तथा चारों पार्श्व फलक (*side faces*) ऊर्ध्वाधर होते हैं। साथ ही, ऊपरी और निचले फलक के आठों किनारे क्षैतिज होते हैं तथा शेष चार किनारे ऊर्ध्वाधर होते हैं। इस स्थिति में निचला फलक घनाभ (या घन) का **आधार** (*base*) कहलाता है तथा इसकी लम्बाई और चौड़ाई क्रमशः घनाभ (या घन) की **लम्बाई** और **चौड़ाई** कहलाती हैं। ऊर्ध्वाधर किनारों की लम्बाई घनाभ (या घन) की **ऊँचाई** कहलाती है।

17.7 लम्ब प्रिज्म



आकृति 17.10 : लम्ब प्रिज्म

आइए आकृति 17.10 में दिए हुए ठोस को देखें। इसके पाँच फलक हैं। इनमें दो सर्वांगसम एवं समांतर त्रिभुज ABC और DEF हैं तथा तीन आयत $BCFE$, $CADF$ और $ABED$ हैं। इसके नौ किनारे और छः शीर्ष हैं। यदि ठोस को एक मेज पर इस प्रकार रखा जाए कि दोनों त्रिभुजाकार फलक क्षैतिज हों तो किनारे AD, BE और CF ऊर्ध्वाधर होंगे। हम इस ठोस को **त्रिभुजाकार आधार का लम्ब प्रिज्म** (*right prism on a triangular base*) या केवल **त्रिभुजाकार लम्ब प्रिज्म** (*triangular right prism*) कहते हैं।

चतुर्भुज, पंचभुज, इत्यादि आधार वाले प्रिज्म भी हो सकते हैं। **प्रत्येक लम्ब प्रिज्म में उसका आधार और सम्मुख फलक दो सर्वांगसम बहुभुज होते हैं जबकि शेष सभी फलक आयत होते हैं।** ऊर्ध्वाधर किनारों, जो कि सभी बराबर और समांतर हैं, में से किसी एक की लम्बाई को प्रिज्म की **ऊँचाई** (*height*) या **लम्बाई** कहा जा सकता है।

हम देखते हैं कि घनाभ एक प्रिज्म है। (क्यों?) वास्तव में, कभी कभी घनाभ को आयताकार लम्ब प्रिज्म (*rectangular right prism*) भी कहते हैं।

प्रश्नावली 17.2

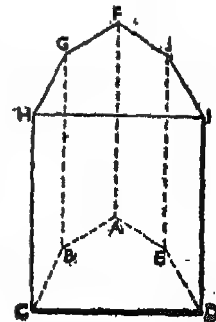
1. आकृति 17.10 में हम देखते हैं कि किनारा EF, फलकों DEF और $BCFE$ में उभयनिष्ठ हैं। अन्य किनारों में से प्रत्येक, फलकों के एक युग्म में उभयनिष्ट है। उन फलकों के युग्मों के नाम बताइए जिनमें निम्नलिखित किनारे उभयनिष्ठ हैं :

   (i) AD (ii) CF (iii) AB

2. आकृति 17.10 में देखिए कि प्रत्येक शीर्ष पर तीन किनारे मिलते हैं। उन तीनों किनारों के नाम बताइए जोकि निम्न शीर्षों पर मिलते हैं :

   (i) A (ii) E (iii) C

3. पंचभुजीय आधार (*pentagonal base*) के प्रिज्म को देखिए। (आकृति 17.11) इसके सभी फलकों, किनारों और शीर्षों के नाम बताइए।



आकृति 17.11

*4. बिना आकृति खींचे, निम्न आधारों के प्रिज्मों में फलकों, किनारों और शीर्षों की संख्याएँ बताइए :

   (i) षड्भुज (ii) अष्ठभुज

[क्या अब आप इस नियम का अनुमान लगा सकते हैं कि 'यदि

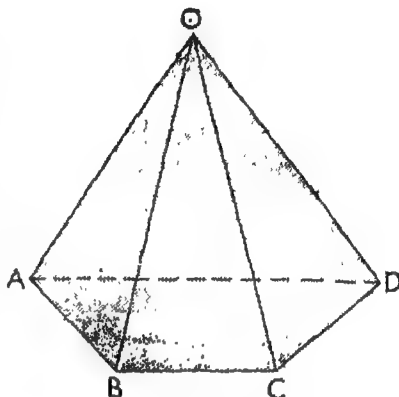
आधार के बहुभुज की भुजाओं की संख्या m हो तो प्रिज्म के फलकों की संख्या $m+2$, किनारों की संख्या $3m$ तथा शीर्षों की संख्या $2m$ होती है' ?]

17.8 पिरैमिड

आइए आकृति 17.12 में दिए हुए ठोस को देखें। इसके पाँच फलक हैं। इन फलकों में एक चतुर्भुजीय आधार $ABCD$ तथा चार त्रिभुजाकार फलक OAB, OBC, OCD और ODA हैं। त्रिभुजाकार फलक एक सर्वनिष्ठ बिंदु O पर मिलते हैं। हम इस ठोस को चतुर्भुजीय आधार का **पिरैमिड** (*pyramid*) कहते हैं।

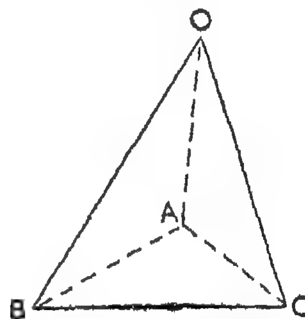
O पिरैमिड का **शीर्ष** कहलाता है। इस पिरैमिड के आठ किनारे OA, OB, OC, OD, AB, BC, CD और DA तथा पाँच शीर्ष O, A, B, C और D हैं।

अन्य आधारों जैसे कि त्रिभुज, पंचभुज, षड्भुज, इत्यादि के भी पिरैमिड हो सकते हैं।



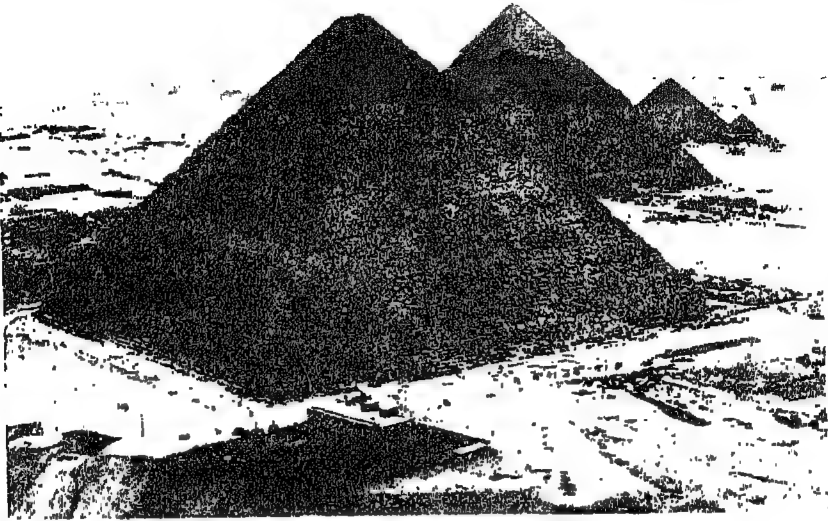
आकृति 17.12 : पिरैमिड

त्रिभुजाकार आधार का पिरैमिड **चतुष्फलक** (*tetrahedron*) कहलाता है। (देखिए आकृति 17.13) इसके चार फलक होते हैं और इनमें से प्रत्येक एक त्रिभुज होता है। चतुष्फलक के छः किनारे और चार शीर्ष होते हैं। चूँकि चतुष्फलक के सभी फलक त्रिभुज हैं इसलिए इनमें से किसी एक को उसका आधार माना जा सकता है तथा इस फलक के बाहर वाले कोने को उसका शीर्ष। यदि किसी चतुष्फलक के सभी किनारे बराबर हों तो वह एक **सम चतुष्फलक** (*regular tetrahedron*) कहलाता है।



आकृति 17.13 : चतुष्फलक

आपने मिस्र के महान पिरैमिडों के बारे में सुना होगा जो कि 3000-2000 ई० पू० काल में बनाए गए थे। ये वर्गीय आधारों (*square bases*) पर पिरैमिडों के आश्चर्यजनक और यथार्थ उदाहरण हैं। (देखिए आकृति 17.14) ये किस प्रकार बनाए गए? कोई नहीं जानता। क्या ये कब्रिस्तान थे? कोई नहीं जानता। क्या इनमें कोई गुप्त रहस्य छिपा हुआ है? कोई नहीं जानता।



आकृति 17.14 : मिस्र के पिरैमिड

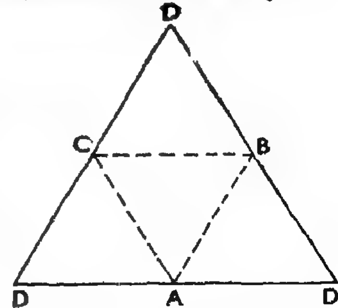
संयुक्त अरब गणराज्य के नई दिल्ली स्थित दूतावास के प्रेस ब्यूरो के सौजन्य से

प्रश्नावली 17.3

1. आकृति 17.13 में दिए चतुष्फलक के सभी फलकों, किनारों और शीर्षों के नाम बताइए।

2. चतुष्फलक के दो फलक OAB तथा OAC किनारे OA पर प्रतिच्छेद करते हैं तथा शेष दो फलक OBC तथा ABC किनारे BC पर प्रतिच्छेद करते हैं। (देखिए आकृति 17.13) OA और BC **चतुष्फलक के सम्मुख किनारे (*opposite edges*) कहलाते हैं। सम्मुख किनारों के अन्य दो युग्मों के नाम बताइए।**

3. एक गत्ते के टुकड़े पर 6 सेमी की भुजा का एक समबाहु त्रिभुज खींचकर उसे काट लीजिए। इसके प्रत्येक शीर्ष को D से व्यक्त कीजिए। माना A, B और C क्रमशः इस त्रिभुज की भुजाओं के मध्य-बिंदु हैं। इनको बिंदुंकित रेखाओं द्वारा जोड़िए। (देखिए आकृति 17.15) हमें एक चतुष्फलक $ABCD$ का जाल प्राप्त होता है जिसका प्रत्येक किनारा 3 सेमी लम्बा है।

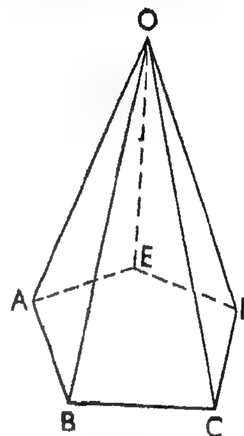


आकृति 17.15: चतुष्फलक का जाल

गत्ते को बिंदुकित रेखाओं BC, CA और AB के अनुदिश मोड़िए ताकि तीनों कोने D पास पास आ जाएँ। हमें एक चतुष्फलक का प्रतिरूप प्राप्त हो जाता है जिसका प्रत्येक फलक 3 सेमी भुजा का एक समबाहु त्रिभुज है। क्या यह एक सम चतुष्फलक है ?

4. 4 सेमी के किनार वाले एक सम चतुष्फलक का जाल खींचिए तथा उसका प्रतिरूप बनाइए।

5. पंचभुजीय आधार के पिरैमिड को देखिए। (आकृति 17.16) इसके फलकों, किनारों और शीर्षों के नाम बताइए तथा उनकी संख्या ज्ञात कीजिए।



आकृति 17.16.

*6. बिना आकृति खींचे निम्न आधारों के पिरैमिडों में फलकों, किनारों और शीर्षों की संख्याएँ बताइए :

(i) षड्भुज (ii) सप्तभुज

[क्या अब आप इस नियम का अनुमान लगा सकते हैं कि 'यदि आधार में m भुजाएँ हों तो पिरैमिड के फलकों कि संख्या $m+1$, किनारों की संख्या $2m$ तथा शीर्षों की संख्या $m+1$ होती है ?]

17.9 बहुफलक

वे सभी ठोस जिनके आकारों के विषय में हमने अभी तक पढ़ा है, बहुभुजीय फलकों के हैं। वह ठोस जिसकी सतह (*surface*) बहुभुजीय फलकों की बनी हो बहुफलक (*polyhedron*) कहलाता है। यदि किसी बहुफलक में कोई छेद (*hole*) न हो तो वह सरल (*simple*) बहुफलक कहलाता है। हमने अभी तक जितने बहुफलक पढ़े हैं वे सभी सरल बहुफलक हैं।

आइए निम्न सारणी को पूरा करें। माना F, E, और V क्रमश: उन बहुफलकों के फलकों, किनारों और शीर्षों की संख्या व्यक्त करते हैं, जोकि पहले स्तंभ में दिए हुए हैं।

बहुफलक का नाम	F	E	V	$F-E+V$
घनाभ				
त्रिभुजाकार प्रिज्म				
पंचभुजीय प्रिज्म				
चतुष्फलक				
चतुर्भुजीय आधार पर पिरैमिड				

आइए अंतिम स्तंभ पर विशेष ध्यान दें। हम क्या देखते हैं?

स्विटज़रलैण्ड के एक महान गणितज्ञ ऑयलर (1703-1783), जिसने अपना अधिकांश समय रूस में वहाँ के राजा (czar) से प्राप्त वृत्ति (stipend) पर व्यतीत किया, ने यह खोज की कि प्रत्येक सरल बहुफलक के लिए

$F-E+V=2$ होता है।

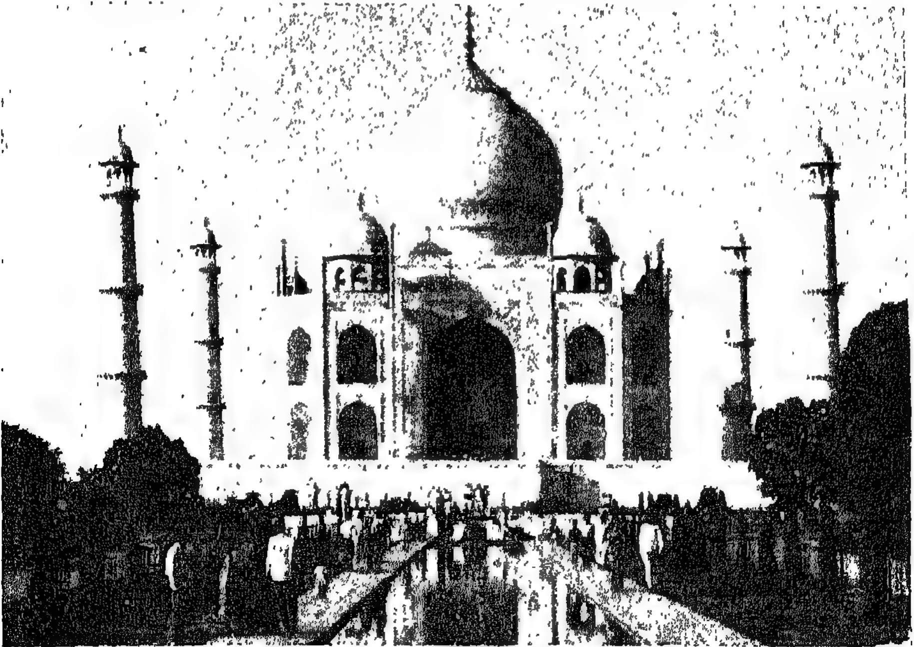
यह **ऑयलर का सूत्र** (*Euler's Formula*) कहलाता है।

एकक XVIII

रैखिक सममिति

18.1 भूमिका

हम कुछ सममित आकृतियों (*symmetrical figures*) से पहले से ही परिचित हैं। सममिति (*symmetry*), कला में एक बहुत ही महत्वपूर्ण भूमिका (role) अदा करती है। यह सुन्दर डिजाइनें बनाने में कलाकार की सहायता करती है। क्या आपको अपनी देखी हुई सुन्दर इमारतों जैसे कि मंदिर, मस्जिद, महल तथा अन्य ऐतिहासिक स्मारक चिन्हों (*historical monuments*) के बारे में कुछ याद है? इनमें से प्रत्येक में कलाकार ने सममिति की कल्पना का प्रयोग किया है। आगरे का ताजमहल इनका एक बहुत ही सुन्दर उदाहरण है। (देखिए आकृति 18.1)

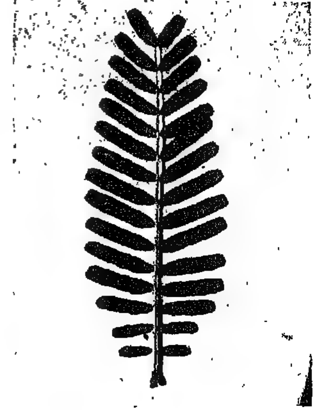


आकृति 18.1 : ताजमहल

प्रकृति की बहुत सी वस्तुओं में भी हमें सममिति देखने को मिलती है। आकृति 18.2 में ऐसी दो सममित वस्तुओं के चित्र दिए हैं। मनुष्य सममित वस्तुओं का एक अन्य उदाहरण है।



(i)



(ii)

आकृति 18.2

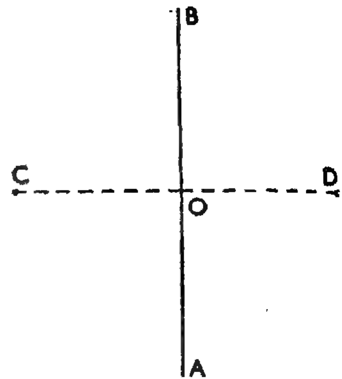
सममिति कई प्रकार की होती है उदाहरणार्थ बिंदु के सापेक्ष सममिति (*symmetry about a point*), रेखा के सापेक्ष सममिति, तल के सापेक्ष सममिति, घूर्णन सममिति (*rotational symmetry*) इत्यादि। इस एकक में हम ज्यामितीय आकृतियों की रेखाओं के सापेक्ष सममिति, जो कि **रैखिक सममिति** (*linear symmetry*) कहलाती है, का अध्ययन करेंगे और सममित आकृतियों की रचना करने की कुछ विधियों का वर्णन करेंगे।

18.2 रेखा के सापेक्ष सममिति

आइए, निम्न प्रयोगों का अध्ययन करें।

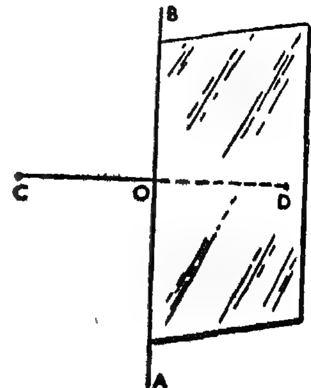
प्रयोग 1 : हम एक कागज़ का पन्ना लेते हैं और उसे मोड़ लेते हैं। हमें एक मोड़ का निशान प्राप्त हो जाता है। माना यह AB है। (देखिए आकृति 18.3) मोड़ी हुई स्थिति में ही, आइए एक नुकीली पिन से कागज़ में छेद करें जिससे कि मोड़ के निशान के दोनों ओर एक एक छेद हो जाता है। अब हम कागज़ को खोल लेते हैं। हम देखते हैं कि हमारे पास एक रेखा AB है और इसके विपरीत ओर पिन द्वारा किए हुए दो छेद हैं जो कि मान लीजिए दो बिंदु C और D हैं। अब हम C और D को बिंदुंकित रेखा द्वारा जोड़ते हैं जैसा कि आकृति 18.3 में दिखाया गया है। माना CD, AB को बिंदु O पर काटती है।

आइए CO और OD को मापें। क्या $CO=OD$ है? पुनः अब हम $\angle COA$, $\angle DOA$, $\angle COB$ तथा $\angle DOB$ को मापें। क्या ये सभी समकोण हैं?



आकृति 18.3

हम देखते हैं कि AB, CD को O पर समद्विभाजित करती है तथा साथ ही $CD \perp AB$ है। हम कहते हैं कि बिंदु C और D रेखा AB के सापेक्ष **सममित स्थित** (*symmetrically situated*) हैं या यह कि C और D, रेखा AB के सापेक्ष **सममित** (*symmetric*) हैं। हम यह भी कहते हैं कि रेखा AB के सापेक्ष C, D का **सममित** है तथा D, C का **सममित** है ।



आकृति 18.4

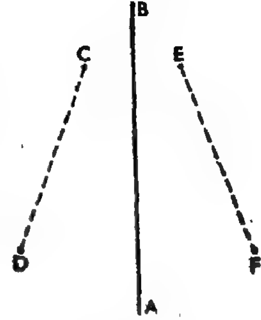
अब आइए एक समतल दर्पण (*mirror*) लें और उसे इस प्रकार खड़ा करें कि उसका सीधा किनारा AB के सम्पर्क में रहे तथा बिंदु C दर्पण के सामने रहे। (देखिए आकृति 18.4) C का प्रतिबिम्ब कहाँ पर स्थित है ? क्या यह D पर प्रतीत होता है ?

अब दर्पण को इस प्रकार खड़ा करें कि उसका सीधा किनारा AB के सम्पर्क में रहे तथा बिंदु D दर्पण के सामने रहे। D का प्रतिबिम्ब कहाँ स्थित है ? क्या यह C पर प्रतीत होता है ?

अतः हम देखते हैं कि यदि AB एक दर्पण हो, तो C और D, AB में एक दूसरे के प्रतिबिम्ब हैं। इस प्रकार, **यदि दो बिंदु एक रेखा के सापेक्ष सममित हों, तो हम यह भी कह सकते हैं कि वे उस रेखा में एक दूसरे के दर्पण प्रतिबिम्ब** (*mirror images*) **हैं।**

प्रयोग 2 : आइए एक दूसरा कागज लें और उसे मोड़ें। फिर मुड़ी हुई स्थिति में ही एक पिन से कागज में दो विभिन्न स्थानों पर छेद करें। अब हम कागज को खोल लेते हैं। हम देखते हैं

कि हमारे पास एक मोड़ का निशान है जो कि हम मान लेते हैं कि रेखा AB है तथा साथ ही पिन द्वारा किए हुए चार छेद हैं जो कि हम मान लेते हैं कि बिंदु C, D, E और F हैं। साथ ही C और D, AB के एक ओर स्थित हैं तथा E और F दूसरी ओर जैसा कि आकृति 18.5 में दिखाया गया है। क्या हम कह सकते हैं कि C और E, AB के सापेक्ष सममित हैं? D और F के बारे में आप क्या सोचते हैं?

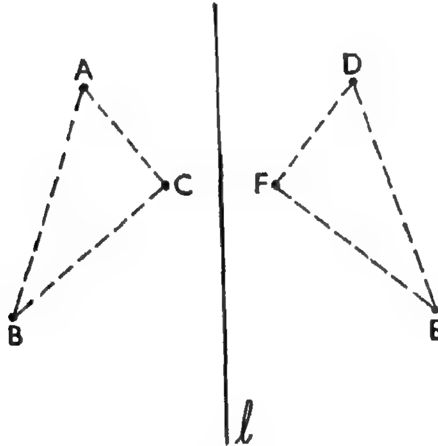


आकृति 18.5

आइए अब रेखाएं CD और EF खींचें। यदि हम कागज़ को पुनः AB पर मोड़ें तो क्या EF पूर्णतया CD पर गिरेगी? हाँ। हम कहते हैं कि रेखाएँ CD और EF, AB के सापेक्ष **सममित** हैं। हम यह भी कहते हैं कि AB के सापेक्ष, CD, EF का **सममित** है तथा EF, CD का **सममित** है।

पहले की तरह हम यह भी कह सकते हैं कि CD और EF एक दूसरे के AB में **दर्पण प्रतिबिम्ब** हैं।

प्रयोग 3: आइए एक कागज़ का नया पन्ना लें और प्रयोग 2 को केवल इस अंतर के साथ दोहराएं कि हम कागज़ में दो के स्थान पर तीन जगहों पर छेद करें जैसा कि आकृति 18.6 में दिखाया गया है।



आकृति 18.6

जब हम कागज़ को खोलते हैं तो देखते हैं कि हमें एक रेखा, मान लीजिए, l तथा छ:बिंदु, मान लीजिए, A, B, C, D, E और F प्राप्त होते हैं। अब हम BC, CA, AB तथा EF, FD, DE खींचते हैं। अब हमारे पास एक रेखा l तथा दो त्रिभुज ABC और DEF हैं। ऐसे दो त्रिभुज l के सापेक्ष परस्पर **सममित** कहलाते हैं। क्या $\triangle ABC$, l में $\triangle DEF$ का **दर्पण प्रतिबिम्ब** है? $\triangle DEF$ और $\triangle ABC$ के बारे में हम क्या कह सकते हैं?

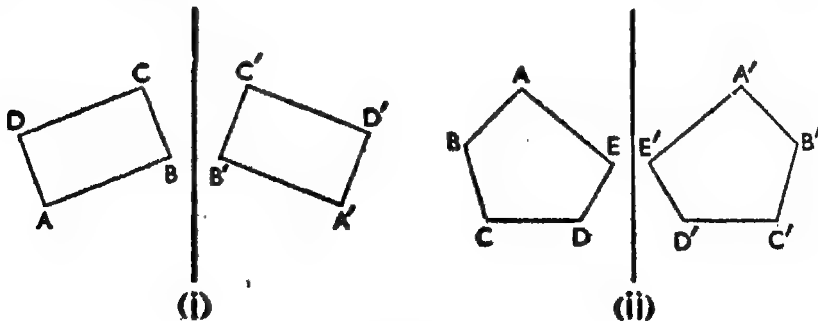
यदि हम कागज़ को पुन: l पर मोड़ें तो क्या $\triangle DEF$ पूर्णतया $\triangle ABC$ पर गिरेगा जिससे कि D, A पर; E, B पर तथा F, C पर गिरे? क्या हम कह सकते हैं कि त्रिभुज ABC और DEF सर्वांगसम हैं?

उपर्युक्त प्रत्येक प्रयोग में रेखा AB या l **सममिति रेखा** (*line of symmetry*) या **सममिति अक्ष** (*axis of symmetry*) कहलाती है।

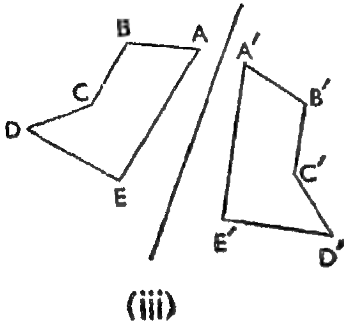
अब (रेखा के सापेक्ष) सममिति की कल्पना ो चतुर्भुज, पंचभुज, इत्यादि तथा व्यापक रूप में बिंदुओं, रेखाओं या रेखाखंडों से बनी किसी भी ज्यामितीय आकृति के लिए सरलता से लागू किया जा सकता है। प्रत्येक दशा में, दोनों आकृतियाँ सममित रेखा में एक दूसरे के दर्पण प्रतिबिम्ब होते हैं तथा यदि उस कागज को, जिस पर ये बनी हुई हैं, सममित रेखा पर मोड़ा जाए तो पहली आकृति, पूर्णतया दूसरी पर गिरती है। दूसरे शब्दों में, कागज़ को सममित अक्ष के अनुदिश मोड़कर एक को दूसरे के साथ संपाती (*coincide*) किया जा सकता है।

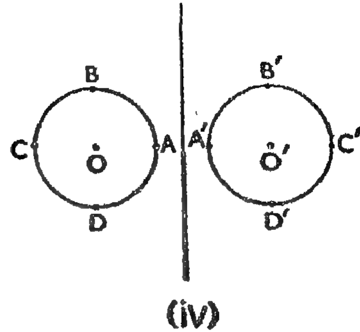
अत: हम कहते हैं कि **दो आकृतियाँ एक दी हुई रेखा के सापेक्ष सममित होती हैं** यदि वह रेखा में एक दूसरे के दर्पण प्रतिबिम्ब हों अर्थात् यदि कागज को रेखा के अनुदिश मोड़ा जाए तो दोनों आकृतियाँ परस्पर संपाती हो जाएँ। तब यह रेखा, **सममिति अक्ष** कहलाती है।

हम नीचे सममित आकृतियों के कुछ युग्म तथा उनके सममिति अक्ष दे रहे हैं। (देखिए आकृति 18.7)



आकृति 18.7



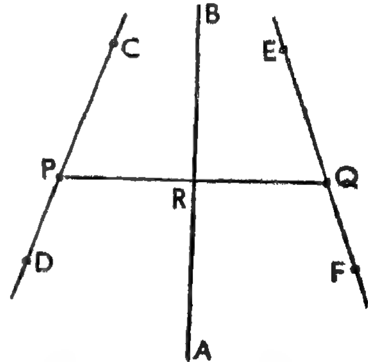


आकृति 18.7 : सममित आकृतियाँ

प्रत्येक आकृति में सममित बिंदुओं के युग्मों को A, A'; B, B'; इत्यादि से व्यक्त किया गया है।

18.3 सममित आकृतियों के युग्मों के कुछ गुण

18.3.1 प्रयोग 2 में हमें सममित बिंदुओं के दो युग्मों C, E और D, F से सममित रेखाओं CD और EF का एक युग्म प्राप्त हुआ था। आइए, CD पर कोई अन्य बिंदु P लें। AB के सापेक्ष P का प्रतिबिम्ब कहाँ स्थित होगा? अब हम $PR \perp AB$ खींचते हैं तथा PR को बढ़ाते हैं जिससे कि वह EF से Q पर मिले। (देखिए आकृति 18.8) क्या $PR = RQ$ है? क्या Q, P का सममित है?



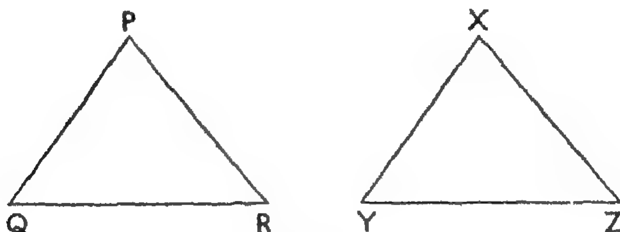
आकृति 18.8

इसी प्रकार यदि प्रयोग 3 में हम $\triangle ABC$ की किसी भी भुजा पर कोई बिंदु P लें तो हम देखेंगे कि उसका सममित Q, $\triangle DEF$ की संगत भुजा पर स्थित होगा। वास्तव में यह सममित आकृतियों के युग्मों का एक व्यापक गुण है। **सममित आकृतियों के युग्म, सममित बिंदुओं के युग्मों के बने होते हैं।** दूसरे शब्दों में, यदि हमारे पास सममित आकृतियों का एक युग्म हो तथा एक बिंदु इनमें से किसी एक के अनुदिश चलता है तो इस बिंदु का सममित दूसरी आकृति के अनुदिश चलेगा।

माना आकृति 18.6 में एक बिंदु A से B, B से C तथा वापिस C से A की ओर चलता है। इस बिंदु का सममित किस प्रकार चलता है? स्पष्ट है कि D से E, E से F तथा वापिस F से D की ओर। साथ ही हम यह देखते हैं कि दोनों बिंदु विपरीत अभिदिशाओं (*opposite senses*) के दो त्रिभुज बनाते हैं। पुनः यह भी सममित आकृतियों के युग्मों का एक व्यापक गुण है। **सममित आकृतियों के युग्म विपरीत अभिदिशाओं में बनते हैं।**

18.3.2 सर्वांगसमता और सममित आकृतियों के युग्म

हम दो ज्यामितीय आकृतियों की सर्वांगसमता से पहले ही से परिचित हैं। उदाहरणार्थ, आकृति 18.9 में दोनों त्रिभुज PQR और XYZ सर्वांगसम हैं। $\triangle PQR$ को $\triangle XYZ$ के संपाती किया जा सकता है। QR और YZ, RP और ZX, PQ और XY बराबर भुजाओं के युग्म हैं।



आकृति 18.9

यदि एक बिंदु पहले त्रिभुज की भुजाओं QR, RP और PQ के अनुदिश चले तथा एक दूसरा बिंदु दूसरे त्रिभुज की संगत भुजाओं YZ, ZX और XY के अनुदिश चले तो हम देखते हैं कि दोनों बिंदु दोनों त्रिभुजों की भुजाओं के अनुदिश एक ही दिशा में चलते हैं।

अब हम आकृति 18.6 को लेते हैं। वहाँ हमने देखा था कि यदि हम कागज़ को l के अनुदिश मोड़ें तो $\triangle DEF$, $\triangle ABC$ के संपाती हो जाता है। अतः हमें त्रिभुजों ABC और DEF को सर्वांगसम त्रिभुज मानना चाहिए। परन्तु हम यह भी देख चुके हैं कि ये दोनों त्रिभुज एक बिंदु और उसके सममित द्वारा विपरीत अभिदिशाओं में बनाए गए हैं।

उपर्युक्त दोनों स्थितियों में परस्पर भेद दिखाने के लिए हम कहते हैं कि आकृति 18.9 के त्रिभुज PQR और XYZ **सीधे सर्वांगसम** (*directly congruent*) हैं जबकि आकृति 18.6 के दोनों त्रिभुज ABC और DEF **विपरीत सर्वांगसम** (*oppositely congruent*) हैं।

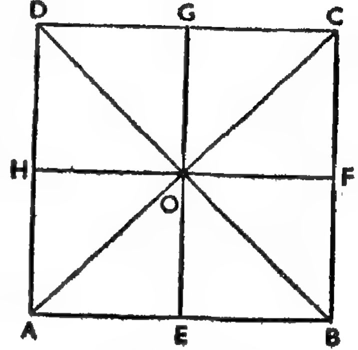
चूंकि सममित रेखा पर कागज़ को मोड़कर सममित आकृतियों के युग्मों को परस्पर संपाती किया जा सकता है तथा साथ ही चूंकि ये आकृतियाँ विपरीत अभिदिशाओं में बनती हैं इसलिए हम कह सकते हैं कि **सममित आकृतियों के युग्म विपरीत सर्वांगसम होते हैं।**

विद्यार्थी को चाहिए कि वह आकृति 18.7 में दिए हुए आकृति युग्मों के लिए इस कथन की सत्यता की जाँच करें।

18.4 सममित आकृतियाँ

अनुच्छेद 18 1 की भूमिका में सममिति की संकल्पना को दर्शाने के लिये हमने ताजमहल, एक तितली और एक पत्ती के चित्र दिए हैं। ये चित्र स्वयं में सममित हैं। इनमें से प्रत्येक चित्र में हम एक मध्य रेखा देख सकते हैं जो चित्र को दो भागों में इस प्रकार बांटती है कि यह दोनों भाग इस रेखा के सापेक्ष एक दूसरे के साथ सममित हैं। हम नीचे कुछ सरल ज्यामितीय आकृतियां दे रहे हैं जो स्वयं में सममित हैं।

18.4.1 वर्ग :



आकृति 18.10

आकृति 18.10 में $ABCD$ एक वर्ग है तथा E, F, G और H क्रमशः उसकी भुजाओं AB, BC, CD और DA के मध्य-बिंदु हैं। आइए E और G को जोड़ें। क्या $AEGD$ और $BEGC$ आयत हैं?

अब हम वर्ग को EG पर मोड़ते हैं। क्या आयत $AEGD$, आयत $BEGC$ के संपाती हो जाता है? हाँ, ऐसा ही होता है। शीर्ष A, B के संपाती है तथा D, C के संपाती है। इस प्रकार, $AEGD$ और $BEGC$ रेखा EG के सापेक्ष आयतों का एक सममित युग्म है। हम केवल यही कहते हैं कि **वर्ग ABCD रेखा EG के सापेक्ष सममित है** जो कि सम्मुख भुजाओं के एक युग्म के मध्य-बिंदुओं को मिलाती है तथा यह कि EG, वर्ग के लिए एक **सममिति रेखा या अक्ष है**।

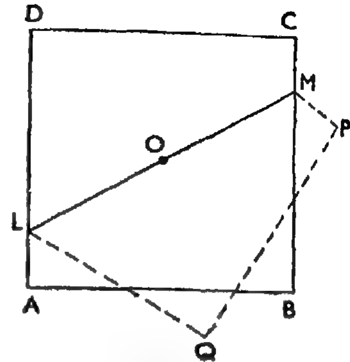
क्या यह वर्ग रेखा FH, जो कि सम्मुख भुजाओं के दूसरे युग्म के मध्य-बिंदुओं को मिलाती है, के सापेक्ष सममित है?

क्या वर्ग की कोई और सममिति अक्ष भी है? आइए विकर्ण AC को देखें।

यदि हम वर्ग को AC पर मोड़ें तो क्या होगा? क्या वर्ग AC के सापेक्ष सममित है? हाँ!

अब हम दूसरा विकर्ण BD लेते हैं। हम देखते हैं कि वर्ग BD के सापेक्ष भी सममित है।

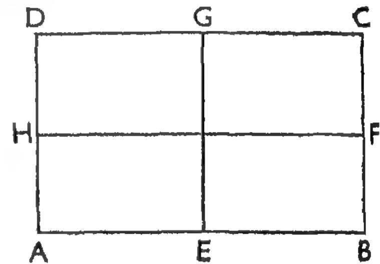
क्या वर्ग की अब भी कोई और सममिति अक्ष है? अभी तक ज्ञात चारों सममिति अक्षों में से प्रत्येक, वर्ग के केन्द्र O से होकर जाती है। आइए केन्द्र O से जाती हुई कोई और रेखा LM को देखें जो कि भुजा AD को L पर तथा BC को M पर काटती है। (देखिए आकृति 18.11) यदि हम वर्ग को



LM पर मोड़ें तो क्या होता है? क्या चतुर्भुज *LMCD*, चतुर्भुज *LMBA* के संपाती हो जाती है? नहीं! यह एक भिन्न स्थिति *LMPQ* धारण कर लेता है। दूसरे शब्दों में, **दिया हुआ वर्ग ABCD, *LM* के सापेक्ष सममित नहीं है।**

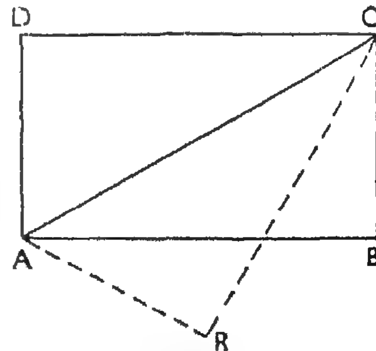
आकृति 18.11

संक्षेप में वर्ग अपनी सम्मुख भुजाओं के युग्मों के मध्य-बिंदुओं को जोड़ने से बनने वाली दो रेखाओं के सापेक्ष सममित होता है तथा साथ ही दोनों विकर्णों के सापेक्ष भी।



आकृति 18.12

18.4.2 आयत : आकृति 18.12 में, *ABCD* एक आयत है तथा *E*, *F*, *G* और *H* क्रमश: उसकी भुजाओं के मध्य-बिंदु हैं। इसकी सरलता से जाँच की जा सकती है कि आयत, *EG* और *FH* जो कि उसकी सम्मुख भुजाओं के युग्मों के मध्य-बिंदुओं को जोड़ने वाली रेखाएँ हैं, के सापेक्ष सममित है।



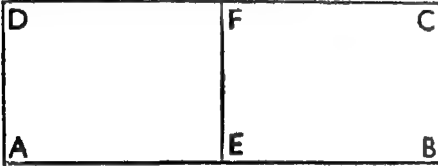
आकृति 18.13

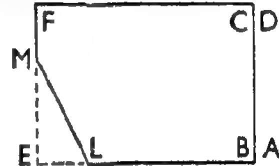
परन्तु क्या आयत विकर्ण *AC* के सापेक्ष सममित है? आइए *ABCD* को *AC* पर मोड़ें। $\triangle ACD$, $\triangle ACB$ के संपाती नहीं होता। वस्तुत: यह एक भिन्न स्थिति $\triangle ACR$ धारण कर लेता है। (देखिए आकृति 18.13) अत: आयत विकर्ण *AC* के सापेक्ष सममित नहीं है। इसी प्रकार यह दूसरे विकर्ण *BD* के सापेक्ष भी सममित नहीं है।

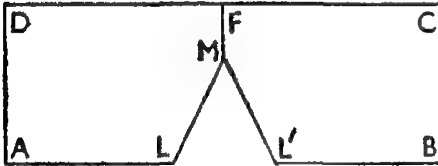
वास्तव में आयत केवल अपनी सम्मुख भुजाओं के युग्मों के मध्य-बिन्दुओं से बनी दोनों रेखाओं के सापेक्ष सममित होता है।

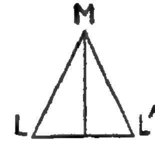
अब आइए कल्पना करें कि आकृति 18.13 के दो त्रिभुज ABC और CDA पतले गत्ते के बने हैं। क्या हम $\triangle CDA$ को $\triangle ABC$ पर इस प्रकार रख सकते हैं कि वे परस्पर संपाती हो जाएँ? हाँ, हम ऐसा कर सकते हैं तथा इसमें C, A पर गिरेगा, D, B पर गिरेगा और A, C पर गिरेगा। दूसरे शब्दों में, $\triangle ABC$ और $\triangle CDA$ सर्वांगसम हैं। अतः हम देखते हैं कि विकर्ण AC, आयत $ABCD$ को दो सर्वांगसम भागों में विभाजित करता है। परन्तु फिर भी AC सममिति अक्ष नहीं है। क्यों? क्योंकि दोनों सर्वांगसम त्रिभुज विकर्ण में एक दूसरे के दर्पण प्रतिबिम्ब नहीं हैं।

इस प्रकार एक रेखा किसी आकृति को दो सर्वांगसम भागों में विभाजित कर सकती है परन्तु यह सममिति अक्ष केवल तभी होगी जबकि दोनों भाग इस रेखा में एक दूसरे के दर्पण प्रतिबिम्ब हों।







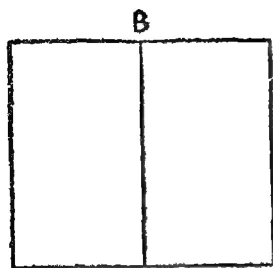


आकृति 18.14

18.4.3 समद्विबाहु त्रिभुज :

आइए एक कागज़ का पन्ना $ABCD$ लें जिसमें AB और CD के मध्य-बिंदु क्रमशः E और F हैं। [देखिए आकृति 18.14(i)] फिर हम कागज़ को EF पर मोड़ते हैं जैसा कि आकृति 18.14(ii) में दिखाया गया है। अब हम कैंची की सहायता से एक कोना ELM काटते हैं और कागज़ को खोल लेते हैं। [देखिए आकृति 18.14(iii)] साथ ही आइए कटे हुए भाग को भी खोल लें जैसा कि आकृति 18.14(iv) में दिखाया गया है। पन्ने में से समद्विबाहु त्रिभुज के आकार का एक भाग $LL'M$ कट गया है। (क्यों?) साथ ही, $\angle LEM = \angle L'EM = 90°$। क्या $\triangle LL'M$ असमान भुजा LL' के मध्य-बिंदु को सम्मुख शीर्ष M से मिलाने वाली रेखा ME के सापेक्ष सममित है? हाँ! हम कहते हैं कि **प्रत्येक समद्विबाहु त्रिभुज असमान भुजा के मध्य-बिंदु को उसके सम्मुख शीर्ष से मिलाने वाली रेखा के सापेक्ष सममित होता है।**

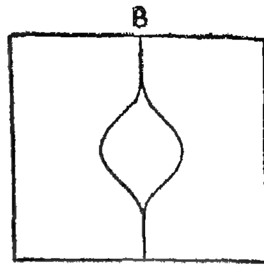
क्या कागज का शेष बचा हुआ भाग [देखिए आकृति 18.14 (iii)] FM के सापेक्ष सममित है? हाँ!



(i)



(ii)



(iii)

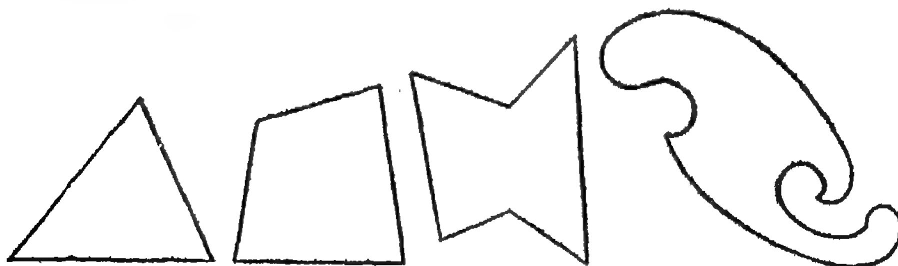
आकृति 18.15

18.4.4 पुनः आइए एक कागज़ का पन्ना लें और उसे रेखा AB के अनुदिश मोड़ें। [देखिए आकृति 18.15 (i)] एक कैंची की सहायता से अब हम छायामय भाग काट लेते हैं जैसा कि आकृति 18.15(ii) में दिखाया गया है और कागज़ को खोल लेते हैं। [देखिए आकृति 18.15(iii)] कटा हुआ भाग तथा कागज़ का शेष बचा हुआ भाग दोनों ही रेखा AB के सापेक्ष सममित हैं।

अतः (तल में) **कोई ज्यामितीय आकृति सममित तब कहलाती है जबकि (तल में) कोई ऐसी रेखा ज्ञात हो सकती हो जो इस आकृति को दो सर्वांगसम भागों में विभाजित करे तथा प्रत्येक भाग इस रेखा में एक दूसरे का दर्पण प्रतिबिम्ब हो जाए।** तब आकृति को इस रेखा के सापेक्ष **सममित** कहा जाता है तथा रेखा **सममिति अक्ष** कहलाती है।

ऐसी ज्यामितीय आकृति जिसकी कोई सममिति अक्ष न हो असममित (*non-symmetric*) कहलाती है।

असममित ज्यामितीय आकृतियों के कुछ उदाहरण नीचे दिए जा रहे हैं:

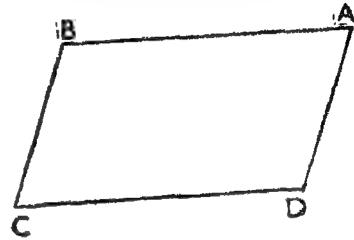


आकृति 18.16

प्रश्नावली 18.1

1. ABC एक समबाहु त्रिभुज है और D, E तथा F क्रमशः उसकी भुजाओं BC, CA और AB के मध्य-बिंदु हैं। त्रिभुज की कितनी सममिति अक्ष हैं? इनके नाम बताइए।

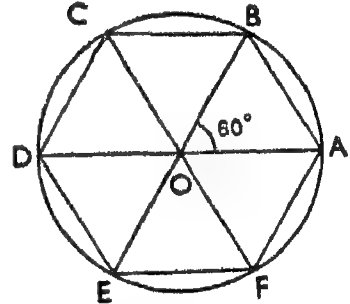
2. आकृति 18.17 में $ABCD$ एक समांतर चतुर्भुज है। क्या इसकी कोई सममिति अक्ष है?



आकृति 18.17

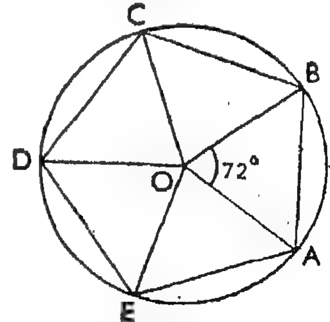
3. जाँच कीजिए कि वृत्त अपने प्रत्येक व्यास के सापेक्ष सममित होता है।

4. केन्द्र O का एक वृत्त लीजिए और इसे $60°$ के कोण के छः त्रिज्यखंडों में विभाजित कीजिए जैसा कि आकृति 18.18 में दिखाया गया है। AB, BC इत्यादि को जोड़िए और एक सम षड्भुज $ABCDEF$ प्राप्त कीजिए। इसकी छः सममिति अक्ष हैं। इनमें से तीन विकर्ण AD, BE और CF हैं। क्या आप शेष तीन विकर्ण ज्ञात कर सकते हैं? उनके नाम बताइए।



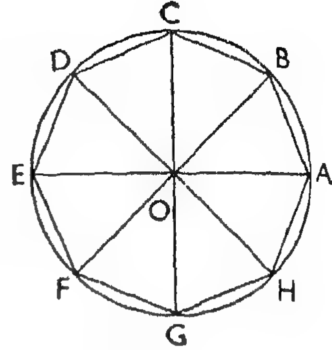
आकृति 18.18

5. केन्द्र O का एक वृत्त लीजिए और चाँदे की सहायता से इसे $72°$ के कोण के पाँच त्रिज्यखंडों में विभाजित कीजिए जैसा कि आकृति 18.19 में दिखाया गया है। AB, BC, इत्यादि को जोड़िए और एक सम पंचभुज $ABCDE$ प्राप्त कीजिए। इसकी पाँच सममिति अक्ष हैं। क्या आप इनको खोज कर इनके नाम बता सकते हैं?



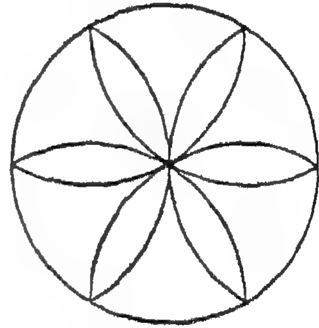
आकृति 18.19

6. केन्द्र O का एक वृत्त लीजिए और इसके दो लम्ब व्यास AOE और COG खींचिए। कोणों AOC और COE को क्रमशः व्यासों BOF और DOH से समद्विभाजित कीजिए (देखिए आकृति 18.20)। क्या ये व्यास कोणों EOG और GOA को भी समद्विभाजित करेंगे? क्या अब वृत्त $45°$ के कोण के आठ त्रिज्यखंडों में विभाजित हो गया है? AB, BC इत्यादि को जोड़िए और एक सम अष्टभुज $ABCDEFGH$ प्राप्त कीजिए। सम अष्टभुज की कितनी सममिति अक्ष होती है? क्या आप इन सबके नाम बता सकते हैं?



आकृति 18.20

7. परकार की सहायता से अपनी कापी पर संलग्न आकृति (आकृति 18.21) खीचिए। इसकी कितनी सममिति अक्ष हैं?



आकृति 18.21

8. 2 सेमी और 3 सेमी की त्रिज्याओं के दो वृत्त इस प्रकार खींचिए कि उनके केन्द्रों के बीच की दूरी 6 सेमी रहे। इस आकृति का सममिति अक्ष ज्ञात कीजिए।

9. 2 सेमी त्रिज्या के दो वृत्त इस प्रकार खींचिए कि उनके केन्द्रों के बीच की दूरी 5 सेमी रहे। इस आकृति के सममिति अक्ष ज्ञात कीजिए।

10. एक ही केन्द्र तथा 3 सेमी और 4 सेमी त्रिज्याओं के दो वृत्त खीचिए। इस आकृति के कितने सममिति अक्ष हैं?

11. अंग्रेजी वर्णमाला के बड़े (capital) अक्षरों में कौन से अक्षर ऊर्ध्वाधर रेखा के सापेक्ष सममित हैं? कौन से क्षैतिज रेखा के सापेक्ष? कौन से दोनों रेखाओं के सापेक्ष?

12. हिन्दी वर्णमाला के कौन से अक्षर सममित हैं? उसकी (उनकी) सममिति अक्ष भी बताइए।

13. एक वृत्त और उसकी एक जीवा खींचिए। इस प्रकार बने वृत्त के दीर्घ खंड (*major segment*) और लघु खंड (*minor segment*) के क्रमशः सममिति अक्ष ज्ञात कीजिए। आप क्या देखते हैं?

14. एक अर्धवृत्त का सममिति अक्ष ज्ञात कीजिए।
15. वृत्त का एक त्रिज्यखंड दिया है। उसका सममिति अक्ष ज्ञात कीजिए।

18.5 रचनाएँ

18.5.1 एक दी हुई रेखा के सापेक्ष एक दिए बिंदु के सममित एक बिंदु की रचना करना

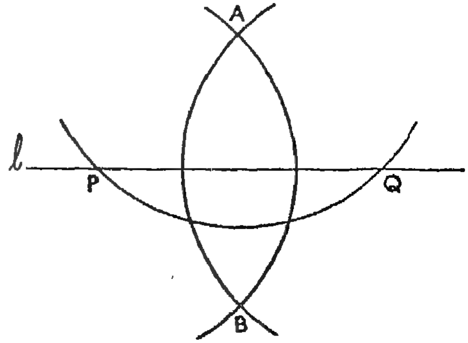
माना l दी हुई रेखा है तथा A दिया हुआ बिंदु है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : A को केन्द्र मानकर और एक उपयुक्त त्रिज्या लेकर हम l को P और Q पर काटता हुआ एक चाप खींचते हैं।

चरण 2 : अब P को केन्द्र मानकर तथा PA त्रिज्या लेकर हम एक चाप खींचते हैं जो कि A से होकर जाएगा।

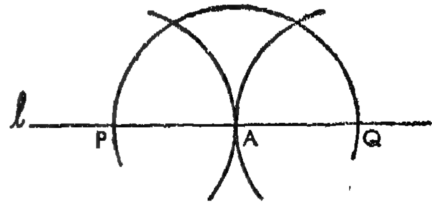
चरण 3 : अब Q को केन्द्र मानकर और चरण 2 वाली ही त्रिज्या (PA के बराबर) लेकर हम एक और चाप खींचते हैं। यह चाप भी बिंदु A से होकर जाएगा (क्यों?) तथा पहले चाप को A के दूसरी ओर एक अन्य बिंदु, मान लीजिए, B पर काटेगा।

तब B ही वांछित बिंदु है। (देखिए आकृति 18.22) A और B रेखा l के सापेक्ष सममित हैं।



आकृति 18.22

यदि A, रेखा l पर स्थित हो तो क्या होगा? क्या हमें आकृति 18.23 से कुछ संकेत मिलता है?



आकृति 18.23

क्या B, A के संपाती होगा? हम देखेंगे कि

यदि कोई बिंदु सममिति अक्ष पर स्थित हो तो वह स्वयं के सममित होता है।

18.5.2 दो दिए हुए बिंदुओं की सममिति अक्ष की रचना करना

माना A और B दो दिए हुए बिंदु हैं।

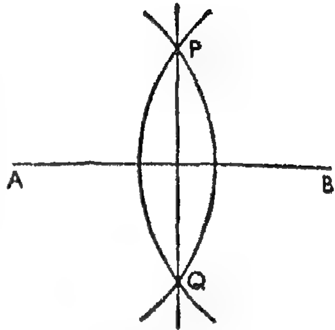
हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : A को केन्द्र मानकर और $\frac{1}{2}AB$ से अधिक त्रिज्या लेकर हम एक चाप खींचते हैं।

चरण 2 : तब B को केन्द्र मानकर और वही त्रिज्या लेकर जो चरण 1 में ली थी हम पहले चाप को बिंदुओं P और Q पर काटता हुआ एक अन्य चाप खींचते हैं।

चरण 3 : अब हम रेखा PQ खींचते हैं। तब PQ ही दिए हुए बिंदुओं A और B की वांछित सममिति रेखा है। (देखिए आकृति 18.24)

[यदि हम चरण 1 में त्रिज्या $\frac{1}{2}AB$ से अधिक न लें तो रचना में क्या कठिनाई आएगी ?]



आकृति 18.24

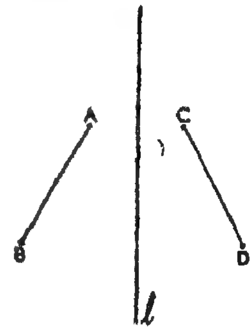
18.5.3 एक दी हुई सममिति रेखा के सापेक्ष एक दिए हुए रेखाखंड के सममित रेखाखंड की रचना करना।

माना AB दिया हुआ रेखाखंड है और l सममिति रेखा है। हम यह रचना निम्न चरणों में करते हैं :

चरण 1 : हम पहले l के सापेक्ष A के सममित बिंदु की रचना करते हैं। (देखिए अनुच्छेद 18.5.1) मान लीजिए यह बिंदु C है।

चरण 2 : फिर हम l के सापेक्ष B के सममित बिंदु D की रचना करते हैं।

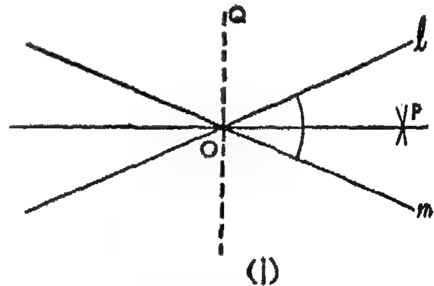
चरण 3 : अब हम C और D को जोड़ते हैं।
तब CD ही l के सापेक्ष रेखाखंड AB का वांछित सममित रेखाखंड है। (देखिए आकृति 18.25)



आकृति 18.25

18.5.4 दो दी हुई रेखाओं की सममिति रेखा की रचना करना।

स्थिति 1 : माना l और m दो रेखाएँ हैं और वे बिंदु O पर प्रतिच्छेद करती हैं। हम l और m के बीच के कोण का समद्विभाजक OP खींचते हैं। तब OP रेखाओं l और m के लिए एक सममिति रेखा है। [देखिए आकृति 18 26(i)]

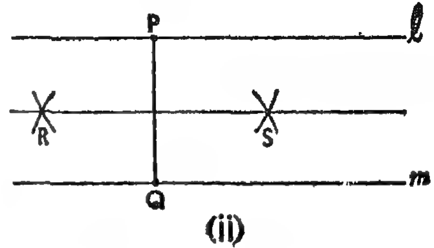


आकृति 18.26

आइए O पर $OQ \perp OP$ खींचे। क्या l और m के लिए रेखा OQ भी एक सममिति रेखा है? हाँ!

स्थिति 2 : माना रेखाएं l और m समांतर हैं। आइए l पर कोई बिंदु और P लें $PQ \perp l$ खींचें जो कि m से Q पर मिले। (क्या Q पर $PQ \perp m$ है?) अब हम P और Q की सममिति रेखा RS खींचते हैं। (देखिए अनुच्छेद 18.5.2)

तब RS ही l और m की सममिति रेखा है। [देखिए आकृति 18.26 (ii)]



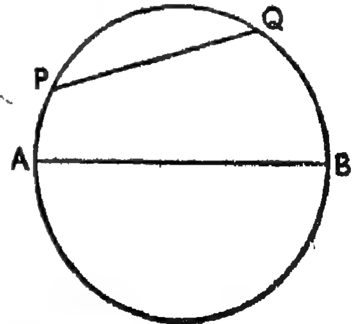
आकृति 18.26

प्रश्नावली 18.2

1. एक त्रिभुज ABC और रेखा l खींचिए। उस त्रिभुज की रचना कीजिए जो कि l के सापेक्ष $\triangle ABC$ के सममित हो।

2. एक वृत्त पर कोई बिंदु P लीजिए और वृत्त का एक व्यास खींचिए जो P में से होकर न जाता हो। इस व्यास के सापेक्ष बिंदु P के सममित बिंदु Q की रचना कीजिए। क्या Q भी वृत्त पर स्थित है?

3. आकृति 18.27 में PQ, वृत्त की एक जीवा तथा AB एक व्यास है। क्या आप एक जीवा, मान लीजिए, RS की रचना कर सकते हैं जो AB के सापेक्ष PQ के सममित हो?



आकृति 18.27

4. केन्द्र O वाले वृत्त पर दो बिंदु P और Q स्थित हैं। P और Q की सममिति रेखा खींचिए। क्या यह O से होकर जाती है?

उत्तरमाला

एकक I

प्रश्नावली 1.1

1. (*i*) 70 (*ii*) 7 (*iii*) 7000000 (*iv*) 70000000
2. 28000 3. 198 4. 1
5. (*i*) 5326 (*ii*) 7080952 (*iii*) 2003801
6. (*i*) $2\times10^3+4\times10^2+6\times1$ (*ii*) $1\times10^4+5\times10^3+9\times10^2+6\times10+8\times1$
   (*iii*) 4×10^5 (*iv*) $1\times10^7+1\times10^6+1\times10^4+1\times10^3+1\times1$
7. योग=3330 9. 4 11. 10

प्रश्नावली 1.2

1. 8; 15; 190; 8646 2. 1 3. कोई नहीं 4. 6 5. 1
6. (*i*) $73>61$ (*ii*) $18<29$ (*iii*) $57<59<63$ (*iv*) $0<n$ जबकि n एक धनपूर्णांक है
7. (*i*) 68708 (*ii*) 78888 (*iii*) 3000 (*iv*) 0
8. 100005, 999992 9. 6530, 3056 10. 975421, 124579

एकक II

प्रश्नावली 2.1

1. (क) योग सम है (ख) योग सम है
2. (क) 8386 (ख) 4400 (ग) 11669 3. 4, 3
4. (क) 12, 8, 19 (ख) नहीं, नहीं 5. (क) 15 (ख)

6	1	8
7	5	3
2	9	4

प्रश्नावली 2.2

1. (क) 522 (ख) 274 (ग) 2737 2. (क) 2245
   (ख) 171240 3. (क) $\begin{array}{r} 895 \\ -291 \\ \hline 604 \end{array}$ (ख) $\begin{array}{r} 5376 \\ -1859 \\ \hline 3517 \end{array}$ 4. 1
5. 1089 क्विंटल 6. 42 रु० 7. 218 रु० 8. 1089 लाख

प्रश्नावली 2.3

1. (क) 607920 (ख) 1245616 2. (क) 10172486 (ख) 104023689
3. (क) 20300 (ख) 2451600 4. 2881 पैसे
5. 999900 6. 400 लाख या 4 करोड़ टन 7. 31392 रु०
8. 36135504 9. (क) 50, 36, 72. (ख) नहीं, नहीं

प्रश्नावली 2.4

1. 6400 2. 10000 3. 6695 4. 8910 5. 6030
6. 2400 7. 13000

प्रश्नावली 2.5

2. (क) 1607 (ख) 408 (ग) 608 3. नहीं 4. 8050×302
5. 5 6. 22, 0 7. 15 पंक्तियाँ 8. 49, 74, 99,... 9. शनिवार

विविध प्रश्नावली I

(एकक I और II पर)

1. (i) $<$ (ii) $=$ (iii) $>$ 2. (क) 1, 4, 7, 15, 85 (ख) 85, 15, 7, 4, 1
3. (क) 2, 4, 7 4. (i) $7 \times (8+9) = 7 \times 8 + 7 \times 9$
(ii) $25 \times (75-15) = 25 \times 75 - 25 \times 15$ (iii) $35 \times (7+3) = 35 \times 7 + 35 \times 3$
(iv) $a \times (b+c) = a \times b + a \times c$ (v) $a(b-c) = ab - ac$ 5. 8050
6. 10 7. नहीं 8. 75 9. 125 10. हाँ, एकक स्थान
11. 0 12. 300 13. (क) अंतर सम है (ख) अंतर सम है (ग) योग विषम है, अंतर विषम है
14. 1000 15. (क) 1005 (ख) 9995 16. 7520, 2057 17. 1999
18. अंतर 8853086421 है 19. (क) 1485 (ख) 24950 20. बृहस्पतिवार 21. मंगलवार
22. (क)

$$\begin{array}{r} 123 \\ \times \quad 7 \\ \hline 861 \\ \hline \end{array}$$

(ख)

$$\begin{array}{r} 527 \\ \times \quad 27 \\ \hline 3689 \\ 1054 \\ \hline 14229 \\ \hline \end{array}$$

एकक III

प्रश्नावली 3.1

1. (i) मूल्य में कमी (ii) दक्षिण को जाना (iii) जनसंख्या में वृद्धि
(iv) बैंक से रुपया निकालना (v) वजन बढ़ना 2. (i) +7°C (ii) −7°C
3. (i) −25 (ii) +110 4. लाभ (रुपयों में) : +10, +6, +5, −7, −22, −3, −3
5. (i) समुद्र सतह से 300 मीटर ऊपर (ii) समुद्र सतह से 100 मीटर नीचे
(iii) समुद्र सतह से 70 मीटर ऊपर 7. (i) −390 (ii) +8840

प्रश्नावली 3.2

1. (*i*) $>$ (*ii*) $<$ (*iii*) $<$ (*iv*) $<$ (*v*) $<$ (*vi*) $>$ (*vii*) $>$ (*viii*) $<$
2. −41, −10, −3, 16, 17 3. 21, 0, −3, −4, −5, −10 4. (*i*) 140 (*ii*) 120 (*iii*) 140 (*iv*) −120 5. (*i*) −4948 (*ii*) −10785 (*iii*) −120610 (*iv*) 999 6. 9, 3 7. 1, 0, 8, 7, 13

एकक IV

प्रश्नावली 4.1

2. (*i*) −6 (*ii*) −10 (*iii*) −29 (*iv*) −8 (*v*) −90 (*vi*) 37
3. (क) हाँ (ख) हाँ (ग) −3, 3; −2, 2; −1, 1; 0, 0
(घ) शून्य और किसी संख्या का योग स्वयं वह संख्या होता है

प्रश्नावली 4.2

1. (*i*) −30 (*ii*) −34 (*iii*) 20 (*iv*) −80 (*v*) 900 (*vi*) 186 (*vii*) −1111 (*viii*) −6683 2. (*i*) −1289 (*ii*) 462 (*iii*) −1325 (*iv*) 0 (*v*) 2004 (*vi*) 0

प्रश्नावली 4.3

2. (*i*) −245 (*ii*) −481 (*iii*) −1100 (*iv*) −52
3. (क) −3, 8, −4 (ख) हाँ, हाँ

प्रश्नावली 4.4

1. −17, 17; 8, −8; −6, 6; −4, 4; −3, 3
2. 200, 100, 65, 48, −84, −95, 0, 1 4. (*i*) 4 (*ii*) −8 (*iii*) 0 (*iv*) 5 (*v*) 7 (*vi*) 0

प्रश्नावली 4.5

1. (*i*) 11 (*ii*) −2 (*iii*) −1 (*iv*) −9 2. (*i*) 3938 (*ii*) −8656 (*iii*) −122 (*iv*) 236 (*v*) −236 (*vi*) 732 (*vii*) −732 (*viii*) −42920 3. (*i*) $<$ (*ii*) $>$ (*iii*) $>$
4. −50, 50; नहीं 5. 8°C 6. 12 मी 7. (*i*) −7 (*ii*) 11 (*iii*) −8 (*iv*) 8 (*v*) 151 8. −1037 9. 28 10. −25 11. (*i*) 1 (*ii*) 0

प्रश्नावली 4.6

1. (*i*) −30 (*ii*) −143 (*iii*) −126 (*iv*) 72 (*v*) 0 (*vi*) 0 2. (क) हाँ (ग) हाँ (घ) −3, 0; −2, 0; −1, 0; 0, 0 1. 0; 2, 0; 3, 0 3. (*i*) −1953 (*ii*) −3591 (*iii*) 11625 (*iv*) 0

प्रश्नावली 4.7

1. (i) 360 (ii) 0 (iii) 13176 (iv) 24750
2. (i) -1470000 (ii) -720 (iii) 23040 (iv) 0
3. (i) ऋणात्मक (ii) धनात्मक (iii) धनात्मक (iv) ऋणात्मक
4. $(-1)\times 40$; $(-1)\times(-16)$; $(-1)\times 54$; $(-1)\times 68$; $(-1)\times 0$
5. -480 6. (i) $(11+9)\times 10 > 11+(9\times 10)$ (ii) $(41-3)\times 10 > 41-(3\times 10)$

प्रश्नावली 4.8

1. (i) -4 (ii) 3 (iii) -3 (iv) 14 (v) 0
2. (i) 33 (ii) -7 (iii) 120 (iv) 12 (v) -10
3. (i) 1 (ii) -1 (iii) -1 (iv) 1

विविध प्रश्नावली II

(एकक III और IV पर)

1. (i) $-7 > -17$ (ii) $10 > 0$ (iii) $-3 < 0$ (iv) $-3 < 3$
2. 1, -1 3. (i) $(-1)\times(-5)$ (ii) $(-1)\times 13$ (iii) $(-1)\times(-9)$ (iv) $(-1)\times 1$ (v) $(-1)\times(-1)$ (vi) $(-1)\times 0$ 4. $|-3| = 3$; $-|3| = -3$;
5. (i) -4 (ii) 10 6. 16°C 8. (i) 15 (ii) -16
9. (i) 5 (ii) -5 (iii) 6 (iv) -6
10. (i) 12 (ii) 10 (iii) 0 11. (i) 6 (ii) 0 (iii) 6
12. (क) 50 (ख) 25 13. (i) -58 (ii) -87 (iii) 819 (iv) 90
14. (क) 29, 3, -3, -1 (ख) हाँ 15. 50 सेमी, 16

एकक V

प्रश्नावली 5.1

1. (i) 8 (ii) 4 (iii) 1 (iv) 2 (v) 644
2. (i) 256 (ii) -64 (iii) 196 (iv) 81
3. (i) 7^1 (ii) $(-3)^6$ (iii) 10^8 (iv) $(-26)^1$
4. (i) 2500 (ii) 64 (iii) 81 (iv) 1 (v) 1 (vi) -1 (vii) 1 (viii) -32 (ix) 32 (x) 1024
5. (i) 72 (ii) -108 (iii) -100 (iv) -576 (v) -2500 (vi) 1000 (vii) 90000 (viii) 432 6. (i) -1728 (ii) -15625 (iii) -729 (iv) 1000000 (v) 8000
7. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 8. 1, 8, 27, 64, 125, 216, 343, 512, 729

प्रश्नावली 5.2

1. (i) 10 (ii) 16 (iii) 15 (iv) 13 (v) 14 (vi) 18 (vii) 100 (viii) 26 (ix) 1000 2. 17, 20, 60, 28
3. 121, 144, 11025 4. 0

एकक VI

प्रश्नावली 6.1

1. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60   2. (क) विषम (ख) सम (ग) 4 के गुणज
3. (*i*) गुणनखंड (*ii*) 3, 9 (*iii*) 1, 2, 4, 19, 38, 76 4. No; 1, 2, 3, 5, 6, 10, 15, 30

प्रश्नावली 6.2

1. 2   2. हां, 9   3. 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; 71, 73
5. 139   6. 90, 91, 92, 93, 94, 95, 96

प्रश्नावली 6.3

2. $90 = 3^2 \times 2 \times 5$; $108 = 3^3 \times 2^2$; $9000 = 5^3 \times 2^3 \times 3^2$; $221 = 17 \times 13$;
$7325 = 5^2 \times 293$; $8712 = 2^3 \times 3^2 \times 11^2$; $13915 = 5 \times 23 \times 11^2$ 3. $5^2 \times 2^2$

प्रश्नावली 6.4

1. (*ii*) और (*iv*)   3. (*i*) हाँ (*ii*) नहीं
4. 2 से विभाज्य : (*i*), (*ii*), (*iv*), (*v*), (*vi*) और (*vii*)
3 से विभाज्य : (*i*), (*ii*), (*iii*), (*iv*), (*v*) और (*vi*)
5 से विभाज्य : (*i*), (*v*) और (*vi*)
9 से विभाज्य : (*ii*), (*iii*) और (*vi*)
10 से विभाज्य : (*i*), (*v*) और (*vi*)
5. हाँ   6. हाँ   7. (*i*) और (*ii*)

विविध प्रश्नावली III

(एकक V और VI पर)

1. 10000, 676, 10201, −1331, 59049, −1, 1
2. 21, 125, 32   3. 12, 24, 36, 48, 60, 72, 84
4. $3 \times 5^2 \times 107$ 5. 151   6. 101, 103, 107, 109
7. $819 = 3^2 \times 13 \times 7$; $3105 = 5 \times 3^3 \times 23$; $153549 = 3^3 \times 11^2 \times 47$
10. 28   13. 21, 28, 36, 45
14. (*i*) 25   (*ii*) 36   (*iii*) 784

एकक VII

प्रश्नावली 7.1

1. (*ii*) और (*iv*) एकपदी हैं; (*i*) द्विपद हैं; (*iii*), (*vi*) और (*vii*) त्रिपद हैं
2. (*i*), (*iii*)   3. $-3y$, 4, −1, m, $17yz$

प्रश्नावली 7.2

1. (i) $18x^2y$ (ii) 0 (iii) $17abc$
2. (i) $-15y^2$ (ii) $-18ab$ (iii) $6a^3$
3. (i) $6x^2$ (ii) $2b$ (iii) $-3x^2-2y-4$ (iv) $a-3b+2c$ (v) $7m^2-7m-8$
4. (i) $23y-22z$ (ii) $-7x^2y$ (iii) $18x^3+2x^2$ (iv) $12a+4b-25c+1$
5. (i) c^3+3a^3-2abc (ii) $-3x^2+7xy-3y^2$ (iii) $4m^3-6mn+8$
6. $a-c-3$
7. $x^2+2xy-y^2$
8. $-24x+21y-15a$
9. $-x^2+2xy+3y^2$
10. $-9m-4n+2p$

प्रश्नावली 7.3

1. (i) $x-5y$ (ii) $14m-13l$ (iii) $9a-7b$ (iv) $3x^2-x$ (v) $-2y+1$
(vi) $2m-l$ (vii) $-3x-2y-4$ (viii) $3x^2-2y+3x-4$ (ix) $7-8a+2b+3c$
(x) $-7x^3+2x^2+2x-2$ (xi) $2ab-3a-3b$ (xii) $y-13x-15$
2. (i) $9x+6z-(4y+8)$ (ii) $2x^3+4y^3-(3z-9)$ (iii) $a-b-(4d+5)$

प्रश्नावली 7.4

1. (i) x^7 (ii) x^5 (iii) x^5y^5 (iv) x^5y^4 (v) $-52x^3y$ (vi) $-72abx$
(vii) $18xy^3$ (viii) $-12a^4b^4$ (ix) $6m^3n^3$ (x) $18a^3b^9$ (xi) $120a^5b^6c^6$
2. (i) $-14x^2+7xy$ (ii) a^3b-3ab^3 (iii) $-2b^2a^3+ab^3$ (iv) $-32x^3y+8x^2y^3$
(v) $13x^3y+13x^3y^2-39x^3y$ (vi) $2ba^2-4b^2y+10by^2$
(vii) $-12m^3n^2-24m^3n^3+12m^2n$ (viii) $8x^6-16x^5-16x^4+40x^3$
3. (i) $3a^2+13a-10$ (ii) $2m^2-5mn-3n^2$ (iii) $12a^2+7ab-12b^2$
(iv) $4x^4+12x^2+9$ (v) $4x^4-12x^2+9$ (vi) x^4+3x^2+2
4. (i) $13xy-2xz$ (ii) $10ab$ (iii) a^2+b^2 (iv) $4ab$ (v) $xy+6y^2$
(vi) x^2+2x (vii) b^2+2ab (viii) a^2+2ab (ix) $2a^2+3ab+3b^2$

प्रश्नावली 7.5

1. (i) -3 (ii) 0 (iii) -2 (iv) 3 (v) 4 (vi) 20
(vii) -4 (viii) -24

एकक VIII

प्रश्नावली 8.1

1. (i) $x=5$ (ii) $y=4$ (iii) $m=3$ (iv) $x=4$ (v) $y=4$
(vi) $z=-2$ (vii) $x=3$

प्रश्नावली 8.2

1. $x=5$ 2. $x=2$ 3. $y=2$ 4. $x=3$ 5. $y=4$ 6. $y=-3$ 7. $x=3$
8. $y=-1$ 9. $x=2$ 10. $z=10$ 11. $y=-20$ 12. $m=-1$
13. $y=3$ 14. $x=-2$ 15. $x=0$ 16. $x=3$ 17. $x=2$

प्रश्नावली 8.3

1. 21 2. 30 3. 21, 32 4. 14, 42 5. 7, 8, 9
6. 23 मी, 19 मी 7. 36 मी 8. 25 पैसे के सिक्के : 20 और 10 पैसे के सिक्के : 40
9. 100.00 रु० के पुरस्कार : 19 और 25.00 रु० के पुरस्कार : 44 10. 5 वर्ग

विविध प्रश्नावली IV

(एकक VII और VIII पर)

1. (*i*) एकपदी है; (*iii*) और (*v*) द्विपद हैं; (*ii*) और (*iv*) त्रिपद हैं
2. (*i*) $2-y$ (*ii*) y^2 (*iii*) $-7y$ (*iv*) $4a^2+2$
3. (*i*) $a+b+c$ (*ii*) $-x^2-y^2-z^2$ (*iii*) $x+1$
4. (*i*) $2y^2x-2x^3-2z$ (*ii*) $-2a-2b$ (*iii*) $-2x^2+6x+9$ 5. $3x+3y$
6. (*i*) $x^2-6xy-3y^2-10x$ (*ii*) $2xy^2-5y^3+x^3-x^2-7$ 7. $6b$ एकक
8. (*i*) $3x^2-6y^2$ (*ii*) $a-b$ (*iii*) $b-a-3c$
9. (*i*) 3 (*ii*) -3 (*iii*) -1 (*iv*) 5 (*v*) -8
10. (*i*) 3 (*ii*) 9 (*iii*) 1 (*iv*) 0 (*v*) 0
11. (*i*) x^6 (*ii*) x^3+x (*iii*) x^2-2x (*iv*) $5x^2-x^3$ (*v*) $-x^3+x^5$
12. (*i*) x^2-y^2 (*ii*) $x^2+xy+x+y$ (*iii*) $x^2-5xy+6y^2$ (*iv*) $y^2+xy-2x^2$
13. (*i*) 4 (*ii*) $4y^2$ (*iii*) -7 14. $z-y$, 0 16. $4a$ एकक
17. $-x^2-4y^2-2xy-2x+8y-8$ 18. (*i*) -1 (*ii*) 3
19. (क) (*i*) $p^3r^2oe^2tis$ (*ii*) $com^2ut^3ai^2vy$ (*iii*) $c^2a^2n^2el^2tio$
19. (ख) (*i*) *proportion* (*ii*) *addition* (*iii*) *symmetry* 20. (*i*) $x=13$ (*ii*) $x=9$ (*iii*) $x=-1$ (*iv*) $x=14$ (*v*) $x=-4$ (*vi*) $x=-2$ (*vii*) $x=0$ (*viii*) $x=1$ (*ix*) $x=9$ 21. 25 22. राम : 220, श्याम : 100; जीवन : 230
23. 230 24. 602, 86 25. 34 सेमी 26. 300 किमी

एकक IX

प्रश्नावली 9.1

1. (*i*) 1 : 8 (*ii*) 5 : 4 (*iii*) 2 : 1 3. (*i*) 1 : 3 (*ii*) 5 : 3 (*iii*) 2 : 3 (*iv*) 3 : 5 (*v*) 3 : 20 (*vi*) 3 : 4 (*vii*) 4 : 1 (*viii*) 1 : 4 (*ix*) 4 : 1 4. 3 : 2 5. 4 किमी 7. 25 घन सेमी 8. 4 9. 800

प्रश्नावली 9.2

2. (*iv*) और (*v*) समानुपात में हैं 3. (क) हाँ (ख) हाँ
5. 5 6. 33 7. 360 8. 9 9. -21

प्रश्नावली 9.3

1.

समय (मिनटों में)	2	3	7	25	155
ऊँचाई (मीटर में)	24	36	84	300	1860

2.

लम्बाई (मीटर में)	3	4	7	15	20
मूल्य (रुपयों में)	78	104	182	390	520

3. 39, 65, 78, 130 किलोग्राम 4. -6 5. 45 : 1, 8 घंटे, 90 किमी
6. 18 मीटर

प्रश्नावली 9.4

1. (i)

x	36	72	108	144
y	48	24	16	12

(ii)

x	50	75	100	150	200
y	300	200	150	100	75

2. (i) 80 (ii) 60 3. 56 किमी प्रति घंटा 4. 8 5. 14
6. 10 दिन 7. 6 घंटे 8. (i) 300 (ii) 60 सेमी

प्रश्नावली 9.5

1. (i) 1 : 5 (ii) 3 : 10 (iii) 6 : 25 (iv) 19 : 20 (v) 3 : 2
2. (i) 140 रु० (ii) 600 (iii) 70 गैलन (iv) 24 मीटर (v) 21
3. 2.00 रु० 4. 180.00 रु० 5. 1568
6. 7500 रु०, 8750 रु०, 8750 रु० 7. 132 8. 12%

प्रश्नावली 9.6

1. 210 रु० 2. लाभ : 64 रु०, 8% 3. 1320 रु० 4. 50 रु०
5. 240 रु० 6. 1200 रु० 7. 594 रु०, 94 रु० 8. 34500 रु०

प्रश्नावली 9.7

1. 525 रु० 2. 72 रु०, 360 रु० 3. 45 रु० 4. 2600 रु०
5. 350 रु०, 4950 रु० 6. 10%

एकक X

प्रश्नावली 10.1

1. कितनी ही रेखाएं
2. एक 3. (i) एक (ii) तीन 4. 6 5. (i) BA, BC, BD
(ii) CB, CD, CA (iii) DA, DC, DB 6. 3 7. 6, 6
8. $A, E, D; B, F, D; C, F, E$ 9. 10 10. 1

एकक XI

प्रश्नावली 11.1

1. (i) 3 (ii) 10 (iii) 7 2. 6

प्रश्नावली 11.2

1. (*i*) $AB > CD$ (*ii*) $AB = CD$ (*iii*) $AB < CD$
2. हाँ 3. b

प्रश्नावली 11.3

1. (*i*) 300 सेमी (*ii*) 240 सेमी (*iii*) 435 सेमी (*iv*) 520 सेमी
2. (*i*) 60 मिमी (*ii*) 64 मिमी (*iii*) 2000 मिमी (*iv*) 3400 मिमी
   (*v*) 4520 मिमी 4. 3 सेमी

प्रश्नावली 11.4

3. 6.5 सेमी 5. 7.5 सेमी

एकक XII

प्रश्नावली 12.1

1. $\angle ABC$ या $\angle B$; $\angle BCA$ या $\angle C$ 3. $\angle COB$, $\angle BOD$, $\angle AOD$ 4. $\angle NPM$

प्रश्नावली 12.3

3. उत्तर 4. दक्षिण 7. $40°$, $20°$, $10°$, $60°$, $45°$
8. $30°$, $130°$, $110°$, $100°$ 9. (*i*), (*ii*), (*iii*), (*vii*) और (*viii*) पूरक हैं; (*iv*), (*v*), (*vi*) और (*ix*) संपूरक हैं 11. $45°$ 12. $90°$ 13. (क) सत्य (ख) असत्य 14. न्यून कोण 15. $132°$

प्रश्नावली 12.7

1. O और N अभ्यंतर में हैं; M बहिर्भाग में है
2. नहीं, नहीं 3. नहीं, नहीं

एकक XIII

प्रश्नावली 13.2

1. (*i*) $\angle CQE$ (*ii*) $\angle FQD$ 2. $\angle PQD$

प्रश्नावली 13.3

1. $45°$ 2. $75°$ 3. $110°$

प्रश्नावली 13.5

2. हाँ

एकक XIV

प्रश्नावली 14.1

1. 5.

प्रश्नावली 14.2

1. नहीं 2. नहीं 3. (*i*) $80°$ (*ii*) $60°$ (*iii*) $30°$ (*iv*) $\angle A = \angle C = 45°$
4. (क) (*i*), (*ii*) (ख) (*iv*) (ग) (*iii*) 5. (*i*) और (*ii*) सत्य हैं
7. 6.6 सेमी 8. हाँ 9. $30°, 60°, 90°$; समकोण त्रिभुज
10. $45°, 45°, 90°$

प्रश्नावली 14.3

2. समबाहु 3. $30°, 30°$ 4. $45°, 45°$

एकक XV

प्रश्नावली 15.1

3. (क) और (ग) सत्य हैं 4. हाँ 6. हाँ 8. $260°$

प्रश्नावली 15.2

4. नहीं

एकक XVII

प्रश्नावली 17.1

1. *ABFE* और *DCGH*, *BCGF* और *ADHE* 2. वर्ग 3. वर्ग
4. वर्ग 5. (*i*) *ABFE* और *BCGF* (*ii*) *ABCD* और *CDHG* (*iii*) *EFGH* और *ADHE* 6. (*i*) *BA, BF, BC* (*ii*) *DA, DH, DC* (*iii*) *FG, HG, CG*
7. *B, H*; *BH*। *C, E*; *CE*। *D, F*; *DF*।

प्रश्नावली 17.2

1. (*i*) *ACFD* और *ABED* (*ii*) *EFCB* और *DFCA* (*iii*) *ABED* और *ABC*
2. (*i*) *AB, AD, AC* (*ii*) *EB, ED, EF* (*iii*) *AC, BC, FC*
4. (*i*) 8, 18, 12 (*ii*) 10, 24, 16

प्रश्नावली 17.3

2. *BA, OC*; *CA, OB* 3. हाँ 5. 6, 10, 6 6. (*i*) 7, 12, 7
(*ii*) 8, 14, 8

एकक XVIII

प्रश्नावली 18.1

1. 3, *AD, BE, CF* 2. नहीं
4. सम्मुख भुजाओं के तीन युग्मों के मध्य-बिंदुओं को जोड़ने वाले रेखाएं
5. शीर्षों को उनकी सम्मुख भुजाओं के मध्य-बिंदुओं से जोड़ने वाले रेखाएं। 6. हाँ, हाँ, 8
7. 6 8. उनके केन्द्रों को मिलाने वाली रेखा
9. उनके केन्द्रों को मिलाने वाली रेखा। साथ ही, उनके केन्द्रों को जोड़ने वाले रेखाखंड का लम्ब समद्विभाजक
10. कितनी ही 12. ठ, इसकी ऊर्ध्वाधर रेखा
13. जीवा का लम्ब समद्विभाजक 14. व्यास का लम्ब समद्विभाजक 15. केन्द्रीय कोण का समद्विभाजक

प्रश्नावली 18.2

2. हाँ 4. हाँ

पारिभाषिक शब्द-सूची

अंक	digit
अंकगणित	arithmetic
अंकगणित की आधारभूत प्रमेय	Fundamental Theorem of Arithmetic
अंकित मान	face value
अंत: एकांतर कोण	alternate interior angles
अंत: कोण	interior angle
अंत बिंदु	end point
अंतर द्वारा तुलना	comparison by difference
अंतराष्ट्रीय अंकन पद्धति	International System of Numeration
अंश	degree
अंशांकन	graduations
अंशांकित रूलर	graduated ruler
अक्स	trace
अक्स करने का कागज	tracing paper
अक्षर गुणनखंड	literal factor
अक्षर संख्याएँ	literal numbers
अज्ञात	unknown
अद्वितीय	unique
अधिक कोण	obtuse angle
अधिक कोण त्रिभुज	obtuse triangle
अर्ध वृत्त	semicircle
अनुक्रमानुपात	direct proportion/direct variation
अनुक्रमानुपाती	directly proportional/vary directly
अनुपात	ratio
अनुपात के पद	terms of the ratio
अनुप्रयोग	applications
अभाज्य गुणनखंडन	prime factorization
अभाज्य गुणनखंडन गुण	*Prime Factorization Property*
अभाज्य युग्म	twin primes
अभाज्य संख्या	prime number
अभिगृहीत	axiom
अभिदिशा	sense
अभ्यंतर	interior
अम्ल	acid
अवरोही क्रम	descending order
असंरेखी	non-collinear
असममित	non-symmetric

असमान	unequal
असमान चिन्ह	unlike signs
असमान पद	unlike terms
अष्टभुज	octagon
आकृति	figure
आधार	base
आधारभूत संक्रियाएँ	fundamental operations
आपतन गुण	incidence property
आयत	rectangle
आयतन	volume
आयताकार लम्ब प्रिज्म	rectangular right prism
आयताकार समांतर षड्फलक	rectangular parallelopiped
आरोही क्रम	increasing order
आलम्ब	fulcrum
आसन्न कोण	adjacent angles
इरेटोसथीन्स की सीव	Sieve of Eratosthenes
उत्तरी ध्रुव	North Pole
उभयनिष्ठ/सर्वनिष्ठ	common
उभयनिष्ठ सीमा	common boundary
ऊँचाई	height
ऊर्ध्वाधर तल	vertical plane
ऊर्ध्वाधर दिशा	vertical direction
ऊर्ध्वाधर रेखा	vertical line
ऊपर	above
ऋजुकोण	straight angle
ऋण	minus/loan
ऋणात्मक	negative
ऋणात्मक पूर्णांक	negative integer
एकक/मात्रक	unit
एकक स्थान	unit's place
एकपदी	monomial
एकांतर कोण	alternate angles
एल्गोरिथ्म	algorithm
कम्पन आवृत्ति	vibration frequency
किनारा	edge
किरण	ray
केन्द्र	centre
कोण	angle
क्रमविनिमेय	commutative
क्रय मूल्य	cost price
क्षैतिज तल	horizontal plane
क्षैतिज रेखा	horizontal line
खगोलशास्त्री	astronomer

खाता	account
गणन	count
गणन संख्याएं	counting numbers
गणित	mathematics
गतिविधि	activity
गमन	transition
गहराई	depth
गुणज	multiple
गुणन	multiplication
गुणनखंड	factor
गुणनखंडन	factorization
गुणनफल	product
गुणन सारणियाँ	multiplication tables
गुणांक	coefficient
घन	cube
घनमूल	cube root
घनाभ	cuboid
घात	power
घातांक	exponent/index
घूर्णन	rotation
घूर्णन सममिति	rotational symmetry
चतुर्थमूल	fourth root
चतुर्भुज	quadrilateral
चतुष्फलक	tetrahedron
चाँदा	protractor
चाप	arc
चिन्ह	sign/mark
चौड़ाई	breadth/width
जीवा	chord
डिवाइडर	divider (ins)
तत्समक अवयव	identity element
तल	plane
तापमान	temperature
तिर्यक रेखा	transversal
तुला	balance
त्रिज्यखंड	sector
त्रिज्य रेखाखंड	radial segment
त्रिज्या	radius
त्रिपद	trinomial
त्रिभुज	triangle
त्रिभुजाकार क्षेत्र	triangular region
त्रिभुजाकार संख्याएँ	triangular numbers
थोक विक्रेता	wholesaler

दक्षिण पक्ष	Right Hand Side
दर्पण	mirror
दर्पण प्रतिबिम्ब	mirror image
दर प्रतिशत	rate per cent
दशांश	tenth
दस का स्थान	ten's place
द्विपद	binomial
दीर्घ खंड	major segment
दीर्घ चाप	major arc
धनपूर्णांक	natural number
धनात्मक पूर्णांक	positive integers
धारा	current
निर्माता	manufacturer
निरपेक्ष मान	absolute value
नीचे	below
न्यून कोण	acute angle
न्यून कोण त्रिभुज	acute triangle
पंचभुज	pentagon
पंचमूल	fifth root
पक्ष	side
पग	step
पद	term
पद का गुणांक	coefficient of the term
परकार	compasses
परमाणु ऊर्जा	atomic energy
परवर्ती	successor
परिधि	circumference
परिमाण	magnitude
परिमाप	perimeter
पार्श्व फलक	side face
पिरैमिड	pyramid
पुनर्व्यवस्थितिकरण गुण	rearrangement property
पुस्तकालय	library
पूर्ण घन	perfect cube
पूर्ण वर्ग	perfect square
पूर्ण संख्या	whole number
पूर्ण संख्याओं का समुच्चय	set of whole numbers
पूर्णांक	integers
पूर्णांकों का समुच्चय	set of integers
पूर्णांकीय घात	integral powers
पूरक	complement
पूरक कोण	complementary angles
प्रतिकूल	opposites

प्रतिच्छेद	intersect
प्रतिच्छेद बिंदु	point of intersection
प्रतिबन्धित समीकरण	conditional equation
प्रतिरूप	model/pattern
प्रतिरूप भिन्न	Representative fraction
प्रतिरूप विधि	pattern approach
प्रतिलोम	inverse
प्रतिवर्ती कोण	reflex angle
प्रतिशत/प्रतिशतता	per cent/percentage
प्रतिस्थापन	substitution
प्रयत्न और भूल	trial and error
प्रसारित संकेतन	expanded notation
प्रारम्भिक बिंदु	initial point
प्रिज्म	prism
प्रोत्साहन	motivation
फलक	face
बराबर/समान	equal
बहिर्भाग	exterior
बहुफलक	polyhedron/polyhedra(pl)
बहुभुज	polygon
बार बार योग	repeated addition
बार बार व्यवकलन	repeated subtraction
बाह्य : कोण	exterior angle
बिंदु	point
बिंदुकित प्रतिरूप	dot pattern
बीजगणित	algebra
बीजीय व्यंजक	algebraic expression
ब्याज	interest
भागफल	quotient
भाज्य	dividend
भाज्य संख्या	composite number
भुजा	arm/side
भूमध्य रेखा	Equator
भौतिकी	physics
मध्य	between
मध्य के पद	middle terms
मध्य-बिंदु	middle point
मध्यस्थिति	betweenness
मानक	standard
मानक कोण	standard angle
मापन	measure/measurement
मिश्रधन	amount
मीटर छड़	metre rod

मुख्य विकर्ण	main diagonal
मूल	root/original
मूलधन	principal
मैजिक वर्ग	majic square
मोटाई	thickness
मोड़ का निशान	crease
मृत्यु समुद्र	Dead Sea
युग्म	pair
योग	addition/sum
योग पर वितरित	distributive over addition
योज्य प्रतिलोम	additive inverse
रसायन	chemistry
रिक्त	empty
रूलर	ruler
रेखा	line
रेखाखंड	line-segment/segment
रैखिक युग्म	linear pair
रैखिक सममिति	linear symmetry
लघु खंड	minor segment
लघु चाप	minor arc
लम्ब	perpendicular
लम्ब प्रिज्म	right prism
लम्बाई	length
लाभ	profit
लौंद का वर्ष	leap year
वर्ग	square
वर्गमूल	square root
वर्ग संख्याएँ	square numbers
वाम पक्ष	Left Hand Side
वार्षिक	per annum
विकर्ण	diagonal
विक्रय मूल्य	selling price
वितरण गुण	distributive property
विपरीत अभिदिशाएँ	opposite senses
विपरीत किरणें	opposite rays
विपरीत दिशाएँ	opposite directions
विपरीत सर्वांगसम	oppositely congruent
विभवांतर	potential difference
विभाजक	divisor
विभाजन	division
विभाजन द्वारा तुलना	comparison by division
विभाज्य	divisible
विभाज्यता	divisibility

विभाज्यता की जाँच	divisibility test
विभाजित	divide
विमाएँ	dimensions
विषमबाहु त्रिभुज	scalene triangle
विषम संख्या	odd number
वैज्ञानिक अनुसंधान	scientific research
वृत्त	circle
वृत्तखंड	segment of a circle
वृत्तीय क्षेत्र	circular region
व्यवकलन	subtraction
व्यवकलित करना	subtract
व्यास	diameter
व्युत्क्रमानुपात	inverse proportion/inverse variation
व्युत्क्रमानुपाती	inversely proportional/vary inversely
शाब्द समस्याएँ या प्रश्न	word problems
शीर्ष	vertex
शीर्ष कोण	vertical angle
शीर्षाभिमुख कोण	vertically opposite angles
शून्य	zero
शून्य का योज्य गुण	addition property of zero
शून्य कोण	zero angle
शून्येतर	non-zero
शेष	remainder
संकल्पना	concept
संक्रिया सारणी	operation table
संकेतन	notation
संख्या रेखा	number line
संख्यांक	numeral
संख्यात्मक गुणनखंड	numerical factor
संगत कोण	corresponding angles
संग्रह	collection
संगामी	concurrent
संपाती	coincide
संपूर्ण कोण	complete angle
संपूर्ण संख्या	perfect number
संपूरक	supplement
संपूरक कोण	supplementary angles
संयोग	combination/association
संरेखी	collinear
सप्तभुज	septagon
सपाट	flat
सम अंक	even digit
समकोण	right angle

समकोण त्रिभुज	
समद्विबाहु त्रिभुज	
समद्विभाजक	
सम बहुभुज	regular polygon
समबाहु त्रिभुज	equilateral triangle
सममित	symmetric/symmetrical
सममित स्थित	symmetrically situated
सममिति	symmetry
सममिति अक्ष	axis of symmetry
सममिति रेखा	line of symmetry
सम्मुख	opposite
सम संख्या	even number
समांतर	parallel
समांतर चतुर्भुज	parallelogram
समान चिन्ह	like signs
समान पद	like terms
समानुपात	proportion
समानुपातिकता स्थिरांक	constant of proportionality
समिका	equality [illegible]
समीकरण	equation [illegible]
समुद्र सतह	
समूहन संकेत	grouping symbols
सरलतम रूप	simplest form
सर्वांगसम	congruent
सहचारी	associative
साधारण ब्याज	simple interest
साहचर्य गुण	associative property
साहुल	plumb-line
सिरों के पद	extreme terms
सीधापन	straightness
सीधे सर्वांगसम	directly congruent
सूचित योग	indicated sum
सूत्र	formula
सौ का स्थान	hundred's place
स्तम्भ विधि से योग	column addition
स्थान धारक	place holder
स्थानीय मान	place value
स्वयं सिद्ध प्रमाण	postulate
षड्भुज	hexagon
हल	solution
हल्का	dilute
हानि	loss
हिन्दू अरेबिक अंकन पद्धति	Hindu Arabic Numeration System

the struggle for that new world order, the Committee are eager and desirous to help in every way. But the Committee cannot associate themselves or offer any co-operation in a war which is conducted on imperialist lines and which is meant to consolidate imperilism in India and elsewhere.

11. In view, however, of the gravity of the occasion and the fact that the pace of events during the last few days has often been swifter than the working of men's minds, the Committee desire to take no final decision at this stage, so as to allow for the full elucidation of the issues at stake, the real objectives aimed at, and the position of India in the present and in the future. But the decision cannot long be delayed, as India is being committed from day to day to a policy to which she is not a party and of which she disapproves

12 The Working Committee therefore invite the British Government to declare in unequivocal terms what their war aims are in regard to democracy and imperialism and the new order that is envisaged, in particular, how these aims are going to apply to India and to be given effect to in the present Do they include the elimination of imperialism and the treatment of India as a free nation whose policy will be guided in accordance with the wishes of her people? A clear declaration about the future, pledging the Government to the ending of Imperialism and Fascism alike, will be welcomed by the people of all countries, but it is far more important to give immediate effect to it, to the largest possible extent, for only this will convince the people that the declaration is meant to be honoured The real test of any declaration is its application in the present, for it is the present that will govern action today and give shape to the future.

13 War has broken out in Europe and the prospect is terrible to contemplate. But war has been taking its heavy toll of human life during the past year in Abyssinia, Spain and China. Innumerable innocent men, women and children have been bombed to death from the air in open cities, cold-blooded massacres, torture and utmost humiliation have followed each other in quick succession during these years of horror That horror grows, and violence and the threat of violence shadow the world, and, unless checked and ended, will destroy the precious inheritance of past ages. That horror has to be checked in Europe

and China, but it will not end till its root causes of Fascism and Imperialism are removed To that end, the Working Committee are prepared to give their co-operation. But it will be infinite tragedy if even this terrible war is carried on in the spirit of imperialism and for the purpose of retaining this structure which is itself the cause of war and human degradation.

14 The Working Committee wish to declare that the Indian people have no quarrel with the German people or the Japanese people or any other people. But they have a deep-rooted quarrel with systems which deny freedom and are based on violence and aggression. They do not look forward to a victory of one people over another or to a dictated peace, but to a victory of real democracy for all the people of all countries and a world freed from the nightmare of violence and imperialist oppression.

15. The Committee earnestly appeal to the Indian people to end all internal conflict and controversy and, in this grave hour of peril, to keep in readiness and hold together as a united nation, calm of purpose and determined to achieve the freedom of India with the larger freedom of the world.

Wardha, 14-9-1939

Gandhiji's Comment on the Manifesto

The Working Committee's statement on the world crisis took four days before it received final shape. Every member expressed his opinion freely on the draft that was, at the Committee's invitation, prepared by Pandit Jawaharlal Nehru I was sorry to find myself alone in thinking that whatever support was to be given to the British should be given unconditionally. This could only be done on a purely non-violent basis. But the Committee had a tremendous responsibility to discharge. It could not take the purely non-violent attitude. It felt that the nation had not imbibed the non-violent spirit requisite for the possession of the strength which disdains to take advantage of the difficulty of the opponent. But in stating the reasons for its conclusion the Committee desired to show the greatest consideration for the English.

The author of the statement is an artist. Though he cannot be surpassed in his implacable oppositon to imperialism in any shape or form, he is a friend of the English people. Indeed he is more English than Indian in his thoughts and make-up. He

is often more at home with Englishmen than with his own countrymen. And he is a humanitarian in the sense that he reacts to every wrong, no matter where perpetrated. Though, therefore, he is an ardent nationalist, his nationalism is enriched by his fine internationalism. Hence the statement is a manifesto addressed not only to his own countrymen, not only to the British government and the British people, but it is addressed also to the nations of the world including those that are exploited like India. He has compelled India, through the Working Committee, to think not merely of her own freedom, but of the freedom of all the exploited nations of the world.

The same time that the Committee passed the statement it appointed a Board of his choice with himself as Chairman to deal with the situation as it may develop from time to time.

I hope that the statement will receive the unanimous support of all the parties among Congress. The strongest among them will not find any lack of strength in it. And at this supreme hour in the history of the nation the Congress should believe that there will be no lack of strength in action, if action becomes necessary. It will be a pity, if Congressmen engage in petty squabbles and party strife. If anything big or worthy is to come out of the Committee's action, the undivided and unquestioned loyalty of every Congressman is absolutely necessary. I hope too that all other political parties and all communities will join the Committee's demand for a clear declaration of their policy from the British Government with such corresponding action as is possible amidst martial conditions Recognition of India, and for that matter of all those who are under the British Crown, as free and independent nations seems to me to be the natural corollary of British professions about democracy. If the war means anything less, the co-operation of dependent nations can never be honestly voluntary, unless it were based on non-violence.

All that is required is a mental revolution on the part of British statesmen To put it still more plainly, all that is required is honest action to implement the declaration of faith in democracy made on the eve of the war, and still being repeated from British platforms. Will Great Britain have an unwilling India dragged into the war or a willing ally co-operating with her in the prosecution of a defence of true democracy? The

Congress support will mean the greatest moral asset in favour of England and France. The Congress fights not with violent but with non-violent means, however imperfect, however crude the non-violence may be.

Sevagram, 15-9-39
Harijan, 23-9-1939

86

A POLISH SISTER'S AGONY

> "In spite of all I am going to try to reach Poland — sailing tonight — to Basra in Iraq, then through Turkey and Rumania The inner call is irresistible It may seem madness from the ordinary human point of view Now I am not going for my mother's sake or for my dearest friends who are now on the battle-fields — ready to die at any moment — it is for Poland itself I believe countries have also souls Souls of nations are a reality, not a theory, for me If I reach the soil of Poland, I will feel satisfied, even if I do not find those whom I love. It is the soul (and body) of the nation itself that is in its supreme hour of martyrdom I believe Poland bleeds and struggles not only for her own rights but for the Right, the Just, the True, for the freedom of all nations, India including I feel Hindu to the bottom of my heart, Indian as much as Pole, both motherlands are to be in my soul to the last day of my life. But I could not live if I would not do what is humanly possible to reach the feet of the Mother who is now bleeding in agony of pain. I shall write from the way, but not when I reach the war zone; I shall only think often about you and send mental messages as well as I can Bapuji, do pray in all fervour of your great loving heart for those thousands of innocent people who suffer incredible pain and misery in Poland It is these that need most sympathy and blessings and tender thoughts"

This is the letter a Polish sister wrote from Bombay harbour I have known her for some years. She has become as much Indian as she is Polish. She had decided to work at Maganwadi in the Magan Museum. But the rumours of war upset her She has an aged mother in Poland whom she could not bring out owing to passport difficulties. When the war actually broke out, she calmed down so far as her mother was concerned. But her highly strung nature would not let her rest

whilst her nearest and dearest were in peril of their lives for no offence of theirs. She is herself a believer through and through in non-violence But her very non-violence made her restless. Her whole soul has rebelled against the wrong, as she thinks, that is being perpetrated against her motherland So she has gone to find the Poland of her imagination fighting to the last ditch, not for merely preserving her own freedom but for the freedom of all those nations who have lost it And in this she naturally includes her second love, India. May her dream prove true. If Poland has that measure of uttermost bravery and an equal measure of selflessness, history will forget that she defended herself with violence Her violence will be counted almost as non-violence

Sevagram, 18-9-39
Harijan, 23-9-1939

87

IS INDIA A MILITARY COUNTRY ?

In the interesting broadcast delivered by the Commander-in-Chief of the Defence Forces in India on the 5th inst. there occurs this passage.

> "India is a military country and I am a soldier. It will, therefore, perhaps not be amiss, if I give you some personal impressions of what the effect of modernization will be on the personnel of the future Army in India They are not just guess-work but based on what has already been done With new scientific weapons and with modern vehicles, there will inevitably come new ideas and a new outlook Modernization is likely to give increased impetus to the already high rate of education in the Indian Army; and when nearly every soldier on discharge returns to his home with a knowledge of motor cars and machinery, there may well be a perceptible effect upon the age-old methods of agriculture and ways of living. Modernization in the Army may therefore have a considerable indirect effect upon the life of India Many of those who hear me will regret the passing of the horse No one regrets it more than myself. But as a soldier who knows the fate which awaits the horse in modern warfare, I rejoice for its sake, that one of the

greatest and best of friends of man is in future to be spared the horrors of war."

I must wholly, though respectfully, dissent from the view that India is a military country. And I thank God that it is not. It may be that the Commander-in-Chief has a special meaning for the term which I do not know. Or is it that his India is composed of only the Defence Forces under his command ? For me the Defence Forces are of the least importance in the make-up of the nation I need not be reminded that life would be in constant peril if the forces were withdrawn. The forces notwithstanding, life is not free from peril. There are riots, there are murders, there are dacoities, there are raids The Defence Forces avail little in all these perils. They generally act after the mischief is done. But the gallant Commander-in-Chief looks at things as a soldier I and, with me, the millions are untouched by the military spirit From ages past India has had a military caste in numbers wholly insignificant That caste has had little to do with the millions. This, however, is not the occasion for examining its contribution to the making of India. All I want to state, with the utmost emphasis at my command, is that the description of India as a military country is wrong Of all the countries in the world India is the least military. Though I have failed with the Working Committee in persuading them, at this supreme moment, to declare their undying faith in non-violence as the only sovereign remedy for saving mankind from destruction, I have not lost the hope that the masses will refuse to bow to the Moloch of war but will rely upon their capacity for suffering to save the country's honour. How has the undoubted military valour of Poland served her against the superior forces of Germany and Russia ? Would Poland unarmed have fared worse if it had met the challenge of these combined forces with the resolution to face death without retaliation ? Would the invading forces have taken a heavier toll from an infinitely more valorous Poland ? It is highly probable that their essential nature would have made them desist from a wholesale slaughter of innocents.

Of all the organisations of the world the Congress is the best fitted to show it the better way, indeed the only way, to the true life Its non-violent experiment will have been in vain if, when India wakes up from the present fear, she does not show

to the world the way of deliverance from the blood bath. The criminal waste of life and wealth that is now going on will not be the last, if India does not play her natural part by showing that human dignity is best preserved not by developing the capacity to deal destruction but by refusing to retaliate. I have no manner of doubt that, if it is possible to train millions in the black art of violence which is the law of the beast, it is more possible to train them in the white art of non-violence which is the law of regenerate man. Anyway, if the Commander-in-Chief will look beyond the Defence Forces, he will discover that the real India is not military but peace-loving.

Nor do I contemplate without uneasiness the prospect of Indian soldiers, trained after the modern manner, taking the motor spirit to their homes. Speed is not the end of life. Man sees more and lives more truly by walking to his duty.

On the train to Simla, 25-9-39
Harijan, 30-9-1939

88

CONUNDRUMS

Thus asks a well-known Congressman:

"1. What is your personal attitude towards this war consistent with non-violence?

2. Is it the same as, or different from, your attitude during the last war?

3. How could you with your non-violence actively associate with and help the Congress whose policy is based on violence in the present crisis?

4. What is your concrete plan based on non-violence to oppose or prevent this war?"

These questions conclude a long friendly complaint about my seeming inconsistencies or my inscrutability. Both are old complaints, perfectly justified from the standpoint of the complainants, wholly unjustified from my own. Therefore my complainants and I must agree to differ. Only this let me say. At the time of writing I never think of what I have said before. My aim is not to be consistent with my previous statements on a given question, but to be consistent with truth as it may

present itself to me at a given moment. The result has been that I have grown from truth to truth , I have saved my memory an undue strain , and what is more, whenever I have been obliged to compare my writing even of fifty years ago with the latest, I have discovered no inconsistency between the two But friends who observe inconsistency will do well to take the meaning that my latest writing may yield unless, of course, they prefer the old. But before making the choice they should try to see if there is not an underlying and abiding consistency between the two seeming inconsistencies.

So far as my inscrutability is concerned, friends should take my assurance that there is never any attempt on my part to suppress my thought when it is relevant. Sometimes it arises from my desire to be brief. And sometimes it must be due to my own ignorance of the subject on which I may be called upon to give an opinion

To give a typical instance, a friend, between whom and me there never is any mental reservation, thus writes in anguish rather than anger .

> "In the not-improbable event of India being a theatre of war, is Gandhiji prepared to advise his countrymen to bare their breasts to the enemy's sword ? A little while ago I would have pledged my word he would do so, but I am not confident any more "

I can only assure him that, notwithstanding my recent writings, he can retain his confidence that I would give the same advice as he expects I would have given before, or as I gave to the Czechs or the Abyssinians. My non-violence is made of stern stuff It is firmer than the firmest metal known to the scientists. Yet, alas, I am painfully conscious of the fact that it has still not attained its native firmness If it had, God would have shown me the way to deal with the many local cases of violence that I helplessly witness daily This is said not in arrogance but in the certain knowledge of the power of perfect non-violence I will not have the power of non-violence to be underestimated in order to cover my limitations or weaknesses.

Now for a few lines in answer to the foregoing questions

1 My personal reaction towards this war is one of greater horror than ever before I was not so disconsolate before as I am today. But the greater horror would prevent me today

from becoming the self-appointed recruiting sergeant that I had become during the last war. And yet, strange as it may appear, my sympathies are wholly with the Allies. Willy nilly this war is resolving itself into one between such democracy as the West has evolved and totalitarianism as it is typified in Herr Hitler. Though the part that Russia is playing is painful, let us hope that the unnatural combination will result in a happy though unintended fusion whose shape no one can foretell. Unless the Allies suffer demoralization, of which there is not the slightest indication, this war may be used to end all wars, at any rate of the virulent type that we see today. I have the hope that India, distraught though it is with internal dissensions, will play an effective part in ensuring the desired end and the spread of cleaner democracy than hitherto. This will undoubtedly depend upon how the Working Committee will ultimately act in the real tragedy that is being played on the world stage. We are both actors in and spectators of the drama. My line is cast. Whether I act as a humble guide of the Working Committee or, if I may use the same expression without offence, of the Government, my guidance will be for the deliberate purpose of taking either or both along the path of non-violence, be the step ever so imperceptible It is plain that I cannot force the pace either way. I can only use such power as God may endow my head or heart with for the moment.

2 I think I have covered the second question in answering the first

3 There are degrees of violence as of non-violence The Working Committee has not wilfully departed from the policy of non-violence. It could not honestly accept the real implications of non-violence. It felt that the vast mass of congressmen had never clearly understood that in the event of danger from without they were to defend the country by non-violent means All that they had learnt truly was that they could put up a successful fight, on the whole non-violent, against the British Government. Congressmen have had no training in the use of non-violence in other fields Thus, for example, they had not yet discovered a sure method of dealing successfully in a non-violent manner with communal riots or *goondaism* The argument is final inasmuch as it is based on actual experience. I would not serve the cause of non-violence, if I deserted my best

co-workers because they could not follow me in an extended application of non-violence I therefore remain with them in the faith that their departure from the non-violent method will be confined to the narrowest field and will be temporary.

4. I have no ready-made concrete plan. For me too this is a new field. Only I have no choice as to the means It must always be purely non-violent, whether I am closeted with the members of the Working Committee or with the Viceroy Therefore what I am doing is itself a part of the concrete plan. More will be revealed to me from day to day, as all my plans always have been. The famous non-co-operation resolution came to me within less than 24 hours of the meeting of the A.I C C at which it was moved in Calcutta in 1920, and so did practically the Dandi March The foundation of the first civil resistance under the then known name of passive resistance was laid by accident at a meeting of Indians in Johannesburg in 1906 convened for the purpose of finding the means of combating the anti-Asiatic measure of those days. I had gone to the meeting with no pre-conceived resolution. It was born at the meeting. The creation is still expanding But assuming that God had endowed me with full powers (which He never does), I would at once ask the English to lay down arms, free all their vassals, take pride in being called "little Englanders", and defy all the totalitarians of the world to do their worst Englishmen will then die unresistingly and go down to history as heroes of non-violence I would further invite Indians to co-operate with Englishmen in this godly martyrdom It will be an indissoluble partnership drawn up in letters of the blood of their own bodies, not of their so-called enemies But I have no such general power. Non-violence is a plant of slow growth. It grows imperceptibly but surely And even at the risk of being misunderstood, I must act in obedience to "*the still small voice*".

On the train to Simla, 25-9-39
Harijan, 30-9-1939

89

INDIA'S ATTITUDE

On the 27th August last, i.e. just before the senseless war broke out, Shrimati Kamaladevi Chattopadhyaya wrote to me as follows:

> "I have sent you an appeal through *The Bombay Chronicle*, asking you to voice the attitude of India and of the exploited peoples of the East on the present situation. What I meant was not a mere reiteration of our old position that we can have nothing to do with this imperialist war, but something more than that. The present conflict is mainly centred round the usual scramble for colonies, or spheres of influence as they are now called in polite phraseology. On this question the world thinks there are only two opinions, for it hears only two views: one which believes in the maintaining of the *status quo*; the other which wants a change but on the same basis, in other words, a redistribution of the loot and the right to exploit, which of course means war. It is in the very nature of things that such a redistribution can never come about without an armed conflict. Whether there will be anyone or anything left to enjoy, of course, is another question. But the world is mainly torn between these two. If the thesis of the one is accepted, then that of the other should also be. For, if England and France have the right to rule over large tracts and big nations, then Germany and Italy have an equal right. There is as little moral justification in the former countries crying halt to Hitler as there is in his what he calls his rightful claims.
>
> That there is a third view the world hardly seems to think, for it rarely hears it. And it is so essential that it should find expression: the voice of the people who are mere pawns in the game. Neither Danzig nor the Polish Corridor is the issue. The issue is the principle on which the whole of this present Western civilization is based: the right of the strong to rule and exploit the weak. Therefore it is centred round the whole colonial question, and Hitler and Mussolini are never tired of reminding the world of that. And that is exactly the reason why England has raised the cry of the Empire in danger. This question, therefore, vitally concerns all of us.
>
> We are against the *status quo*. We are fighting against it for we want a change in it. But our alternative is not war, for we know that the real solution does not lie there. We have an alternative to offer which is the only solution of this horrible muddle and the key to future world peace. It is this which

I would like to be placed before the world It may seem today like a cry in the wilderness; still we know that it is the voice which will ultimately prevail; and it is those hands which seem so feeble before these mailed fists that will finally reshape a battered humanity

You are eminently fitted to give voice to it India has, I think, a peculiar place today in the colonies of the world. It has both a moral prestige and organizational strength enjoyed by few colonies The others look to it for a lead in many matters It has already shown to the world a superior technique of struggle whose moral value the world is bound to appreciate some day India has, therefore, to tell a very distraught and maddened world that there is another path that humanity must tread, if it would save itself from these periodical disasters and bring peace and harmony to a bleeding world It is only those who have suffered so much against this system and who are bravely struggling to change it that can speak with all the conviction and moral basis necessary, speak not only for themselves but all the exploited peoples of the world"

I am sorry I had not seen Shrimati Kamaladevi's letter to *The Chronicle* Try as I will, I simply cannot do adequate justice to the reading of newspapers The letter then remained on my file for want of time to deal with it But I think delay has not affected the object of her letter Perhaps this is the psychological moment for me to express what is or should be India's attitude. I agree with Kamaladevi's analysis of the motives of the parties to the war. Both are fighting for their existence and for the furtherance of their policies. There is, however, this great difference between the two. however incomplete or equivocal the declarations of the Allies are, the world has interpreted them to mean that they are fighting for saving democracy. Herr Hitler is fighting for the extension of the German boundaries, although he was told that he should allow his claims to be submitted to an impartial tribunal for examination. He contemptuously rejected the way of peace or persuasion and chose that of the sword. Hence my sympathy for the cause of the Allies. But my sympathies must not be interpreted to mean endorsement, in any shape or form, of the doctrine of the sword for the defence even of proved right Proved right should be capable of being vindicated by right means as against the rude, i e sanguinary, means. Man may and should shed his own blood for establishing what he considers to be his 'right'. He may not shed the blood of his opponent who disputes his 'right'.

India as represented by the Congress has been fighting in order to prove her 'right', not by the sword but by the non-violent method. And she has carved out for herself a unique place and prestige in the world, although she is yet far—let us hope, not very far—from the independence of her dream. Her novel method has evidently struck the imagination of the world It has the right to expect India to play a decisive part in this war, which no people of the world have wanted, by insisting that the peace this time is not to be a mockery designed to share among the victors the spoils of war and to humiliate the vanquished. Pandit Jawaharlal Nehru, who has a right to speak for the Congress, has said in stately language that the peace must mean freedom for those who are held in bondage by the imperialist powers of the world. I have every hope that the Congress will also be able to show the world that the power that armaments give to defend right is nothing compared to the power that non-violence gives to do the same thing and that too with better show of reason. Armaments can show no reason, they can make only a pretence of it.

Sevagram, 9-10-39
Harijan, 14-10-1939

90

ON TRIAL

In the course of the conversation with the members of the Working Committee, I discovered that their non-violence had never gone beyond fighting the British Government with that weapon I had hugged the belief that congressmen had appreciated the logical result of the practice of non-violence for the past twenty years in fighting the biggest imperialist power in the world But in great experiments like that of non-violence, hypothetical questions have hardly any play. I myself used to say in answer to questions that when we had actually acquired independence we would know whether we could defend ourselves non-violently or not. But today the question is no longer hypothetical Whether there is on the part of the British Government a favourable declaration or not, the Congress has to decide upon the course it would adopt in the event of an invasion of India For though there may be no settlement with the Government, the Congress has to declare its policy and say whether it would fight the invading host violently or non-violently.

So far as I can read the Working Committee's mind after a fairly full discussion, the members think that congressmen are unprepared for non-violent defence against armed invasion.

This is tragic Surely the means adopted for driving an enemy from one's house must, more or less, coincide with those to be adopted for keeping him out of the house If anything, the latter process must be easier. The fact, however, is that our fight has not been one of non-violent resistance of the strong It has been one of passive resistance of the weak Therefore there is no spontaneous response in our hearts, at this supreme moment, to an undying faith in the efficacy of non-violence. The Working Committee, therefore, wisely said that they were not ready for the logical step The tragedy of the situation is that, if the Congress is to throw in its lot with those who believe in the necessity of armed defence of India, the past twenty years will have been years of gross neglect of the primary duty of congressmen to learn the science of armed warfare And

I fear that history will hold me, as the general of the fight, responsible for the tragedy. The future historian will say that I should have perceived that the nation was learning not non-violence of the strong but merely passivity of the weak, and that I should have, therefore, provided for congressmen's military training.

Being obsessed with the idea that somehow or other India will learn true non-violence, it would not occur to me to invite my co-workers to train themselves for armed defence. On the contrary, I used to discountenance all sword-play and the display of stout *lathis* Nor am I even now repentant for the past. I have the unquenchable faith that, of all the countries in the world, India is the one country which can learn the art of non-violence, that if the test were applied even now, there would be found, perhaps, thousands of men and women who would be willing to die without harbouring malice against their persecutors I have harangued crowds and told them repeatedly that they might have to suffer much including death by shooting. Did not thousands of men and women brave hardships during the salt campaign equal to any that soldiers are called upon to bear? No different capacity is required from what has been already evinced, if India has to contend against an invader. Only it will have to be on vaster scale

One thing ought not to be forgotten. India unarmed would not require to be destroyed through poison gas or bombardment. It is the Maginot line that has made the Siegfried line necessary And vice versa. Defence of India by the present methods has been necessary because she is an appendage of Britain Free India can have no enemy. And if her people have learnt the art of saying resolutely 'no' and acting up to it, I dare say, no one would want to invade her. Our economy would be so modelled as to prove no temptation for the exploiter.

But some congressmen will say · "Apart from the British, India has so many martial races within her border that they will want to put up a fight for the country which is as much theirs as ours." This is perfectly true I am therefore talking, for the moment, only of congressmen. How would they act in the event of an invasion? We shall never convert the whole of India to our creed unless we are prepared to die for it

The opposite course appeals me. Already, the bulk of the army is manned by the Mussalmans of the North, Sikhs and

Gurkhas. If the masses of the South and the Centre wish to become militarized, the Congress, which is supposed to represent them, will have to enter into competition with the former. The Congress will then have to be party to an enormous military budget. There may be all these things without the Congress consent. It will make all the difference in the world whether the Congress is party to them or not. The world is looking for something new and unique from India The Congress will be lost in the crowd, if it wears the same old outworn armour that the world is wearing today. The Congress has a name because it represents non-violence as a political weapon *par excellence*. If the Congress helps the Allies as a representative of non-violence, it will give to the Allied cause a prestige and a power which will be invaluable in deciding the ultimate fate of the war. But the members of the Working Committee have honestly and bravely not made the profession of such non-violence.

My position is, therefore, confined to myself alone. I have to find out whether I have any fellow-traveller along the lonely path. If I am in the minority of one, I must try to make converts. Whether one or many, I must declare my faith that it is better for India to discard violence altogether even for defending her borders. For India to enter into the race for armaments is to court suicide With the loss of India to non-violence the last hope of the world will be gone. I must live up to the creed I have professed for the last half a century, and hope to the last breath that India will make non-violence her creed, preserve man's dignity, and prevent him from reverting to the type from which he is supposed to have raised himself.

Sevagram, 10-10-39
Harijan, 14-10-1939

91

A POSER

A Britisher has written to Deenabandhu Andrews a letter on the war expounding his own views He is an ardent pacifist. Deenabandhu has shared the letter with me. In it occur the following paragraphs:

> "For India too I think that this is a very critical time The danger I see is that Britain may promise full Dominion Status or something of the kind, and as a result India will raise an army and become one more military-minded nation Her witness for the way of non-violence and soul force would then be largely discounted
>
> How can Gandhiji as a believer in non-violence ask for clarification of war aims with a view to getting India's support for Britain in this way of war? The only thing that he can do and that we should all be doing is to build up an army of men and women who are committed to the way of love and forgiveness and to receive, but never to return, violence. We have to work this out to see how it will alter our daily life as well as all our thinking and acting towards other communities and nations. We have to be disciplined in this and also to learn to act together as one man. Along this line I see tremendous possibilities
>
> Of course, we should also use all the influence we can to urge Britain to acknowledge and put into practice full democracy in India, as it is a high principle quite apart from whether India helps Britain in the war or not"

The danger that the writer senses is real. I dealt with it last week. The writer cavils at my sympathy with the Allies. I have shown it as an out-and-out believer in non-violence, even because of my belief. Whilst all violence is bad and must be condemned in the abstract, it is permissible for, it is even the duty of, a believer in *ahimsa* to distinguish between the aggressor and the defender. Having done so, he will side with the defender in a non-violent manner, i e. give his life in saving him. His intervention is likely to bring a speedier end to the duel, and may even result in bringing about peace between the combatants. Applying the argument to the present war, if the Congress actively sides with the Allies in a non-violent way, the Congress assistance will lift the Allied cause to a high moral

plane, and the Congress influence will be effectively used in the cause of peace. What is more it will be the special business of the Congress to see that, if the war is fought to a finish, no humiliation is heaped upon the vanquished That is the role I have conceived for the Congress The declaration of independence has become a necessity The question having been raised, the Congress cannot help Britain, if Britain is secretly fighting for imperialism while it declares to the world that the fight is for saving democracies For Britain to be in the right, a clear declaration of her war aim is a necessity, irrespective of the Congress policy.

Sevagram, 16-10-39
Harijan, 21-10-1939

92

HINDU-MUSLIM CLASHES

If proof were wanted to show that the non-violence of the Congress was in effect violence in suspension or inactive violence, it is furnished by the effective though quite indisciplined violence exhibited in Hindu-Muslim clashes Had the thousands of Hindus and Muslims who took part in the Khilafat agitation been non-violent at heart, they could never have been violent towards one another as they are continually found to be Nor can it be said that those who take part in the clashes are not congressmen If we rule out all those who resort to violence as non-congressmen, the Congress will cease to be a mass organization. For the combatants in these clashes are derived from the masses. Moreover one finds today violence resorted to by rival congressmen at Congress meetings. The gross indiscipline and fraud practised at Congress elections are all illustrations of Congress violence It is difficult to say, therefore, who if any, are non-violent congressmen. If they were in a majority in the Congress and if they played an effective part in Hindu-Muslim clashes, they could stop them or at least give their lives in stopping them. If the bulk of congressmen were truly non-violent, Muslims would be obliged to confess that congressmen could not be accused of anti-Muslim bias. It is not enough for

congressmen to say that they have not been found guilty of incorrect attitude. I may be proved to be legally correct, but may fail miserably if my action was examined in non-violent scales. But this non-violence has to be non-violence of the brave and the strong It must come from inward conviction. I have, therefore, not hesitated to say that it is better to be violent, if there is violence in our breasts, than to put on the cloak of non-violence to cover impotence. Violence is any day preferable to impotence. There is hope for a violent man to become non-violent There is no such hope for the impotent

Sevagram, 17-10-39
Harijan, 21-10-1939

93

UNCONDITIONAL SUPPORT ?

Several British friends are puzzled and ask, "Have you really said you will give unconditional support to Great Britain as you are reported to have said ?" The origin of this report lies in my very first statement after my talk with the Viceroy. The phrase 'unconditional support' does not occur in that statement. But had the Congress appreciated the position I took up, Congress support would have been unconditional in the sense that the Congress would not have asked for a clarification of Britain's war aims But the Working Committee could not honestly take up that position. Therefore it passed, as it had every right and justification for passing, the resolution it did Events have shown the prudence of the course adopted by the Working Committee. But had it waived the demand for the declaration of war aims, the support would have been unconditional. Remember, according to its constitution, the Congress is a non-violent body. Therefore its support would have been purely moral. It has no soldiers to supply, it has no money to give. But it has its good wishes. In my opinion those good wishes would have been more than many battalions. The Congress support would have put the British cause on an unassailable moral basis It would have made the British case just, beyond measure All the dependent races of the earth would have felt an indescribable glow of freedom. British moral stock would

have shot up high at a bound. That is what the Congress unconditional support would have meant.

But God had willed otherwise. British statesmen had not faith enough in the Congress profession of non-violence. I must confess that the Congress could not pass the test to the satisfaction of a strict though just examiner. Anyway my British friends should have no difficulty in understanding my position. Of course it is open to them to argue that as a war resister I could not even give moral support. I have already said in these columns that I do not hold such a view. It is open to a war resister to judge between two combatants and wish success to the one who has justice on his side. By so judging he is more likely to bring peace between the two than by remaining a mere spectator.

Sevagram, 30-10-39
Harijan, 4-11-1939

94

THE HOUR OF TRIAL

"If India takes up the doctrine of the sword, she may gain momentary victory. Then India will cease to be the pride of my heart. I am wedded to India because I owe my all to her. I believe absolutely that she has a mission for the world. She is not to copy Europe blindly. India's acceptance of the doctrine of the sword will be the hour of my trial. I hope I shall not be found wanting. My religion has no geographical limits. If I have a living faith in it, it will transcend my love for India herself. My life is dedicated to service of India through the religion of non-violence which I believe to be the root of Hinduism."

"The religion of non-violence is not meant merely for the *rishis* and saints. It is meant for the common people as well."

With the Working Committee

I quote these words from an article Gandhiji wrote in August 1920, but it looks as though they were uttered today. With that living faith in non-violence as the peculiar contribution of India to the world, he has tried to steer the course of our ship in fair weather and foul. Often enough the sky has been

darkened by spray, but we have never lost sight of the star. The present storm threatens to obliterate the star from our view, but the pilot has made it his business to warn us day in and day out against losing sight of the star which alone can guide us to the end of the voyage.

At the termination of the Working Committee meeting, therefore, Gandhiji asked the members to consider the question again, and decide once for all what the Congress and congressmen were going to do. The question affected not only our relations with government but the relations between community and community.

Gandhiji's suggestion was discussed for many hours, but no conclusion could be reached. At the next meeting of the Working Committee the question will come up again and a final decision taken. In the meanwhile members will have had enough time to think over the matter.

With the Gandhi Seva Sangh

The question has been engaging all Gandhiji's time, so much so that he has gone into perpetual silence except with people whom he sees by appointment, and often he wakes up in the early hours of the morning thinking about it. On the 25th morning he woke up at one o'clock, and was thinking about what he should say to the members of the executive of the Gandhi Seva Sangh whom he was to meet in the afternoon. So to them he said :

"The problem haunts me. It gives me no rest. I have described in *Harijan* the position of the junior members of the Working Committee. Theirs was a difficult position. They were torn between loyalty to a principle and loyalty to their colleagues. But their eagerness to make their position clear to me was most welcome It shows that we are all votaries of Truth, and even our mental struggles and conflicts arise out of our anxiety to be faithful to it. We had a very fine discussion in the Working Committee yesterday, and we discussed threadbare the position of the members as individuals and as representatives of the Congress and the people The question with you is different For you are here in your individual capacity, and you have to decide your conduct no matter what Congress or congressmen may think. The question is thus much simpler for you. Will you adopt an attitude of brotherliness to one who

has done your dear ones a grievous injury? Supposing Rajendra Babu was attacked, would you retaliate or rather stand between Rajendra Babu and the assailant and bear cheerfully the blows meant for him? You will do the latter, if you have shed all fear of death and injury to limbs, and all considerations for the domestic ties that bind you. But unless you have nothing but brotherliness for those that despitefully use you, your resolution that you would stand by the principle of non-violence through thick and thin will have no meaning It would be far better to wind up the Sangh than to have an empty resolution.

"Non-violence is not a cloistered virtue, confined only to the *rishi* and the cave-dweller. It is capable of being practised by the millions, not with full knowledge of its implications, but because it is the law of our species. It distinguishes man from the brute. But man has not shed the brute in him. He has to strive to do so This striving applies to the practice of non-violence, not to the belief in it I cannot strive to believe in a principle: I either believe in it or I do not And if I believe in it, I must bravely strive to practise it *Ahimsa* is an attribute of the brave. Cowardice and *ahimsa* do not go together any more than water and fire. It is that *ahimsa* that every member of the Gandhi Seva Sangh has to make a conscious effort to develop in himself.

"We have often thought about this question, but the hour of our trial has arrived today, as much with reference to war as with the struggle for Swaraj and equally with reference to Hindu-Muslim unity. Remember also that your non-violence cannot operate effectively unless you have faith in the spinning wheel. I would ask you to read *Hind Swaraj* with my eyes and see therein the chapter on how to make India non-violent. You cannot build non-violence on a factory civilization, but it can be built on self-contained villages. Even if Hitler was so minded, he could not devastate seven hundred thousand non-violent villages He would himself become non-violent in the process. Rural economy as I have conceived it eschews exploitation altogether, and exploitation is the essence of violence. You have, therefore, to be rural-minded before you can be non-violent, and to be rural-minded you have to have faith in the spinning wheel."

Posers

The members slept over this discourse and met Gandhiji again the next day. Numerous questions were troubling them, as they should everyone who is a votary of *ahimsa*. But out of regard for Gandhiji's time they limited themselves to a few.

"How can a believer in the non-violence of your conception be a minister?"

"I fear he cannot in the present state of things," said Gandhiji. "We have seen that our ministers have had to resort to violence even as the British government in the pre-autonomy days. It was inevitable perhaps. Had congressmen been truly non-violent, there would have been no resort to force But the Congress majorities were not based on unadulterated non-violence. A minister said the other day that, although he had not given up an iota of non-violence, he could not do without resorting to the minimum of firing. He had resorted to it only to the extent that it was unavoidable. He may have said it then; he may not say it again if I can help it. For, if he goes in again, he will have made his position clear, and he will represent a House that is predominantly non-violent. In other words, he will take office, if he is sure that the people would let him carry on the government on a non-violent basis."

"But may it not be that whereas a non-violent minister will confine violence to the lowest minimum, one who does not believe in non-violence would observe no such restraint?"

"That belief is a delusion. All those who are using violence today make the same claim Hitler too would say the same thing. General Dyer was acclaimed as the hero of the hour by the House of Lords because his object was said to be to prevent the spread of mob violence Soviet Russia believes its violence is a transitional stage to the establishment of an order without violence. In the present state of our belief and practice it may be better to wind up the Sangh, leaving each one to grow unfettered."

"But the suggestion is being made," said Kishorlalbhai, "that we may confine the membership to those who are engaged in constructive work."

"That suggestion is good, and we can conceivably convert the Sangh into such a body and then try each of us in our individual capacity to purify ourselves as much as we can. For

non-violence is impossible without self-purification Let us, therefore, be members of a self-purification association, but no association is necessary for that purpose. Therefore let us try, each in our own way, to face difficulties and problems as they come and see how far we can go. In Hudli, two years ago, I asked you to help in the elections and in sending the best possible men to the legislature. I gave advice in the atmosphere as it existed then I cannot give you that advice today. In fact the time may have come when it becomes necessary for such of you as believe in the non-violence of the brave to retire from the Congress as I did in 1934."

"How do you think that the masses can practise non-violence, when we know that they are all prone to anger, hate, ill-will? They are known to fight for the most trivial things."

"They are, and yet I think they can practise non-violence for the common good. Do you think the thousands of women that collected contraband salt had ill-will against anyone? They knew that the Congress or Gandhi had asked them to do certain things, and they did those things in faith and hope. To my mind the most perfect demonstration of non-violence was in Champaran. Did the thousands of ryots who rose up in revolt against the agrarian evils harbour the least ill-will against the government or the planters? Their belief in non-violence was unintelligent, even as the belief in the earth being round with many is unintelligent. But their belief in their leaders was genuine, and that was enough. With those who lead it is another matter. Their belief has got to be intelligent, and they have to live up to all the implications of the belief."

"But then are not the masses the world over like that?"

"They are not, for others have not that background of non-violence."

"But if there was non-violence ingrained in our masses, how should they have come to this state of slavery?"

"*There* indeed is what I flatter myself is going to be my contribution. I want that non-violence of the weak to become non-violence of the brave. It may be a dream, but I have to strive for its realization"

Sevagram, 29-10-39 M D.
Harijan, 4-11-1939

95

SINDH RIOTS

I have been following the riots in Sindh with painful interest. Many people delude themselves with the belief that I possess powers to remedy all wrongs. I wish I had them, though I am not sure that such possession will be an unmixed blessing. I should make people helpless, if I made an indiscriminate use of such powers And they would be of no use, if I might not use them freely As it is, I use what powers I have to the fullest extent Thank God, they are too limited to be harmful My chief work, however, is to teach people to help themselves

Here is a pathetic wire from Shikarpur :

> "Riots, loot, incendiarism. Sukkur district villages Hindus mercilessly butchered, women and girls raped and kidnapped. Hindu life, property unsafe. Situation most critical. Government policy not firm Pray send inquiry committee immediately to see situation personally — President, Hindu General Panchayat"

It is the third of its kind from Sindh. I took no notice of the first two mainly because I was pre-occupied in Allahabad and I had no concrete consolation to offer The Shikarpur Panchayat has come to the wrong person for help. For I am myself helpless. The Congress has not yet sufficiently advanced in non-violence to deal with riots and the like. It must develop it enough to deal with such situations if it is to retain its prestige. I suggested 'peace brigades', but the suggestion proved premature if not unworkable. No doubt the Sindh government should be able to protect life and property of the people within their jurisdiction Evidently the matter has gone beyond their control. Sindh is nominally autonomous and to that extent less able to protect life and property than the preceding government. For it has never had previous training in the policing or the military arts I have shown in previous writings that the central government is impotent to prevent loss of life, property and worse during riots. It is able to check their spread and punish the wrong-doers when it wishes. It is organized solely

for the protection of imperial trade and therefore for the maintenance of peace in so far as it is necessary for the safety of that trade. Hence it is ill equipped for real protection of the people. Such protection involves the training of the people in the art of self-defence and securing their co-operation in quelling riots, etc. This would be putting imperial rule in jeopardy

Now the only effective way in which I can help the Sindhis is to show them the way of non-violence. But that cannot be learnt in a day. The other way is the way the world has followed hitherto, i.e. armed defence of life and property God helps only those who help themselves The Sindhis are no exception. They must learn the art of defending themselves against robbers, raiders and the like. If they do not feel safe and are too weak to defend themselves, they should leave the place which has proved too inhospitable to live in

Sevagram, 28-11-39
Harijan, 2-12-1939

96
IN GOD'S GOOD HANDS

"Dear Friend,

I am unknown to you personally, but my wife and I came very near to being your host and hostess when you visited the Greenfield Mill at Darwen, Lancashire, in 1931 I think I was the first to express the hope that you might be willing to meet personally our cotton operatives during your stay in England that year, a period of distress through widespread unemployment in Lancashire; and this was conveyed to you through my friend C. F. Andrews Just before the time for the visit arrived, however, we left our Darwen home and our work as welfare supervisors at the Greenfield factory, to start a new life in charge of the Quaker International Centre in Berlin; and you were entertained by our friends and successors, Charles and Harriet Haworth

Through 5½ years' residence in Germany with our family of four young children we have come to know and to love very many people in that country, and have also watched and shared at first hand all the tragic developments, moral and political, of this time We were, however, already well acquainted with Germany and her people through the Quaker relief work for

starving children after the last war, in which we had been privileged to take part.

"I read with deep interest and inspiration, in the early days of this war, a few lines written by you in *Harijan*, which have been a great help and comfort to me in these troubled times. They are as follows: 'I personally would not purchase my own country's freedom by violence, even if such a thing were possible... My faith that what is gained by the sword will also be lost by the sword, is imperishable.' I have been encouraged, after mentioning my thoughts to my friend Agatha Harrison, to write and tell you what I have had so much in mind during these first distressful weeks of war. She has kindly let me see something of what you have written on the subject of the war situation, which helped me to understand your attitude towards it, but still leaves me under the sense of a deep concern which I now ask permission to lay before you.

"I meet so often nowadays even fine and convinced friends of peace who, in view of the brutal suppression of independence in countries like Czechoslovakia and Poland, feel themselves in a moral dilemma; who take the view that when such things happen war is inevitable and even right for their country, in the attempt to restore what has been lost — though as individuals they would probably dissociate themselves from it. I have wondered whether this is not the moment when such an acknowledged spiritual prophet and leader as you are, might give a clarion call to the whole world, pointing to another way than the senseless gamble and destruction of war; another way, not merely for the settling of disputes but far more important, for resistance to evil and even for the attainment of political aims. The war method, as we experienced in 1914-1918, seems inevitably to vitiate its own ends, however good these may be at the outset, whereas the moral means of non-violence and redemptive love do but strengthen and purify the aims, in so far as these are based on moral right and justice. This you have taught us. It seems to me that a lead from one with your spiritual authority along the lines of a better way than war to help the stricken and oppressed, whether individuals or nations, might come with redeeming power to many who find themselves in the distress of a moral dilemma at present time, and release spiritual energies which, because of this dilemma, are lamed and useless, or are prostituted in the attempt to supply uplift and inspiration for the pursuit of a holy war for civilization, freedom — or of the mere negative aim of destroying Hitlerism by successful rivalry in its own methods.

"I have no moral right to judge — sadly the contrary — but I know that you appreciate frankness, and I therefore venture to confess myself puzzled that, so far from seizing this opportunity

to proclaim the better way in which you have been such a magnificent pioneer, you appear (I feel sure I am mistaken or misunderstand) to be thinking mainly of what political advantage for the cause of independence in India can be gained from the war situation. One almost has the impression that the opportunity which war presents is welcomed, and that support for the Allies in the attempt to gain their ends through war would be gladly given if an adequate bargain were forthcoming

I have no special knowledge of Indian affairs, but, relying on the judgment of my best friends who have close association with the best opinion in India, I am eager for the realization of their hopes and yours; but I would hope for this as the outcome of a liberal movement of mind in our own people and Government as part of a generous desire and determination for a new and better world instead of one filled with aggression and imperialist domination — rather than as a bargain by imperialist Britain without any change of mind, primarily for the purpose of obtaining the material support of India in winning the war.

My heart cries to you out of its distress and despair caused by the war. I have seen so much that is good in the German people, the self-giving idealism of its youth whom I love, as well as the evil with which I am so intimately acquainted through trying to help the victims. I have been in a Nazi prison, and have believed that through my suffering there has been shown to me a nobler way to help the splendid young men of Germany and of my own country, than the way we are about to employ, of driving millions of them to slaughter one another I believe that many people throughout the world are longing to see the way out of this evil into which we are slowly slipping deeper and deeper, until it may soon be too late. Perhaps you alone can help us. I write to you in deep concern to beg you to consider whether it may not be laid upon you to do so.

With deep esteem and love,

49, Parliament Hill, Your friend sincerely,
London, N. W. 3 CORDER CATCHPOOL"

I know that this letter reflects the attitude of many Englishmen who look to me for showing the better way. Sir Radhakrishnan's collection of tributes on my completing seventy years has deepened the hope of thousands of peace lovers. But I know what a poor instrument I am for the fulfilment of that hope Admirers have given me credit I do not deserve. I am not able to testify that India furnishes the world with a good example of non-violence of the strong and therefore as a substitute for armed resistance against an aggressor. India undoubtedly has

shown the efficacy of passive non-violence as a weapon of the weak. But useful as it is as a substitute of terrorism, I claim no newness or merit for it. It is no contribution to the peace movement.

It is no wonder that my identification with the Congress demand appears to contradict my previous writing quoted by my correspondent. There is, however, no contradiction. Even now as then I would not gain independence at the cost of non-violence. The critic might retort that, if the British Government made the required declaration, I would be helping the Allies and thereby taking part in violence. The retort would be reasonable but for the fact that the additional help that Britain would gain from the Congress would be purely moral. The Congress would contribute neither men nor money. The moral influence would be used on the side of peace. I have already said in these columns that my non-violence does recognize different species of violence — defensive and offensive. It is true that in the long run the difference is obliterated, but the initial merit persists A non-violent person is bound, when the occasion arises, to say which side is just. Thus I wished success to the Abyssinians, the Spaniards, the Czechs, the Chinese and the Poles, though in each case I wished that they could have offered non-violent resistance. In the present case, if the Congress could justify the British case on the high ground professed by Mr Chamberlain, India declared free would throw in her whole moral weight on the side of peace. The part I am playing is, in my opinion. strictly non-violent There is no spirit of bargaining behind the Congress demand which itself is wholly moral There is no desire to embarrass the Government There is to be no precipitate civil disobedience. Care is being taken to meet every just objection to the Congress demand and to smooth every difficulty in the way of Great Britain making the requisite declaration The greatest strain is being put upon impatient congressmen spoiling for a fight though non-violent. I myself want to be able to play an effective part in bringing peace nearer. I might be able to do so if India becomes in fact an independent ally of Britain, though the legal process is delayed till the war is ended

But who am I ? I have no strength save what God gives me. I have no authority over my countrymen save the purely moral.

If He holds me to be a pure instrument for the spread of non-violence in the place of the awful violence now ruling the earth, He will give the strength and show me the way My greatest weapon is mute prayer. The cause of peace is, therefore, in God's good hands. Nothing can happen but by His will expressed in His eternal, changeless Law which is He. We neither know Him nor His Law save through the glass darkly. But the faint glimpse of the Law is sufficient to fill me with joy, hope, and faith in the future

Sevagram, 5-12-39
Harijan, 9-12-1939

97

SINDH TRAGEDY

I have before me several letters from Sindh and a longish report from Dr. Choithram over the recent riots in Sukkur and Shikarpur. The Sindh Hindus should remember that Sindh has a national government. Though for the sake of brevity I have often used the term Congress government, the proper expression is national democratic government as distinguished from foreign bureaucratic government, which it replaces Whilst in discussing domestic differences and party politics we have to speak of Congress and Muslim League governments, we must for all other purposes think and speak in terms of national government. And so those who feel aggrieved must appeal to their provincial national governments and cultivate public opinion in favour of justice and public tranquillity It would be wrong always to think in communal terms I know that we may not shut our eyes to hard facts. But to attribute everything to the communal spirit is a sign of inferiority complex. It may well perpetuate what is yet a temporary distemper in the national life

But, as I have already suggested, contrasted with irresponsible bureaucracy, national governments would be found to be weak in action because of their responsibility to the people in whose name and by whose goodwill alone they can rule. They can, therefore, deal with crimes with more or less success, but

they will be found to be powerless to deal with popular upheavals which communal riots are. British military aid will not always be at their disposal. National governments will cease to be national, if they have to depend on British military aid. Moreover, if the Congress policy of non-violence becomes universal among all parties, military and even police aid must become taboo. Before the other parties can be expected to become non-violent, Congressmen have to express non-violence in ample measure in their daily conduct Be that, however, as it may, I can only advise the afflicted people of Sindh in terms of non-violence.

The question in Sindh is not really one between Hindus and Muslims. It is essentially one between weak people and strong Muslims fight among themselves as badly as with Hindus. Hindus have also been known to fight among themselves. It will be wrong to weigh ferocity in golden scales.

Hinduism has become a synonym for weakness and Islam for physical strength. Hindus, although they have been taught to believe in *ahimsa*, have not shown *en masse* the strength of *ahimsa*, have never shown its superiority, when matched against physical strength. I have maintained that superiority over physical strength, however overwhelming, is the core of *ahimsa*, and I have further maintained that this non-violence can be exercised as well by individuals as by groups of them, yea, even by millions together. The experiment is still in the making. Sufficient evidence has accumulated during the past twenty years to show that the experiment is worth making. Nothing can possibly be lost by continuing it, provided of course that the non-violence is of the standard brand.

Nothing has come under my observation to show that there was in Sukkur or Shikarpur even one person who believed in and practised non-violence of the strong Had there been one, we would surely have known of him as we know of Ganesh Shankar Vidyarthi. One such person can any day give a better account of himself than one armed to the teeth.

There are many Congressmen in Sukkur and Shikarpur, but they are not non-violently organized. It is not their fault. They know no better. As I have been repeatedly saying nowadays, our non-violence has not been of the strong. Weak people

cannot develop it all of a sudden. But I have no other drug in my chest. I can only prescribe what I have and what has never failed. I can only, therefore, say: 'Try and try again until you succeed' In the composition of the truly brave there should be no malice, no anger, no distrust, no fear of death or physical hurt. Non-violence is certainly not for those who lack these essential qualities. Wherever there are such persons they should be able to cover the weak ones, provided of course that they would listen to their helpers.

Let the weak ones never rely upon armed help. Such help will only make them weaker. If they have not the capacity for non-violent resistance, they should learn the art of defending themselves. It does not require a strong body; it requires a stout heart The African Negroes have become, or were 25 years ago, so terror-stricken that they could not face a White lad — a pigmy compared to the giant-framed Negroes. White children were trained from their infancy not to fear the Negroes. The first lesson, therefore, for those who will learn how to defend themselves is to shed the fear of being hurt or being killed I would like them to observe the laws of the game. Just as there is such a thing as honour among thieves, there should surely be honour between combatants. One hears so often of children and old men being butchered, women being outraged. If men must become beasts, there might even then be some decency observed. Religion is outraged when an outrage is perpetrated in its name. Almost all the riots in this unhappy land take place in the name of religion, though they might have a political motive behind them. My chief point is that the existing situation is intolerable. Cowardice should have no place in the national dictionary.

I have suggested *hijrat*. I repeat the suggestion. It is not unpractical. People do not know its value High and mighty have been known to have resorted to it before now. Planned *hijrat* requires courage and forethought. The second book of the Old Testament is known as Exodus. It is an account of the planned flight of the Israelites In exile they prepared for a military career. In modern times we have the example of the flight of the Doukhobours from Russia owing to persecution. Theirs was no military career. On the contrary they were

non-violent. There is, therefore, nothing wrong, dishonourable or cowardly in self-imposed exile. India is a vast country. Though poor, it is well able to admit of inter-migration especially of those who are capable, hard-working and honest. The people of Sukkur and Shikarpur have all the three qualities. They must appeal to the Government. Only they can give very little help. Apart from political pacts local heads among Hindus and Muslims may meet with mutual profits. It can do no body or party any good to promote mutual slaughter and consequent increase in the existing ill-will But if no honourable local settlement is arrived at, and if the local residents do not feel able to defend themselves and their families and possessions non-violently or violently, I have no doubt that they should vacate the place in which they live in perpetual fear of their lives and honour of their womenfolk.

Sevagram, 1-1-40
Harijan, 6-1-1940

98

SINDH TRAGEDY

Q. In your article "Sindh Tragedy" you have advised the oppressed Hindus of Sindh to perform *hijrat* if they cannot protect their honour and self-respect by remaining in Sindh Where do you expect them to go ? Who will provide them the wherewithal in their place of refuge ? May I further ask you if the remedy of *hijrat* is meant for the Hindus only ? Why do you not advise *hijrat* to the Mussalmans in the Congress provinces who complain so loudly of 'oppression' ? As it is you have given them weightage in provinces in which they are in a minority and a statutory majority in the Punjab where they are numerically superior.

A. My advice to migrate is for all who feel oppressed and cannot live without loss of self-respect in a particular place. If the Muslims where they are in a minority were really oppressed and they sought my advice, I should give them the same advice I have given to the Sindh Hindus But, as a general rule, they

are capable of holding their own even when they are in a minority. I have already told the Sindhis that, if they have the bravery to defend themselves even though they are a handful, they should not leave the places where they are settled. My advice is meant for those who, though they are conscious of self-respect, lack the strength that comes from non-violence or the capacity to return blow for blow

The question what the refugees should do after migration is surely secondary. A few thousand of them can be easily absorbed in a vast country like India. Sindhis are enterprising. They are scattered all over the world. I hardly think any public appeal will be necessary Let them know that there are refugees from Limbdi, who are bravely and silently bearing their exile. A keen sense of honour turns every privation into a joy. But perhaps migration will be unnecessary. I see signs of Muslim leaders realizing their responsibility and making arrangements to create among the Hindus concerned a sense of security. If this happens, it would be as it should be.

The question of my befriending the Ali Brothers is surely irrelevant to the great issue I have raised I am not sorry for anything I have done in connection with communal unity. I should repeat the same thing under similar circumstances. Neither community has lost by the unity of the Khilafat days, temporary though it unfortunately proved to be You are wrong in holding me responsible for the Communal Decision. It has nothing to commend itself to any community except the solid fact that we are living under it and that we have not yet found an agreed formula to replace it.

Sevagram, 29-1-40

Harijan, 3-2-1940

A PACIFIST'S DOUBTS

I give these details in order to give the reader an idea of the development and evolution of pacifism. No wonder then that, whilst the pacifists appreciated the position taken up by India, their difficulty was how, if Gandhiji accepted the Allied cause as just and said that he would pray for the victory of Britain, he had advised the Congress not to offer its co-operation to Britain.

Whether the questioner's difficulty arose from pacifism or from the Congress demand for declaration of the war aims was not quite clear. But Gandhiji explained that any co-operation or non-co-operation that the Congress could offer would be moral and not material. Materially Britain's position would not be affected by either attitude, for the contribution in men and money that Britain exacted from India as a subject nation would continue. When a non-violent India wished Britain victory, it was not victory for Britain, right or wrong, but victory for Britain which in going to war against Germany was less in the wrong and more in the right. But if there was no satisfactory declaration of the war aims regarding India, the moral support that Britain would gain would not be forthcoming. The result would be quite different, if the Congress were indifferent as to means. She would in that case foment a rebellion and offer effective obstruction

"Now it would be mental violence and not physical violence ?" the friend asked.

"Maybe," said Gandhiji. "But the non-co-operation that may come would be non-violent. Mental violence has no potency and injures only the person whose thoughts are violent. It is otherwise with mental non-violence. It has potency which the world does not yet know. And what I want is non-violence of the thought and deed.

"Such non-violent support can be available to Britain only if her cause is demonstrably just, even though from a pacifist standpoint her means are violent and therefore bad. Her cause

will not be just if she is not just to India, i.e., if of her own accord and without reference to the parties in India she does not wash her hands clean of the imperialist exploitation of India. Therefore, non-violent India's prayer will be for Britain's victory when she declares in unequivocal terms that India is a free nation in action now, and in law as soon as possible, may be even during the war This she will do from the heart only when she begins to rely more on the efficacy of the moral support of non-violent India than on her own strong arm. If England can take this step, the war will end in peace through moral force, rather than through superiority of arms."

Sevagram, 15-1-40 M. D.
Harijan, 20-1-1940

100

UNIVERSAL CONSCRIPTION

Q You say millions are unarmed and unused to wielding arms. But what is the difficulty, when India is free, in training the whole nation in the use of arms by means of universal conscription ?

A. Of course in theory there is nothing to prevent the training by universal conscription. Only I think that the people of this land would not take to arms easily even though conscription may be resorted to. In any event arming of the millions, or even a few, is outside my province. It is repugnant to me. But what I would conscript is productive labour, skilled and unskilled This, I hold would be the easiest and the most effective method of organizing society on a peaceful footing.

Sevagram, 28-1-40
Harijan, 3-2-1940

101

THE QUESTION OF DEFENCE

So far as defence is concerned, surely it will be the primary concern of free India to make her own arrangements It may well be that India would want elaborate preparation and would want Britain's help, if it is given, in enabling her to do so. Thanks to imperial policy, unarmed India is left wholly unprotected except by the British bayonet and Indian soldiers whom British power has brought into being. It is a position humiliating alike for Britain and for India I am personally unconcerned because, if I could carry India with me, I would want nothing beyond a police force for protection against dacoits and the like But so far as defence is concerned unarmed peaceful India would rely upon the goodwill of the whole world. But I know that that is only a dream at the present moment *

Harijan, 10-2-1940

102

MORE CALUMNY

Q. You did not hesitate to join the Ali Brothers in their intrigue to invite Amanulla Khan to invade India and set up Muslim Raj. You drafted a wire for Maulana Mahomed Ali advising the then Amir not to enter into a treaty with the British. The late Swami Shraddhanandji is reported to have seen the draft And now you want the Hindus of Sindh to make a present of their hearths and homes to their Mussalman oppressors instead of demanding the re-amalgamation of Sindh with the Bombay province, which alone can restore the reign of law to Sindh Why won't you realize that in this age of enlightenment and progress what the minorities expect is effective protection of their due rights, not mere pious counsels of perfection ?

* From a press interview given at Delhi on 6-2-'40

A: I have several such letters. Hitherto I have ignored them. But now I see that the news has gone through a revised and enlarged edition in the Hindu Mahasabha An angry correspondent threatens that persons like him will begin to believe what has been stated so authoritatively For the sake of my reputation, therefore, I must answer the question. But my correspondents should know that life for me would be a burden, if I were to make it a point of controverting every false report about me or distortion of my writing A reputation that requires such a mud wall of protection is not worth keeping. So far as the charge of my intriguing with the Amir is concerned I can say that there is no truth whatsoever in it Further, I know that the Brothers stoutly denied the charge when it was brought to their notice. And I believed them implicitly. I do not remember having drafted any telegram on behalf of Maulana Mahomed Ali to the then Amir The alleged telegram is harmless in itself and does not warrant the deduction drawn from it The late Swamiji never referred the matter to me for confirmation It is wrong to say anything against dead men unless one has positive proof and stating it is relevant The romance has been woven round my writings in *Young India*.* Deductions drawn from them are wholly unjustified. I would not be guilty of inviting any power to invade India for the purpose of expelling the English. For one thing, it would be contrary to my creed of non-violence. For another, I have too great a respect for English bravery and arms to think that an invasion of India can be successful without a strong combination of different powers. In any case, I have no desire to substitute British rule with any other foreign rule. I want unadulterated Home Rule, however inferior in quality it may be My position remains today what it was when I wrote the *Young India* paragraphs now sought to be used against me. Let me further remind the readers that I do not believe in secret methods

As for Sindh my advice stands Re-incorporation of Sindh in the Bombay province may or may not be a good proposition on other grounds, but certainly it is not for the purpose of greater protection of life and property. Every Indian, be he Hindu or any other, must learn the art of protecting himself. It is the condi-

---

* Chapters 3 to 6 (pp 6-14) in this book

tion of real democracy. The State has a duty. But no State can protect those who will not share with it the duty of protecting themselves.

On the way to Delhi, 4-2-40
Harijan, 10-2-1940

103

MY ADVICE TO NOAKHALI HINDUS

Manoranjan Babu and other friends from Noakhali came to see me during my stay in Malikanda about the difficulties of the Hindus in their locality. Manoranjan Babu has been in correspondence with me in the matter for some time. I have not examined the grievances. I had neither the time nor the wish to do so. That is the special province of the Provincial Congress Committee and finally the central body. But I had no difficulty in giving general advice Their case is more or less like the Sukkur case. There is a great difference in degree But I feel sure that no popularly elected government can successfully cope with wide-spread *goondaism* as it is alleged to be in Noakhali. It is essentially a case of self-defence. Self-respect and honour cannot be protected by others. They are for each individual himself or herself to guard. Governments can at best punish offenders after the offence has been committed. They cannot assure prevention except in so far as punishment acts as a deterrent. Self-defence can be violent or non-violent. I have always advised and insisted on non-violent defence. But I recognize that it has to be learnt like violent defence. It requires a different training from that which is required for violent defence. Therefore, if the capacity for non-violent self-defence is lacking, there need be no hesitation in using violent means. But Manoranjan Babu, being an old congressman, said, "You say I cannot retaliate even in self-defence ?" "That is certainly my view," I replied. "There was, however, a resolution passed by the Gaya Congress that the use of force in self-defence was permissible to congressmen. I have never justified the resolution. Non-violence becomes meaningless, if violence is permitted for self-defence. What is it but self-defence in national resistance

against an aggressor nation ? I would, therefore, advise secession from the Congress, if you contemplate the use of force in defending yourselves in the circumstances described by you."

"But," said Manoranjan Babu, "supposing I adopt the Gaya resolution, would I be accused of communalism, if I defended the aggrieved Hindus ?" "Certainly not," I replied. "In the first place, you do not cease to be Hindu because you are a congressman. You will, however, be guilty of communalism, if you sided with Hindus right or wrong. In the case in point you will defend Hindus, not because they are Hindus but because they are afflicted. I would expect you to defend Muslims if you found them molested by Hindus A congressman recognizes or should recognize no communal distinction."

The interviewers then discussed the Congress dissensions, and told me that many Hindus despairing of Congress aid had joined the Hindu Mahasabha, and asked whether they could do likewise I told them that in theory I could see no objection Whether the local circumstances justified the step or not I could not judge. But if I was a congressman and found that as such I could not act effectively, I should not hesitate to join an organization which could render effective assistance. I added, however, that no responsible congressman could hold office in a Congress organization and yet be a member of the Hindu Mahasabha which is frankly a communal organization. The whole question bristles with difficulties. The occasion demands calmness, truthfulness and boldness Communalism is bound to win, if the Congress cannot become effectively non-violent. It will itself become communal in action, if it plays with non-violence. For the majority of congressmen who are Hindus are bound to drift into violence, if they do not know the effective use of non-violence I am quite clear in my mind that the Congress can remain non-communal only if it becomes truly non-violent in all matters. It cannot be non-violent only towards the rulers and violent towards others That way lie disgrace and disaster.

Calcutta, 26-2-40
Harijan, 2-3-1940

104

WHEN THE BRITISH WITHDRAW

> "Unless you adopt an all-party form of government, you are paving the way towards sowing Hindu-Muslim conflict after the British protection is withdrawn. It was not non-violence but your tremendous magnetism plus the backing of British bayonet that kept the Congress in power. Try non-violence without the latter for two or three months, and the truth of the above will be realized"

Thus writes an esteemed correspondent. I have no difficulty in endorsing the remark that it was the British bayonet that kept the Congress ministries in power. My 'magnetism' may have had something to do with the victory at the polls. But it proved utterly useless to keep the ministries in power. The sustaining force was the British bayonet. This only shows that the people at large have not yet imbibed the lesson of non-violence

The remedy is not an all-party government. Such will be no government of the people for the people. It will be the government of a caucus for its own ends The caucus will have no smoother sailing than the Congress ministries had. It will also have to rely upon the British bayonet There can be no manly peace in the land unless the British bayonet is withdrawn. The risk of riots has to be run. Non-violence will be born out of such risks, if at all it is to be part of national life. It is daily becoming crystal clear that real unity will not come so long as the British bayonet crushes the free spirit of the people The peace it imposes is the peace of the grave I feel that riots will be a welcome relief, if that is the price we have to pay for freedom. For out of them I can conceive the possibility of peace coming, not out of the present unreality. The way out of riots on the one hand and British bayonets on the other is frank acceptance of non-violence. To this my life is dedicated, and my faith in its possibility and efficacy will survive the dissolution of my body.

On the train to Wardha, 3-3-40
Harijan, 9-3-1940

105

SOME QUESTIONS

"May I ask one or two more important questions before leaving?" said Chinese friend. "Do you expect to see India independent?"

"Yes, of course," came the reply in no uncertain terms "I *want* to see India free in my lifetime. But God may not consider me fit enough to see the dream of life fulfilled. Then I shall quarrel, not with Him but with myself"

"But without an army how can you ever succeed?"

"Well, we have done so thus far. We are nearing our goal without having fired a single shot It will be a miracle, if we succeed. But there is nothing to make me doubt the efficacy of the weapon of non-violence. Whether, however, we have the requisite degree of it within us has yet to be proved."

"Is there hatred against the British?"

"Yes — alas — but if we remain non-violent, hatred will die as everything does from disuse"

"It is very hard for us to get rid of hatred against Japan."

"Yes, it will take generations for you as you are using violence against them I do not say that you should not have defended yourselves violently, but under those circumstances hatred cannot die"

"Are the British easier to deal with than any other people?"

"They are as easy, in terms of non-violence, to deal with as anyone else. But not having dealt with anyone else I cannot say from practical experience. All conquerors of India have reacted to what is noble in Indian culture and in Indian nature, the Muslims included. I believe the Germans would have done likewise. It may even be that the English reaction has been less than what others' may have been because of their insularity and colour prejudice"

Sevagram, 7-4-40 A. K.
Harijan, 13-4-1940

106

TWO QUESTIONS FROM AMERICA

A friend writing from America propounds the following two questions :

> "1 Granted that *satyagraha* is capable of winning India's independence, what are the chances of its being accepted as a principle of State policy in a free India? In other words, would a strong and independent India rely on *satyagraha* as a method of self-preservation, or would it lapse back to seeking refuge in the age-old institution of war, however defensive its character ? To restate the question on the basis of a purely theoretic problem : Is *satyagraha* likely to be accepted only in an uphill battle, when the phenomenon of martyrdom is fully effective, or is it also to be the instrument of a sovereign authority which has neither the need nor the scope of behaving on the principle of martyrdom?
>
> 2 Suppose a free India adopts *satyagraha* as an instrument of State policy, how would she defend herself against probable aggression by another sovereign State? To restate the question on the basis of a purely theoretic problem : What would be the *satyagrahic* action-patterns to meet the invading army at the frontier? What kind of resistance can be offered the opponent before a common area of action, such as the one now existing in India between the Indian nationalists and the British government, is established ? Or should the *satyagrahis* withhold their action until after the opponent has taken over the country ? "

The questions are admittedly theoretical. They are also premature for the reason that I have not mastered the whole technique of non-violence The experiment is still in the making. It is not even in its advanced stage. The nature of the experiment requires one to be satisfied with one step at a time. The distant scene is not for him to see. Therefore my answers can only be speculative

In truth, as I have said before, now we are not having unadulterated non-violence even in our struggle to win independence.

As to the first question, I fear that the chances of non-violence being accepted as a principle of State policy are very

slight, so far as I can see at present. If India does not accept non-violence as her policy after winning independence, the second question becomes superfluous.

But I may state my own individual view of the potency of non-violence. I believe that a State can be administered on a non-violent basis, if the vast majority of the people are non-violent. So far as I know, India is the only country which has a possibility of being such a State. I am conducting my experiment in that faith. Supposing, therefore, that India attained independence through pure non-violence, India could retain it too by the same means. A non-violent man or society does not anticipate or provide for attacks from without. On the contrary, such a person or society firmly believes that nobody is going to disturb them. If the worst happens, there are two ways open to non-violence To yield possession, but non-co-operate with the aggressor. Thus supposing that a modern edition of Nero descended upon India, the representatives of the State, will let him in, but tell him that he will get no assistance from the people They will prefer death to submission. The second way would be non-violent resistance by the people who have been trained in the non-violent way. They would offer themselves unarmed as fodder for the aggressor's cannons The underlying belief in either case is that even a Nero is not devoid of a heart The unexpected spectacle of endless rows upon rows of men and women simply dying rather than surrender to the will of an aggressor must ultimately melt him and his soldiery. Practically speaking there will be probably no greater loss in men than if forcible resistance was offered ; there will be no expenditure in armaments and fortifications. The non-violent training received by the people will add inconceivably to their moral height. Such men and women will have shown personal bravery of a type far superior to that shown in armed warfare In each case the bravery consists in dying, not in killing. Lastly, there is no such thing as defeat in non-violent resistance. That such a thing has not happened before is no answer to my speculation I have drawn no impossible picture. History is replete with instances of individual non-violence of the type I have mentioned. There is no warrant for saying or thinking that a group of men and women cannot by sufficient training act non-violently as a group

or nation Indeed the sum total of the experience of mankind is that men somehow or other live on — from which fact I infer that it is the law of love that rules mankind. Had violence, i e hate. ruled us, we should have become extinct long ago. And yet the tragedy of it is that the so-called civilized men and nations conduct themselves as if the basis of society was violence. It gives me ineffable joy to make experiments proving that love is the supreme and only law of life. Much evidence to the contrary cannot shake my faith. Even the mixed non-violence of India has supported it. But if it is not enough to convince an unbeliever, it is enough to incline a friendly critic to view it with favour.

Sevagram, 8-4-40
Harijan, 13-4-1940

107

OF WHAT AVAIL IS NON-VIOLENCE ?

An Indian friend writes :

"Yesterday Reuter in pathetic terms related how the population of Norway under the rain of bombs and machine gun bullets was fleeting from towns completely demoralized and in panic. It is shocking that such good people should so suddenly be reduced to this helpless condition simply because they had neither the will nor the resources to develop the technique of destruction into a fine art. The futility of violence, and also, I fear, its temporary efficacy, is being proved Of what avail was the violent defence of Norway? And yet for the time being the bigger violence of Germany seems to have succeeded! Let us hope eventually everyone will see the futility of violence and a new era may dawn But are we really making a non-violent contribution towards the world problem? Of what avail is our non-violence to Norway, Sweden and Denmark? Virtually, are we not giving a handle to Germany? True, we are doing nothing beyond embarrassing Great Britain, and perhaps we may say that such an embarrassment is inevitable and is not caused deliberately But the fact remains that England is in distress, and by our action we are embarrassing not only England but all other good nations who have been victims of aggression We are not likely, it seems, to succeed in changing England's heart And victims like Norway etc. can never appreciate our attitude In the light of our

present attitude, the international world can with justification misinterpret our past help to victims of aggression like China and Spain. Were they more deserving of our help than the present victims? And if not, then why this distinction? Simply because an imperialist power, even for the sake of its own interest, happens to decide to do something which is noble and moral! You have never regretted your action during the last war when you vigorously recruited people for military purposes. This time your attitude appears to be in sharp contrast, although you say that both attitudes are right"

My correspondent is not alone in bemoaning the lot of most cultured and inoffensive people like the Danes and the Norwegians. This war is showing the futility of violence. Supposing Hitler becomes victorious over the Allies, he will never subjugate England and France. It will mean another war. Supposing the Allies are victorious, the world will fare no better. They will be more polite but not less ruthless, unless they learn the lesson of non-violence during the war and unless they shed the gains they have made through violence. The first condition of non-violence is justice all round in every department of life. Perhaps it is too much to expect of human nature. I do not, however, think so. No one should dogmatize about the capacity of human nature for degradation or exaltation.

Indian non-violence has brought no relief to the cultured Western powers because it is still poor stuff. Why travel so far to see its inefficacy ? We in India are torn asunder in spite of the Congress policy of non-violence. The Congress itself is distrusted. Not until the Congress or a similar group of people represents the non-violence of the strong, will the world catch the infection.

India's aid to Spain and China was merely moral. The material aid was but an insignificant token of it. There is hardly an Indian who does not feel the same sympathy for Norway and Denmark who lost their freedom overnight. Though their case is different from that of Spain and China, their ruin is more complete perhaps than that of Spain and China. Indeed there is a material difference even between China and Spain. But there is no difference so far as sympathy is concerned. Pauper India has nothing to send to these countries except her non-violence. But, as I have said, this is not yet a sendable commodity. It will be, when India has gained her freedom through non-violence.

There then remains Britain's case. The Congress has caused no embarrassment. I have declared already that I shall do nothing to embarrass Great Britain. She will be embarrassed, if there is anarchy in India That the Congress, so long as it is under my discipline, will not support.

What the Congress cannot do is to lend its moral influence to Britain. Moral influence is never mechanically given. It is for Britain to take it. Perhaps British statesmen do not think the Congress has any to lend. Perhaps they think that all they need is material aid in this warring world. If they do, they will not be far wrong Morality is contraband in war. My correspondent has given up the whole of his case for Britain when he says, "We are not likely to succeed in changing Britain's heart" I do not wish ill to Britain I shall grieve if Britain goes down. But the moral influence of the Congress cannot avail Britain unless she washes her hands clean of India. It works under its own unalterable condition.

My friend does not see the difference between my recruiting in Kheda and my attitude now During the last war the moral issue had not been raised. The Congress was not pledged to non-violence. It had not the moral hold on the masses it now enjoys. I was acting on my own in all I did. I had even attended the War Conference. And to be true to my declaration I had been recruiting at the cost of my health. I told the people that, if they wanted arms, military service was the surest way to get them. But if they were non-violent like me, my appeal was not to them. There was no non-violent man among my audiences so far as I know. Their reluctance was based on ill-will towards Britain. This was gradually giving place to an enlightened determination to throw off the foreign yoke.

Things have changed since then In spite of the unanimous support that Britain got during the last war from India, the British attitude was translated into the Rowlatt Act and the like. The Congress accepted non-violent non-co-operation to meet the British menace. There is the memory of the Jallianwala Bagh, the Simon Commission, the Round Table Conferences, the emasculation of Bengal for the sake of the misdeeds of a few. The Congress having accepted non-violence, I do not need to go to the people to give recruits Through the Congress I can give

something infinitely better than a few such recruits. Of that evidently Britain has no need. I am willing but helpless

Sevagram, 30-4-40
Harijan, 4-5-1940

108

DEMOCRACY AND NON-VIOLENCE

Q. Why do you say, "Democracy can only be saved through non-violence ?" (The questioner is an American friend)

A Because democracy, so long as it is sustained by violence cannot provide for or protect the weak. My notion of democracy is that under it the weakest should have the same opportunity as the strongest. That can never happen except through non-violence No country in the world today shows any but patronizing regard for the weak The weakest, you say, go to the wall Take your own case Your land is owned by a few capitalist owners The same is true of South Africa. These large holdings cannot be sustained except by violence, veiled if not open. Western democracy, as it functions today, is diluted Nazism or Fascism. At best it is merely a cloak to hide the Nazi and the Fascist tendencies of imperialism Why is there the war today, if it is not for the satisfaction of the desire to share the spoils? It was not through democratic methods that Britain bagged India. What is the meaning of South African democracy? Its very constitution has been drawn to protect the White man against the coloured man, the natural occupant. Your own history is perhaps blacker still, in spite of what the Northern States did for the abolition of slavery. The way you have treated the Negro presents a discreditable record And it is to save such democracies that the war is being fought. There is something very hypocritical about it. I am thinking just now in terms of non-violence and trying to expose violence in its nakedness.

India is trying to evolve true democracy, i e. without violence Our weapons are those of *satyagraha* expressed through the *charka*, the village industries, primary education through handicrafts, removal of untouchability, communal harmony,

prohibition, and non-violent organization of labour as in Ahmedabad. These mean mass effort and mass education. We have big agencies for conducting these activities They are purely voluntary, and their only sanction is service of the lowliest

This is the permanent part of the non-violent effort From this effort is created the capacity to offer non-violent resistance called non-co-operation and civil disobedience which may culminate in mass refusal to pay rent and taxes. As you know, we have tried non-co-operation and civil disobedience on a fairly large scale and fairly successfully. The experiment has in it promise of a brilliant future As yet our resistance has been that of the weak The aim is to develop the resistance of the strong. Your wars will never ensure safety for democracy India's experiment can and will, if the people come up to the mark or, to put it another way, if God gives me the necessary wisdom and strength to bring the experiment to fruition.

Sevagram, 13-5-40
Harijan, 18-5-1940

109

PANIC

Nowadays one reads about panic in the Press and hears more than one reads One friend writes, "You sitting in lonely Sevagram can have no notion of the talks and whispers going on in the busy cities Panic has seized them"

Panic is the most demoralizing state anyone can be in There never is any cause for panic One must keep heart whatever happens. War is an unmitigated evil But it certainly does one good thing, it drives away fear and brings bravery to the surface. Several million lives must have been already lost between the Allies and the Germans. They have been wasting blood like water. Old men, women both old and young, and children in Britain and France are living in the midst of imminent death. But there is no panic there. If they were seized by panic, it would be an enemy more dreadful than German bullets, bombs and poison gas. Let us learn from these suffering nations of the

West and banish panic from our midst. And in India there is no cause whatsoever for panic. Britain will die hard and heroically even if she has to We may hear of reverses, but we will not hear of demoralization Whatever happens will happen in an orderly manner.

Therefore I would say to those who lend a listening ear to me: "Go on with your work or business in the usual way. Do not withdraw your deposits or make haste to turn your paper into cash. If you are cautious, you will run no new risks. Your metal buried underground or in your treasure chests need not be considered safer than in banks or in paper, if anarchy overtakes us There is risk just now in everything. It is best to be as you are in such a condition. Your steadiness, if it multiplied, will steady the market It will be the best preventive against anarchy. There is undoubtedly fear of *goondaism* in such times. You must be prepared to cope with it yourself *Goondas* flourish only in the midst of timid people. They will have no quarter from people who can defend themselves violently or non-violently. Non-violent defence presupposes recklessness about one's life and property. If it is persisted in, it will in the end be a sure cure for *goondaism* But non-violence cannot be learnt in a day It requires practice You can commence to learn it from now. You must be ready to lose your life or property or both. But that is implied in the art of non-violence If you do not know how to defend yourself either way, the government will not be able to save you in spite of its best effort No government, however powerful it may be, can without the active co-operation of the people If even God only helps those who will help themselves, how much more true it must be of perishable governments! Do not lose nerve and think that tomorrow there will be no government and it will be all anarchy. You can be the government now, and you certainly will be in the contingency you contemplate or you will perish"

Sevagram, 4-6-40
Harijan, 8-6-1940

110

HOW TO COMBAT HITLERISM

Whatever Hitler may ultimately prove to be, we know what Hitlerism has come to mean. It means naked ruthless force reduced to an exact science and worked with scientific precision. In its effect it becomes almost irresistible.

In the early days of *satyagraha* when it was still known as passive resistance, *The Star* of Johannesburg, stirred by the sight of a handful of Indians, wholly unarmed and incapable of organized violence even if they wished it, pitting themselves against an overwhelmingly armed government, had a cartoon in which the latter was depicted as a steam-roller representing irresistible force, and passive resistance was depicted as an elephant unmoved and comfortably planting himself in his seat This was marked immovable force. The cartoonist had a true insight into the duel between the irresistible and the immovable forces. It was then a stalmate The sequel we know. What was depicted and appeared to be irresistible was successfully resisted by the immovable force of *satyagraha* — call it suffering without retaliation

What became true then can be equally true now Hitlerism will never be defeated by counter-Hitlerism. It can only breed superior Hitlerism raised to *n*th degree What is going on before our eyes is a demonstration of the futility of violence as also of Hitlerism.

Let me explain what I mean by failure of Hitlerism. It has robbed the small nations of their liberty. It has compelled France to sue for peace. Probably by the time this is in print Britain will have decided upon her course The fall of France is enough for my argument. I think French statesmen have shown rare courage in bowing to the inevitable and refusing to be party to senseless mutual slaughter There can be no sense in France coming out victorious if the stake is in truth lost. The cause of liberty becomes a mockery, if the price to be paid is wholesale destruction of those who are to enjoy liberty. It then becomes an inglorious satiation of ambition. The bravery

of the French soldier is world-known. But let the world know also the greater bravery of the French statesmen in suing for peace. I have assumed that the French statesmen have taken the step in a perfectly honourable manner as behoves true soldiers. Let me hope that Herr Hitler will impose no humiliating terms but show that, though he can fight without mercy, he can at least conclude peace not without mercy.

But to resume the thread of the argument. What will Hitler do with his victory? Can he digest so much power? Personally he will go as empty-handed as his not very remote predecessor Alexander For the Germans he will have left not the pleasure of owning a mighty empire but the burden of sustaining its crushing weight. For they will not be able to hold all the conquered nations in perpetual subjection. And I doubt if the Germans of future generations will entertain unadulterated pride in the deeds for which Hitlerism will be deemed responsible. They will honour Herr Hitler as a genius, as a brave man, a matchless organizer, and much more But I should hope that the Germans of the future will have learnt the art of discrimination even about their heroes. Anyway I think it will be allowed that all the blood that has been spilled by Hitler has added not a millionth part of an inch to the world's moral stature.

As against this imagine the state of Europe today if the Czechs, the Poles, the Norwegians, the French and the English had all said to Hitler: "You need not make your scientific preparation for destruction We will meet your violence with non-violence. You will, therefore, be able to destroy our non-violent army without tanks, battleships and airships." It may be retorted that the only difference would be that Hitler would have got without fighting what he has gained after a bloody fight. Exactly. The history of Europe would then have been written differently. Possession might (but only might) have been then taken under non-violent resistance, as it has been taken now after perpetration of untold barbarities. Under non-violence only those would have been killed who had trained themselves to be killed, if need be, but without killing anyone and without bearing malice towards anybody. I dare say that in that case Europe would have added several inches to its moral

stature. And in the end I expect it is the moral worth that will count. All else is dross

I have written these lines for the European Powers. But they are meant for ourselves If my argument has gone home, is it not time for us to declare our changeless faith in non-violence of the strong and say we do not seek to defend our liberty with the force of arms, but we will defend it with the force of non-violence ?

Sevagram, 18-6-40
Harijan, 22-6-1940

111

BOTH HAPPY AND UNHAPPY

It was on the 18th instant that I expressed the following hope in *Harijan* .

> "If my argument has gone home, is it not time for us to declare our changeless faith in non-violence of the strong and say we do not seek to defend our liberty with the force of arms but we will defend it with the force of non-violence ? "

On the 21st the Working Committee felt unable to enforce such faith in action when the time for it came For the Committee never before had an occasion to test their faith. At the last meeting they had to lay down a course of action for meeting impending anarchy within and danger of aggression from without.

I pleaded hard with the Committee · "If you have faith in non-violence of the strong, now is the time to act up to it. It does not matter that many parties do not believe in non-violence whether of the strong or of the weak. Probably that is all the greater reason for congressmen to meet the emergency by non-violent action For if all were non-violent, there could be no anarchy and there would be no question of anybody arming for meeting aggression from without It is because congressmen represent a party of non-violence, in the midst of parties who do not believe in it, that it becomes

imperative for congressmen to show that they are well able to act up to their faith."

But the members of the Working Committee felt that congressmen would not be able to act up to it It would be a new experience for them They were never before called upon to deal with such a crisis The attempt made by me to form peace brigades to deal with communal riots and the like had wholly failed. Therefore they could not hope for the action contemplated.

My position was different. With the Congress non-violence was always a policy. It was open to it to reject it if it failed. If it could not bring political and economic independence, it was of no use. For me non-violence is a creed I must act up to it whether I am alone or have companions Since propaganda of non-violence is the mission of my life, I must pursue it in all weathers I felt that now was the time for me to prove my faith before God and man. And so I asked for absolution from the Committee. Hitherto I have been responsible for guiding the general policy of the Congress I could no longer do so when fundamental differences were discovered between them and me. They readily recognized the correctness of my attitude. And they gave me the absolution. Once more they have justified the trust imposed in them. They have been true to themselves They had not the confidence in themselves or those whom they represented, that they could express in their actions the required measure of non-violence And so they made the only choice they could honestly make. It was a tremendous sacrifice they made — the sacrifice of the prestige that the Congress had gained in the world for unadulterated non-violence, and the dissolution of the unwritten and unspoken bond between them and me. But though it is a break in the common practice of a common ideal or policy, there is no break in the friendship of over twenty years' standing.

I am both happy and unhappy over the result. Happy because I have been able to bear the strain of the break and have been given the strength to stand alone. Unhappy because my word seemed to lose the power to carry with me those whom it was my proud privilege to carry all these many years which seem like yesterday. But I know that, if God shows me the

way to demonstrate the efficacy of non-violence of the strong, the break will prove to have been temporary. If there is no way, they will have justified their wisdom in bearing the wrench of letting me go my way alone. If that tragic discovery of my impotence is in store for me, I hope still to retain the faith that has sustained me all these years and to have humility enough to realize that I was not fit enough instrument to carry the torch of non-violence any further.

But this argument and doubt are based upon the assumption that the members of the Working Committee represent the feeling of the vast majority of congressmen. They would wish, and I hope, that the vast majority of congressmen had in them the non-violence of the strong. No one would be more glad than they to discover that they had underrated congressmen's strength. The probability, however, is that there is no majority but only a good minority which represents the non-violence of the strong. It should be remembered that the matter does not lend itself to argument. The members of the Working Committee had all the argument before them. But non-violence, which is a quality of the heart, cannot come by an appeal to the brain. Therefore what is required is a quiet but resolute demonstration of non-violent strength. The opportunity comes to everyone almost daily. There are communal clashes, there are dacoities, there are wordy duels. In all these things those who are truly non-violent can and will demonstrate it. If it is shown in an adequate measure, it will not fail to infect their surroundings. I am quite clear that there is not a single congressman who disbelieves in the efficacy of non-violence out of sheer cussedness. Let the congressmen who believe that the Congress should adhere to non-violence in dealing with internal disorders or external aggression, express it in their daily conduct. Non-violence of the strong cannot be a mere policy. It must be a creed, or a passion, if 'creed' is objected to. A man with a passion expresses it in every little act of his. Therefore he who is possessed by non-violence will express it in the family circle, in his dealings with neighbours, in his business, in Congress meetings, in public meetings, and in his dealings with opponents. It is because it has not expressed itself in this way among congressmen that the members of the Working Com-

mittee rightly concluded that congressmen were not ready for non-violent treatment of internal disorders or external aggression. Embarrassment caused by non-violent action would move established authority to yield to popular will. But such action has obviously no play in the face of disorders. We have to court death without retaliation and with no malice or anger towards those who bring about disorder. It is easy enough to see that non-violence required here is of a wholly different type from what the Congress has known hitherto. But it is the only non-violence that is true and that can save the world from self-destruction. This is a certainty sooner or later, sooner rather than later, if India cannot deliver the message of true non-violence to a world which wants to be saved from the curse of wars and does not know how to find the deliverance

Sevagram, 24-6-40

P.S.—After the foregoing was written and typed, I saw Pandit Jawaharlal's statement. His love for and confidence in me peep out of every sentence referring to me. The foregoing does not need any amendment. It is better for the reader to have both the independent reactions. Good must come out of this separation.

Harijan, 29-6-1940

112

WHAT TO DO ?

Q. The situation in the country is becoming graver daily. Everywhere panic is growing In certain parts armed gangs are already organizing themselves to take advantage of the ensuing anarchy in case the central power should disintegrate or weaken The danger may not be imminent, but it would be folly to ignore its possibility With all the education in non-violence which the country has received during the last twenty years, you will agree that it has not developed the sort of non-violence that can be effective in the face of anarchy and gangsterism The government is taking steps to organize the citizens for self-defence. What is the duty of those who look up to you for lead and guidance ? Should they take part in these government activities ? If they should not, what else are they to do ? Surely they cannot sit with folded hands doing nothing

A. I am unable to say what the Congress will exactly do, in view of the recent statement of the Working Committee If you believe in non-violent treatment of anarchy and the like, naturally you will prepare yourself and your neighbours and those whom you can influence for non-violent defence I quite agree with you that no responsible person can sit idly by in these times. Violent preparation would need long previous training. Non-violent preparation means mental adjustment. Possibility of anarchy there undoubtedly is But if you are non-violent, you will not give way to fear. Do not anticipate anarchy just as you do not anticipate death though you know that it is a certainty If you are non-violent, you will believe that there will be no anarchy. But if unfortunately it comes, you and your companions or followers will give your lives to prevent it. Those who give their lives in trying to kill those whom they regard as robbers or mischief-makers, do no better, possibly they do worse. They risk their lives, and there is darkness after they are gone. What is more, they may leave things worse by feeding the fire of violence by counter-violence Those who die unresistingly are likely to still the fury of violence by their

wholly innocent sacrifice. But this truly non-violent action is not possible unless it springs from a heart belief that he whom you fear and regard as a robber, dacoit, or worse, and you are one, and that therefore, it is better that you die at his hands than that he, your ignorant brother, should die at yours.

Sevagram, 24-6-40
Harijan, 29-6-1940

113

A VITAL QUESTION

The next question for consideration is that of providing for internal disorder and external invasion. The raising of private armies will be worse than useless It will never be allowed. No power, whether foreign or Swadeshi, can tolerate private armies Those, therefore, who believe in the necessity of India having armed forces will be driven sooner or later to enlist themselves under the British banner It is the logical outcome of the belief. The Working Committee have taken the decision on the point If it is to abide, I have little doubt that they will presently have to advise congressmen to enlist in the usual way. That would mean an end to the slogan of immediate independence and an end also to non-violence of the true brand I shall hope to the last that, for the sake of themselves, India, aye Britain herself, and humanity, congressmen will resolutely decline to have anything to do with the use of arms for any of the two purposes. I feel strongly that the future of humanity lies in the hands of the Congress. May God give wisdom and courage to congressmen to take the right step *

New Delhi, 1-7-40
Harijan, 6-7-1940

---

* From an article entitled 'Some Vital Questions'.

114

TO EVERY BRITON

In 1896 I addressed an appeal to every Briton in South Africa on behalf of my countrymen who had gone there as labourers or traders and their assistants. It had its effect. However important it was from my viewpoint, the cause which I pleaded then was insignificant compared with the cause which prompts this appeal. I appeal to every Briton, wherever he may be now, to accept the method of non-violence instead of that of war, for the adjustment of relations between nations and other matters. Your statesmen have declared that this is a war on behalf of democracy. There are many other reasons given in justification. You know them all by heart. I suggest that, at the end of the war, whichever way it ends, there will be no democracy left to represent democracy. This war has descended upon mankind as a curse and a warning. It is a curse inasmuch as it is brutalizing man on a scale hitherto unknown. All distinctions between combatants and non-combatants have been abolished. No one and nothing is to be spared. Lying has been reduced to an art. Britain was to defend small nationalities. One by one they have vanished, at least for the time being. It is also a warning. It is a warning that, if nobody reads the writing on the wall, man will be reduced to the state of the beast, whom he is shaming by his manners. I read the writing when the hostilities broke out. But I had not the courage to say the word. God has given me the courage to say it before it is too late.

I appeal for cessation of hostilities, not because you are too exhausted to fight, but because war is bad in essence. You want to kill Nazism. You will never kill it by its indifferent adoption. Your soldiers are doing the same work of destruction as the Germans. The only difference is that perhaps yours are not as thorough as the Germans. If that be so, yours will soon acquire the same thoroughness as theirs, if not much greater. On no other condition can you win the war. In other words, you will have to be more ruthless than the Nazis. No cause,

however just, can warrant the indiscriminate slaughter that is going on minute by minute I suggest that a cause that demands the inhumanities that are being perpetrated today cannot be called just.

I do not want Britain to be defeated, nor do I want her to be victorious in a trial of brute strength, whether expressed through the muscle or the brain. Your muscular bravery is an established fact. Need you demonstrate that your brain is also as unrivalled in destructive power as your muscle? I hope you do not wish to enter into such an undignified competition with the Nazis I venture to present you with a nobler and a braver way, worthy of the bravest soldier I want you to fight Nazism without arms, or, if I am to retain the military terminology, with non-violent arms. I would like you to lay down the arms you have as being useless for saving you or humanity. You will invite Herr Hitler and Signor Mussolini to take what they want of the countries you call your possessions Let them take possession of your beautiful island, with your many beautiful buildings. You will give all these but neither your souls, nor your minds. If these gentlemen choose to occupy your homes, you will vacate them. If they do not give you free passage out, you will allow yourselves man, woman and child, to be slaughtered, but you will refuse to owe allegiance to them

This process or method, which I have called non-violent non-co-operation, is not without considerable success in its use in India. Your representatives in India may deny my claim. If they do, I shall feel sorry for them They may tell you that our non-co-operation was not wholly non-violent, that it was born of hatred If they give that testimony, I will not deny it. Had it been wholly non-violent, if all the non-co-operators had been filled with goodwill towards you, I make bold to say that you who are India's masters would have become her pupils and, with much greater skill than we have, perfected this matchless weapon and met the German and Italian friends' menace with it. Indeed the history of Europe during the past few months would then have been written differently. Europe would have been spared seas of innocent blood, the rape of so many small nations, and the orgy of hatred.

This is no appeal made by a man who does not know his business. I have been practising with scientific precision non-

violence and its possibilities for an unbroken period of over fifty years. I have applied it in every walk of life, domestic, institutional, economic and political I know of no single case in which it has failed. Where it has seemed sometimes to have failed, I have ascribed it to my imperfections. I claim no perfection for myself. But I do claim to be a passionate seeker after Truth, which is but another name for God. In the course of that search the discovery of non-violence came to me. Its spread is my life mission. I have no interest in living except for the prosecution of that mission

I claim to have been a lifelong and wholly disinterested friend of the British people. At one time I used to be also a lover of your empire. I thought that it was doing good to India When I saw that in the nature of things it could do no good, I used, and am still using, the non-violent method to fight imperialism Whatever the ultimate fate of my country, my love for you remains, and will remain, undiminished. My non-violence demands universal love, and you are not a small part of it It is that love which has prompted my appeal to you

May God give power to every word of mine In His name I began to write this, and in His name I close it. May your statesmen have the wisdom and courage to respond to my appeal I am telling His Excellency the Viceroy that my services are at the disposal of His Majesty's Government, should they consider them of any practical use in advancing the object of my appeal

New Delhi, 2-7-40
Harijan, 6-7-1940

"A CRY IN THE WILDERNESS"?

ऊर्ध्वबाहुर्विरौम्येष न च कश्चिच्छृणोति मे।
धर्मादर्थश्च कामश्च स धर्मः किं न सेव्यते? ॥

"With hands upraised I cry.
(But none listens to me)
Dharma[1] yields both Artha[2] and Kama[3],
Why is that Dharma not observed?"

Bapuji Aney on his way back from Simla paid a flying visit to me at Delhi on Saturday. Whether we work together or seem to be working in opposite directions, his love for me endures, and so he never misses an opportunity to look in wherever I may be. He expresses himself freely before me, and often shares with me a verse or two from his inexhaustible store. During his Delhi visit he sympathized with me for my having had to sever my connection with the Congress, but he really congratulated me. "They should, I think, leave you in peace," he said, "and let you go your way. I read your appeal to every Briton. It will fall on deaf ears But that does not matter to you You cannot help telling them what you feel to be their *Dharma* (Duty) But it is not strange that they will not listen to you — seeing that the Congress itself did not listen to you at the critical moment When even Sage Vyasa failed to make himself heard, how should others fare better? He had to conclude his great epic — Mahabharata — with a verse which reveals the cry of his soul" With this he cited the verse I have quoted at the head of this article. He thereby strengthened my faith, and also showed how difficult was the way I had chosen

And yet it has never seemed to me so difficult as it is imagined to be Though the Sardar's way and mine seem to diverge today, it does not mean that our hearts also diverge. It was in my power to stop him from seceding from me. But it did not seem to be proper to do so And it would have been morally wrong to strive with Rajaji in what he firmly regarded

1 Duty   2 Wealth   3 Desire, aspiration

as his clear duty. Instead, therefore, of dissuading Rajaji I encouraged him to follow his course. It was my clear duty to do so If I have the power to carry my experiment of *ahimsa* to success in an apparently new field, if my faith endures, and if I am right in thinking that the masses are fundamentally non-violent, Rajaji and the Sardar will again be with me as before

What are these apparently new fields for the operation of non-violence? Those who have followed the Working Committee's resolutions and writings in *Harijan* are now familiar with these. Non-violence in its operation against constituted authority is one field. We have exercised this up to now with a fair amount of success, and I have always described it as the non-violence of the weak. This non-violence may be said to have come to stay with congressmen.

The other field is the exercise of *ahimsa* in internal disturbances — Hindu-Muslim riots and the like We have not been able to show visible success in the exercise of *ahimsa* in this field What then should the congressmen do when internal chaos is so imminent? Will they return blow for blow, or will they cheerfully bend their heads to receive violent blows? The answer to this is not so easy as we might think Instead of going into the intricacies, I should say that congressmen should try to save the situation by laying down their lives, not by taking any He who meets death without striking a blow fulfils his duty cent per cent. The result is in God's hands

But it is clear that this non-violence is not the non-violence of the weak. It does not give one the joy of jail-going. One can have that joy and also cover thereby the ill-will one harbours in his breast against the government. One can also non-co-operate with the government. But where swords, knives, *lathis* and stones are freely used, what is a man to do single-handed? Is it possible for one to receive these deadly blows with ill-will in one's heart? It is clear that it is impossible to do so, unless one is saturated with charity. It is only he who feels one with his opponent that can receive his blows as though they were so many flowers Even one such man, if God favours him, can do the work of a thousand. It requires soul force — moral courage — of the highest type.

The man or woman who can display this non-violence of the brave can easily stand against external invasion. This is

the third field for the exercise of non-violence. The Congress Working Committee were of opinion that, while it might be possible for us to exercise *ahimsa* in internal disturbances, India has not the strength to exercise *ahimsa* against the invasion of a foreign foe This their want of faith has distressed me. I do not believe that the unarmed millions of India cannot exercise *ahimsa* with success in this wide field. It is for congressmen to reassure the Sardar, whose faith in *ahimsa* of the strong has for the moment been shaken, that *ahimsa* is the only weapon that can suit India in the fields mentioned Let no one ask, "But what about the martial races in India?" For me that is all the more reason why congressmen should train themselves to defend their country with a non-violent army. This is an entirely new experiment. But who, save the Congress, is to try it — the Congress which has tried it successfully in one field? It is my unshakable faith that, if we have a sufficient number of non-violent soldiers, we are sure to succeed even in this new field, apart from the saving of the needless waste of crores of rupees.

I am therefore hoping that all Gujarati Congressites — men and women — will declare their adherence to *ahimsa* and reassure the Sardar that they will never resort to violence Even if there is sure hope of success in the exercise of violence, they will not prefer it to the exercise of non-violence. We are sure to learn by our mistakes. "We fall to rise, are baffled to fight better, sleep to wake"

On the train to Wardha, 7-7-40
Harijan, 13-7-1940

THE DELHI RESOLUTION

I have just seen the news that the Working Committee's fateful resolution has been released to the Press. It was passed in my presence, but I wanted to say nothing before it was actually released for publication It will be a profound mistake to suppose that the members passed the five solid days in wrangling. They had to discharge a heavy responsibility. Though logically the resolution is no departure from the Ramgarh resolution, it is an undoubted departure from its spirit. The letter often remains constant, the spirit changes Up to now, for one reason or another, the Congress policy was : no participation in the war except for the moral influence that the Congress can exercise, if the vital demand was satisfied of Britain's own free will Such was not the position of all the members of the Working Committee. Hence at the critical moment every member had to make up his or her mind independently of the rest. The five days were days of tremendous heart-searching I had produced a draft resolution which almost all thought was the best, if they could bring to bear a living belief in non-violence through and through, or if they could truthfully say that such was the belief of their constituencies Some had neither, and some had individually the required faith. Only Khan Saheb was clear so far as his own faith and that of his beloved Khudai Khidmatgars were concerned. So he had decided even after the last Wardha resolution that he had no place in the Congress He had a special mission and a special duty to his followers. So the Working Committee gladly permitted him to retire from the Congress By retiring he serves the Congress all the more, even as I hope to do Who knows that those of us, who retire, may not be able to give our comrades the faith they seem, for the moment, to have lost ?

Rajaji was the framer of the resolution He was as certain of his position as I was of mine. His persistency, courage and utter humility brought him converts. Sardar Patel was his greatest prize. He would not have even thought of bringing

up his resolution, if I had chosen to prevent him But I give my comrades the same credit for earnestness and self-confidence that I claim for myself I had long known that we were drifting away from each other in our outlook upon the political problems that face us He will not allow me to say that his is a departure from *ahimsa*. He claims that his very *ahimsa* has led him to the point which culminated in his resolution He thinks that I suffer from obsession owing to too much brooding on *ahimsa*. He almost thinks that my vision is blurred. It was no use my returning the compliment, though half joking I did I have no proof, save my faith, to question his counter faith. That is evidently absurd. I could not carry the Committee with me at Wardha and so I got my absolution. I at once saw as clear as daylight that, if my position was not acceptable, Rajaji's was the only real alternative I therefore encouraged him to persist in his effort, though all the while I held him to be hopelessly in the wrong. And by exemplary patience, skill and considerateness towards his opponents he got a good majority, five remaining neutral. I had a fearful moment. Generally such resolutions are not carried by the majority vote. But at this juncture unanimity was not to be expected. I advised that Rajaji's resolution should be enforced And so at the last moment the Committee decided that the resolution should go forth to the world.

It was necessary for the public to have this background to the tremendous step the Committee have taken for good or ill Those Congressmen who have a living faith in the non-violence of the strong will naturally abstain. For the moment, however, what they can do is wholly irrelevant. Rajaji's resolution represents the considered policy of the Congress. Non-congressmen, who were eager for the Congress to be free of my religious bias to adopt a purely political attitude, should welcome the resolution and support it wholeheartedly So should the Muslim League, and even the Princes who think of India more than their principalities.

The British government have to make their choice Independence they cannot withhold unless their wisdom is as much blurred as Rajaji claims that mine is. If independence is recognized, the acceptance of the other part of the resolution follows as a matter of course. The question is : do they want to impress

help from India by virtue of their rulership over India, or will they have help that a free and independent India can give? My individual advice has already gone. My help always has been promised Acceptance of that advice can but enhance their heroism. But if they cannot accept it, I advise, as a disinterested but staunch friend, that the British government should not reject the hand of friendship offered by the Congress.

Sevagram, 8-7-40
Harijan, 13-7-1940

117

BEFORE THE GANDHI SEVA SANGH

If the Rank and File Believe

But if the Working Committee were wrong in their assumption that the rank and file have lost their belief in non-violence and that they cannot come up to the scratch in times of crisis, the rank and file have to reassure the Working Committee. That is what Gandhiji said, with special reference to the Gujaratis, in his article last week. He explained the whole thing at much greater length to the joint meeting of the Gandhi Seva Sangh and the Charkha Sangh held soon after the Wardha meeting of the Working Committee. "Now is the testing time for you," he said. "The Working Committee, let us say, were weighed and found wanting. Can the Gandhi Seva Sangh do anything to repair their failure? The Working Committee's resolution does not mean that you cannot appeal to people to declare their faith in non-violence. You can do so, and then tell the members of the Working Committee, 'You undervalued our faith. We are going to hold by the creed.' I tell you the Working Committee members will not only not resent this, but will simply dance with joy Some of you are members of the Congress. It is the duty of those who are members and who believe in non-violence to reassure the members of the Working Committee, to declare their faith before the meeting of the A. I C. C., and even before the open Congress, if the time comes.

But you may not separate your creed as congressmen and creed as ordinary human beings, your behaviour in Congress affairs and in non-Congress affairs Your non-violence, if it is true, must be part of your normal life, must be in your thought, word and deed, and must colour all your behaviour. Then and then only can you give the Working Committee the assurance I have referred to, and compel them to alter their resolution."

The Test of Genuineness

"But," he added, "let me explain this further You will examine every action of yours in the light of the creed. That does not mean that you will be morbid or pernickety. Your conduct will have to be natural. When I began observing silence it meant an effort on my part. Now it has become part of my nature, and to break my silence means an effort. In the same way acting non-violently must be part of your nature. It is likely that your relating everything to non-violence may be logically wrong, but it is not wrong for you. I may be wrong, in the eyes of others, in my belief that with every thread that I draw I am bringing Swaraj nearer, but for me the belief is as true as the fact that I exist That saves me from losing my sanity This spinning wheel is a symbol of non-violence for me. The wheel as such is lifeless, but when I invest it with symbolism it becomes a living thing for me Its sound, if it is musical, is in tune with non-violence If it is unmusical, it is not in tune with it, for it indicates carelessness on my part The steel spindle one can use as a deadly weapon, but we have put it there for the best possible use So we have to be meticulously careful about every part of the wheel Then and then only will it produce fine music and spinning will be a true sacrificial act.

"But this kind of *sadhana*, you will say, may take thousands of years It may take some a thousand years, and it may take some others only one year. Don't think that, if in spite of my 50 years' practice of it I am still imperfect, it must take you many more years. No, there is no rule of three here You may succeed quicker than I. I meant what I said to Prithwisingh: 'You had at any rate the violence of the brave I had nothing of it. Now if you believe in cultivating non-violence of the brave, you will do so much more quickly than I, and you will leave me behind' This applies to every one of you. In South

Africa I was the first to learn shoemaking, and so I taught it to others. But those others soon left me behind It was because I was a true teacher. Now if I am a true teacher of *ahimsa*, I am sure you will soon leave behind your teacher If that does not happen, it will only mean that I was an unfit teacher. But if my teaching fructifies, there will be teachers of *ahimsa* in every home

"I want to know how many of you are with me If none goes with me, I am ready to tread my path alone For I know that I can never be *alone* as God is there with me You are all companion *sadhaks* (seekers) with me. I am old, but you have many years before you. And yet let me tell you that I do not feel the weight of my years I do not think my power of growth or capacity for research has come to an end

"So you have to go forth and find out how many actual believers in *ahimsa* there are among congressmen The Working Committee members are your representatives If they were mistaken in assessing the faith of their electors, you have to correct their judgment My position was different from theirs I consider myself a confirmed representative of *ahimsa* and so I severed my connection with the Congress in 1934 I could not help doing it If I had not done so, I should have been untrue to my creed.

"No one knows my imperfections better than I, but what little power I possess is derived from my *ahimsa*. What is it but my *ahimsa* that draws thousands of women to me in fearless confidence? But neither you nor I can trade on our capital. We have to be up and doing every moment of our lives, and go forward in our *sadhana*. We have to live and move and have our being in *ahimsa* even as Hitler does in *himsa* It is the faith and perseverance and single-mindedness with which he has perfected his weapons of destruction that commands my admiration. That he uses them as a monster is immaterial for our purpose - We have to bring to bear the same single-mindedness and perseverance in evolving our *ahimsa*. Hitler is awake all the 24 hours of the day in perfecting his *sadhana*. He wins because he pays the price His inventions surprise his enemies. But it is his single-minded devotion to his purpose that should be the object of our admiration and emulation Although he works all his waking hours, his intellect is unclouded and unerring Are our intellects unclouded and unerring? A mere belief in *ahimsa*

or the charkha will not do It should be intelligent and creative. If intellect plays a large part in the field of violence, I hold that it plays a larger part in the field of non-violence"

With this he referred to the work of Richard Gregg in this direction, and described how the latter had come to the conclusion that spinning as a symbol of non-violence is good enough not only for India but for the world.

Continuing he said. "The Working Committee's decision was simply an echo of the atmosphere around them My decision could not be its echo For *ahimsa* is my special *sadhana*, not that of the Congress I congratulate the members on their honesty and their courage, though I am sorry for myself that I could not inspire them with confidence in our creed and in my leadership We have now to show that we have faith in the non-violence of the brave. It does not mean the development of the capacity to go to jail. It means increasing faith in the potency of constructive work to bring about Swaraj, and in constructive work being a vital part of the programme of *ahimsa*."

Why an Imperfect Man Chosen?

To those who have been saying that, if Gandhiji has failed in perfecting his *ahimsa*, failure is certain in their case, he had already given the answer But in the concluding part of the speech he presented another aspect of his imperfection. He said. "In placing civil disobedience before constructive work I was wrong, and I did not profit by the Himalayan blunder that I had committed. I feared that I should estrange my co-workers, and so carried on with imperfect *ahimsa* But I am not sorry for my blunders. My imperfections and failures are as much a blessing from God as my successes and my talents, and I lay them both at His feet Why should He have chosen me, an imperfect instrument, for such a mighty experiment? I think He deliberately did so He had to serve the poor dumb ignorant millions A perfect man might have been their despair. When they found that one with their failings was marching on towards *ahimsa*, they too had confidence in their own capacity. We should not have recognized a perfect man if he had come as our leader, and we might have driven him to a cave Maybe he who follows me will be more perfect and you will be able to receive

his message. Maybe some one of you may be that perfect teacher who is to come."

An Impossible Ideal ?

But are we not being driven to philosophical anarchism ? Is that not an impossible ideal ? These questions were asked by a philosophic friend some months ago, and Gandhiji gave him replies which I think will be useful today.

"Does anyone know true non-violence ?" he asked

Gandhiji immediately replied "Nobody knows it, for nobody can practise perfect non-violence."

"Then how can it be used in politics ?"

"It can be used in politics precisely as it can be used in the domestic sphere. We may not be perfect in our use of it, but we definitely discard the use of violence, and grow from failure to success."

"You would govern non-violently. But all legislation is violence"

"No, not all legislation Legislation imposed by people upon themselves is non-violence to the extent it is possible in society A society organized and run on the basis of complete non-violence would be the purest anarchy"

"Do you think it is a realizable ideal ?"

"Yes It is realizable to the extent non-violence is realizable That State is perfect and non-violent where the people are governed the least. The nearest approach to purest anarchy would be a democracy based on non-violence. The European democracies are to my mind a negation of democracy"

"Do you think that non-violence of the democracy which you visualize was ever realized in the olden times ?"

"I do not know. But if it was not, it only means that we had never made the attempt to realize the highest in us I have no doubt in my mind that at some stage we were wiser, and that we have to grow wiser than we are today in order to find what beauties are hidden in human nature. Perfect non-violence is impossible so long as we exist physically, for we would want some space at least to occupy Perfect non-violence whilst you are inhabiting the body is only a theory like Euclid's point or straight line, but we have to endeavour every moment of our lives."

Sevagram, 15-7-40 M. D.

Harijan, 21-7-1940

118

THE CORRESPONDENCE

The following correspondence between Gandhiji and His Excellency the Viceroy is released for publication .

Gandhiji's letter to Lord Linlithgow dated Delhi, July 3

" You must have seen in the Press my public appeal to every Briton Nevertheless, for the sake of courtesy, I enclose herewith a copy. You will note the last sentence of my appeal Will you please convey the contents of it to the proper quarters ? The appeal and the offer at the end represent my personal and humble contribution to Britain's cause. It is the best I could give "

Lord Linlithgow's letter to Gandhiji dated Simla, July 10 .

" I duly conveyed your letter of July 3 and your appeal to His Majesty's Government I have now heard from them that with every appreciation of your motives they do not feel that the policy which you advocate is one which it is possible for them to consider, since in common with the whole empire they are firmly resolved to prosecute the war to a victorious conclusion."

Harijan, 21-7-1940

119
UNREPENTANT

By writing that 'Appeal to Every Briton' I have invited upon my head an extra load of work which without God's help I would be ill able to bear If it is His will that I should shoulder it, He will give me the strength to carry on

When I decided to confine myself mostly to writing in Gujarati or Hindustani, I had no notion that I would have to write the appeal It came to me like a flash, and the courage to write it came with it I had resisted till then all pressure from English and American friends to give guidance. But I could not see my way. Now having addressed that appeal, I must follow up the reactions to it. A large amount of correspondence is pouring in upon me. Save for one angry telegram, I had nothing but friendly criticism from Englishmen and even appreciation from some

I was grateful to H E. the Viceroy for forwarding my offer to His Majesty's Government The correspondence with regard to it the readers have already seen or will see in this issue Though no better response to the appeal was to be expected, I cannot help saying that it was the knowledge of the determination to carry the war to a victorious end that had prompted my appeal. No doubt the determination is natural and worthy of the best British tradition. Nevertheless the awful slaughter that the determination involves, should induce a search for a better and braver way to achieve the end For peace has its victories more glorious than those of war The non-violent method would have meant no abject surrender It would have confounded all modern tactics of war, indeed rendered them of no use The new world order, which all dream of, would surely have been found. I hold a new order to be impossible, if the war is fought to a finish or mutual exhaustion leads to a patched-up peace

Let me, therefore, examine the argument advanced in a letter received from a friend Here it is .

> "Two English friends, who admire you, say your appeal to every Briton cannot have any effect *just now* It is impossible

> to expect the man in the street to do a complete *volte face* with any degree of understanding—indeed it is impossible for the understanding to do, as you say, without a *heart*-belief in non-violence The time to mould a new world on your lines will be *after* the war They realize your way is the right one, but they say it needs endless preparation and instruction and *big* leadership — none of which they possess Regarding India they say the attitude of the present authority is deplorable Long ago India should have been declared as independent as Canada, and her people should be allowed to work out their own constitution. But what they are extremely perplexed about now is that you want absolute Independence straightaway, and the next step you will take is 'no further help to Britain in the prosecution of war, surrender to Germany, and opposition to her by non-violent means' You must explain what you mean in more detail so as to remove this misunderstanding This is an honest reaction"

The appeal was intended to produce the effect now. It could not come out of the mathematical calculation If the conviction could have come, action was an easy matter The mass mind responds under pressure That the appeal has not produced the intended result shows that either my word has no power or that God has a purpose of which we have no knowledge The appeal has come from an anguished heart I could not suppress it It was not written for the moment I am quite sure that it enunciates a truth of eternal value

If the ground is not prepared from now, there may be no time left after a dismal termination of the war for evolving a new order. Whatever the order, it will be in response to a conscious or unconscious effort from now. Indeed the effort began before my appeal. I hope that it has stimulated it, perhaps given it a definite direction I suggest to the non-official leaders and moulders of British opinion, if they are convinced of the truth of my position, to work for its adoption Compared to the big issue raised in my appeal, the question of Indian independence pales into insignificance But I hold with the two Englishmen that the British Government's attitude is deplorable. The two friends are wholly wrong in the deduction they have drawn from the assumed recognition of India's independence. They forget that I am out of the picture Those who are responsible for the Working Committee's last resolution have meant

free India's co-operation with Britain. With them, there is no question of surrender to Germany or non-violent opposition

But I must not here tarry on Indian independence and its implications, tempting though the subject is.

The cuttings and correspondence before me say that the Congress rejection of my advice to abstain from preparation for military defence of India precludes me from making the appeal to Britain or from expecting a favourable response The argument is plausible, but only plausible The critics say that, if I have failed with my people, I have no right to expect Britain whilst she is in the midst of a life and death struggle to listen to me I am a man with a mission India's millions have never tasted the bitters of war as the British have Britain, if she is to fulfil her declared purpose, needs a radical change in her policy I feel that I know the change that is needed My inability to persuade the Working Committee is irrelevant to the theme under discussion There is no analogy between India's case and Britain's I am, therefore, wholly unrepentant I maintain that in issuing my appeal I have acted wholly as a lifelong friend of Britain

A writer, however, retorts "Address your appeal to Hitler" In the first place, I did write to Herr Hitler My letter was published in the Press some time after I addressed it In the second place, there can be no meaning in my appeal to Herr Hitler to adopt non-violence He is marching from victory to victory I can only appeal to him to desist. That I have done. But to Britain, which is just now on the defensive, I can present the really effective weapon of non-violent non-co-operation Let my method be rejected on merits, not by bringing inapt analogies or untenable argument. The issue raised by me, I venture to think, is of universal importance The usefulness of non-violent method seems to be granted by all the critics. They gratuitously assume the impossibility of human nature, as it is constituted, responding to the strain involved in non-violent preparation But that is begging the question I say, "You have never tried the method on any scale In so far as it has been tried, it has shown promising results"

Sevagram, 17-7-40
Harijan, 21-7-1940

120

KHANSAHEB'S AHIMSA

In the storm that shook most of the members of the Working Committee, Khansaheb Abdul Gaffar Khan stood firm as a rock. He had never any doubt about his position, and his statement, which I reproduce below, should serve as a beacon light to all of us:

> "Some recent resolutions of the Congress Working Committee indicate that they are restricting the use of non-violence to the fight for India's freedom against constituted authority. How far and in what manner this will have to be applied in the future I cannot say The near future will perhaps throw light on this Meanwhile it is difficult for me to continue in the Congress Working Committee, and I am resigning from it I should like to make it clear that the non-violence I have believed in and preached to my brethren of the Khudai Khidmatgars is much wider It affects all our life, and only this has permanent value Unless we learn this lesson of non-violence fully, we shall never do away with the deadly feuds which have been the curse of the people of the Frontier Since we took to non-violence and the Khudai Khidmatgars pledged themselves to it, we have largely succeeded in ending these feuds Non-violence has added greatly to the courage of the Pathans Because they were previously addicted to violence far more than others, they have profited by non-violence much more. We shall never really and effectively defend ourselves except through non-violence Khudai Khidmatgars must, therefore, be what our name implies — pure servants of God and humanity — by laying down our own lives and never taking any life."

It is worthy of the Khansaheb and all that he has stood for during the past twenty years. He is a Pathan, and a Pathan may be said to be born with a rifle or sword in his hand. But the Khansaheb deliberately asked his Khudai Khidmatgars to shed all weapons when he asked them to join the *satyagraha* against the Rowlatt Act He saw that this deliberate giving up of the weapons of violence had a magical effect. It was the only remedy for the blood-feuds which were handed down from sire to son and which had become part of the normal life of a Pathan.

They had decimated numerous families, and non-violence seemed to the Khansaheb to have come as a longed-for salvation. The violent blood-feuds would otherwise have no end and would spell the end of the Pathans He saw as clear as daylight that, if he could persuade his people not to retaliate, the suicidal feuds would cease and the Pathans would be able to give a better account of their bravery. They took up his message, and put into practice what with them became non-violence of the brave.

Being so clear about his own faith and that of the Khudai Khidmatgars, there was for him no escape from resignation of his membership of the Congress Working Committee His continuing on it would have been anomalous and might have meant an end of his life's work He could not ask his people to join as recruits in the army and at the same time forget the law of tribal retaliation. The simple Pathan would have argued with him — and the argument would have been irresistible — that the present war was a war of retaliation and revenge, and that there was no difference between it and their blood-feuds

I do not know how far the Khansaheb has succeeded in carrying his message to his people This I know that with him non-violence is a matter not of intellectual conviction but of intuitive faith. Nothing can, therefore, shake it About his followers he cannot say how far they will adhere to it But that does not worry him He has to do his duty which he owes to them The result he leaves to God. He derives his *ahimsa* from the Holy Quran He is a devout Musalman. During his stay with me for over a year I never saw him miss his *namaz* (prayers) or his *Ramzan* fast except when he was ill But his devotion to Islam does not mean disrespect for other faiths He has read the *Gita*. His reading is slight but selective, and he immediately assimilates what appeals to him He lothes long argument and does not take long to make up his mind If he succeeds in his mission, it would mean the solution of many another problem But the result, no one can predict. "The lot is cast into the lap, but the whole disposing thereof is of the Lord"

Sevagram, 16-7-40
Harijan, 21-7-1940

121

THE BEST FIELD FOR AHIMSA

Last week I wrote about three fields for the operation of *ahimsa* I propose to invite attention today to the fourth and the best field for the operation of non-violence This is the family field, in a wider sense than the ordinary. Thus members of an institution should be regarded as a family. Non-violence as between the members of such families should be easy to practise. If that fails, it means that we have not developed the capacity for pure non-violence. For, the love we have to practise towards our relatives or colleagues in our family or institution, we have to practise towards our foes, dacoits, etc. If we fail in one case, success in the other is a chimera.

We have generally assumed that, though it may not be possible to exercise non-violence in the domestic field, it is possible to do so in the political field This has proved a pure delusion. We have chosen to describe our methods adopted so far as non-violence, and thus caricatured non-violence itself If non-violence it was, it was much poor stuff that it proved useless at the critical moment The alphabet of *ahimsa* is best learnt in the domestic school, and I can say from experience that, if we secure success there, we are sure to do so everywhere else For a non-violent person the whole world is one family He will thus fear none, nor will others fear him

It will be retorted that those who satisfy such a test of non-violence will be few and far between It is quite likely, but that is no reply to my proposition. Those who profess to believe in non-violence should know the implications of that belief And if these scare them away, they are welcome to give up the belief Now that the Congress Working Committee has made the position clear, it is necessary that those who claim to believe in non-violence should know what is expected of them If, as a result, the ranks of the non-violent army thin down, it should not matter. An army, however small, of truly non-violent soldiers is likely some day to multiply itself An army of those who are not truly non-violent is never likely to yield any use whether it increases or decreases.

Let no one understand from the foregoing that a non-violent army is open only to those who strictly enforce in their lives all the implications of non-violence It is open to all those who accept the implications and make an ever-increasing endeavour to observe them. There never will be an army of perfectly non-violent people. It will be formed of those who will honestly endeavour to observe non-violence. For the last fifty years I have striven to make my life increasingly non-violent and to inspire my co-workers in the same direction, and I think I have had a fair amount of success The growing darkness around, far from damping my zeal and dimming my faith, brightens them, and makes the implications of non-violence more clearly visible to me

Sevagram, 15-7-40
Harijan, 21-7-1940

122

HOW TO CULTIVATE AHIMSA ?

Q. What is the good of your crying "*ahimsa, ahimsa*" in season and out of season ? Will it by itself teach people to be non-violent ? Would it not be better, instead, to tell people how pure *ahimsa* or the *ahimsa* of the strong can be cultivated ?

A Yours is a very timely and opportune question. I have attempted before this on more occasions than one to answer it. But my effort has, I confess, been rather desultory. I have not concentrated upon it, or given it the weight I might have. This was all right while I was devoting all my energy to forging means to give battle to the government But it had the result of retarding the growth of pure *ahimsa*, so that today we are not even within ken of the *ahimsa*, of the strong. If we now want to advance further, we ought, at least for some time, to completely forget the idea of offering non-violent resistance to constituted authority. If non-violence in the domestic field is successfully achieved, we shall surely see the non-violence against constituted authority revived in its purified form, and it will be irresistible.

Now that I am no longer in the Congress, I may not offer civil disobedience even in my own person in its name. But I am

certainly free to offer civil disobedience in my individual capacity whenever it may be necessary to No one need suppose that all civil disobedience will necessarily be taboo while the country is still being educated in the *ahimsa* of the strong But those who may want to join the non-violent force of my conception should not entertain any immediate prospect of civil disobedience. They should understand that, so long as they have not realized *ahimsa* in their own person in its pure form, there can be no civil disobedience for them.

Let not the mention of pure *ahimsa* frighten anybody If we have a clear conception of it and have a living faith in its matchless efficacy, it will not be found to be so hard to practise as it is sometimes supposed to be It will be well to remember the immortal Mahabharata verse in this connection The Seer Poet therein loudly proclaims to the whole world that *Dharma* includes within itself both legitimate *Artha* and *Kama*, and asks why men do not follow the royal road of *Dharma* that leads to both earthly and spiritual bliss *Dharma* here does not signify mere observance of externals It signifies the way of truth and non-violence The scriptures have given us two immortal maxims One of these is "*Ahimsa* is the supreme Law or *Dharma*." The other is "There is no other Law or *Dharma* than Truth" These two maxims provide us the key to all lawful *Artha* and *Kama*. Why should we then hesitate to act up to them? Strange as it may appear, the fact remains that people find the easiest of things oftentimes to be the most difficult to follow. The reason, to borrow a term from the science of physics, lies in our inertia Physicists tell us that inertia is an essential, and in its own place a most useful, quality of matter It is that alone which steadies the universe and prevents it from flying off at a tangent But for it the latter would be a chaos of motion But inertia becomes an incubus and a vice when it ties the mind down to old ruts It is this kind of inertia which is responsible for our rooted prejudice that to practise pure *ahimsa* is difficult It is up to us to get rid of this incubus. The first step in this direction is firmly to resolve that all untruth and *himsa* shall hereafter be taboo to us, whatever sacrifice it might seem to involve For, the good these may seem to achieve is in appearance only, but in reality it is deadly poison If our resolve is firm and our

conviction clear, it would mean half the battle won, and the practice of these two qualities would come comparatively easy to us.

Let us confine ourselves to *ahimsa*. We have all along regarded the spinning wheel, village crafts, etc. as the pillars of *ahimsa*, and so indeed they are. They must stand But we have now to go a step further. A votary of *ahimsa* will of course base upon non-violence, if he has not already done so, all his relations with his parents, his children, his wife, his servants, his dependants, etc. But the real test will come at the time of political or communal disturbances or under the menace of thieves and dacoits Mere resolve to lay down one's life under the circumstances is not enough. There must be the necessary qualification for making the sacrifice. If I am a Hindu, I must fraternize with the Musalmans and the rest In my dealings with them I may not make any distinction between my co-religionists and those who might belong to a different faith. I would seek opportunities to serve them without any feeling of fear or unnaturalness. The word 'fear' can have no place in the dictionary of *ahimsa* Having thus qualified himself by his selfless service, a votary of pure *ahimsa* will be in a position to make a fit offering of himself in a communal conflagration. Similarly, to meet the menace of thieves and dacoits, he will need to go among, and cultivate friendly relations with, the communities from which thieves and dacoits generally come

A brilliant example of this kind of work is provided by Ravishankar Maharaj His work among the criminal tribes in Gujarat has evoked praise even of the Baroda State authorities There is an almost unlimited field for this kind of work, and it does not call for any other talent in one besides pure love Ravishankar Maharaj is an utter stranger to English. Even his knowledge of Gujarati is barely sufficient for everyday use But God has blessed him with unlimited neighbourly love His simplicity easily wins all hearts, and is the envy of everybody. Let his example provide a cue and inspiration to all those who may be similarly engaged in other fields of *satyagraha*

Sevagram, 16-7-40
Harijan, 21-7-1940

123

A FLAW IN AHIMSA

Q. You have appealed to Britons to lay down arms and to adopt non-violence But that raises a moral difficulty A's *ahimsa* provokes B to *himsa* and makes him impervious to appeal to his heart If a non-violent man comes up against an inanimate thing, his non-violence will have no effect on it There is, therefore, some flaw somewhere in your belief. It is likely that *ahimsa* may have success in a restricted field. If so, what use is it for universal purposes? Your claim, therefore, of its universal use falls to the ground

A *Ahimsa* cannot be dismissed so lightly as you think *Ahimsa* is the strongest force known. But if all can use the strongest force with equal ease, it would lose its importance We have not been able yet to discover the true measure of the innumerable properties of an article of daily use like water Some of its properties fill us with wonder. Let us not, therefore, make light of a force of the subtlest kind like *ahimsa*, and let us try to discover its hidden power with patience and faith. Within a brief space of time we have carried to a fairly successful conclusion a great experiment in the use of this force As you know I have not set much store by it. Indeed I have hesitated even to call it an experiment in *ahimsa* But according to the legend, as Rama's name was enough to float stones, even so the movement carried on in the name of *ahimsa* brought about a great awakening in the country and carried us ahead. It is difficult to forecast the possibilities when men with unflinching faith carry this experiment further forward To say that those who use violence are all insensible is an exaggeration Some do seem to lose their senses, but we are bound to be mistaken if we try to base a moral law on those exceptions The safest course is to lay down laws on the strength of our usual experience, and our usual experience is that in most cases non-violence is the real antidote of violence, and it is safe to infer from it that the highest violence can be met by the highest non-violence

But let us consider for a moment inanimate objects He will surely break his head who strikes it against a stone But supposing a stone comes against us through space, we can escape it by stepping aside, or if there is nowhere to step aside, we can bravely stay where we are and receive the stone. That will mean minimum injury and, in case it proves fatal, the death will not be as painful as it would be if we made an effort to ward it off.

Extend the thought a little further, and it is easy to see that, if a senseless man is left alone and no one tries to resist him, he is sure to exhaust himself. Indeed, it is not quite inconceivable that the loving sacrifice of many may bring an insane man to his senses Instances are not wanting of absolutely insane people having come back to their senses.

Sevagram, 22-7-40
Harijan, 28-7-1940

124

NOT QUITE SO BAD

A friend quotes from a letter received from an English friend :

> "Do you think that Mahatma's appeal to every Briton is going to produce right reactions in the mind or heart of a single Briton ? That appeal has probably created more ill-will than anything else recently We live in astonishing and critical times, and it is frightfully difficult to decide what should be done. At any rate we should try to avoid obvious dangers So far as I can see, Mahatma's unadulterated policy must inevitably lead to disaster for India How far he himself intends following it I do not know, for he has a wonderful way of adopting himself to his material."

Well, I happen to know that many more than one single heart have been touched by my "Appeal to Every Briton". I know that many English friends were anxious for me to take some such step. But I do not want to take comfort from the approbation, however pleasing in itself, of English friends What is of value for me is to know that at least one Englishman thinks

as stated in the extract Such knowledge should put me on my guard It should make me more careful, if possible, in the selection of the words I use to express my thought. But no displeasure even of the dearest friends can put me off the duty I see clearly in front of me. And this duty of making the Appeal was so peremptory that it was impossible for me to put it off As certain as I am writing this, the world has to come to the state to which I have invited Britain Those who will be witnesses of that happy and not far-off event will recall my Appeal with gladness. I know that the Appeal has hastened its advent.

Why should a single Briton resent an appeal to him to be braver than he is, to be better than he is in every respect ? He may plead inability, but he cannot be displeased by an appeal to his nobler nature

Why should the Appeal breed any ill-will at all ? There is no cause given for it by the manner or the matter of the Appeal. I have not advised cessation of fight. I have advised lifting it to a plane worthy of human nature, of the divinity man shares with God Himself. If the hidden meaning of the remarks is that by making the Appeal I have strengthened Nazi hands, the suggestion does not bear scrutiny. Herr Hitler can only be confounded by the adoption by Britain of the novel method of fighting At one single stroke he will find that all his tremendous armament has been put out of action. A warrior lives on his wars whether offensive or defensive. He suffers a collapse, if he finds that his warring capacity is unwanted.

My Appeal is not from a coward to brave people to shed their bravery, nor is it a mockery from a fair-weather friend to one in distress. I suggest to the writer to re-read my Appeal in the light of my explanation.

One thing Herr Hitler, as every critic, may say I am a fool without any knowledge of the world or human nature. That would be a harmless certificate which need excite neither ill-will nor anger. It would be harmless because I have earned such certificates before now. This one would be the latest of the many editions, and I hope not the last, for my foolish experiments have not yet ended.

So far as India is concerned, my unadulterated policy can never harm her, if she adopts it. If India as a whole rejects it, there can be no harm accruing except to those who may foolishly

pursue it. The correspondent has lighted upon my strong point when he says: "Mahatma has a wonderful way of adapting himself to his material" My instinctive knowledge of my material has given me a faith which cannot be moved. I feel within me that the material is ready. My instinct has not betrayed me once But I must not build much upon past experience. "One step enough for me."

Sevagram, 24-7-40
Harijan, 28-7-1940

125

WHAT OF THE 'WEAK MAJORITY'?

Prof Timur of Islamia College, Peshawar, writes:

"The world is indebted to you for revealing to it in these hard times the hidden values of non-violence. The experiment which you want to make of defending India against foreign aggression without the use of arms would be the boldest moral experiment of all times There are two possible results of such a course Either the conscience of the invaders may be awakened by the love of the invaded and they may repent of their sin Or the proud invaders may take non-violence as a sign of physical weakness and degeneration, and may think it right to subjugate, rule over, and exploit, a weak people This is the doctrine of Nietzsche which is followed in practice by Hitler A great loss is involved in such conquest of the physically weak by the physically strong A few strongwilled members of the conquered nation may refuse to own allegiance to the conquerors, but the large majority always submits and adopts servile manners to preserve its existence. Among them may be found great scientists, philosophers and artists Genius and moral strength are not always combined in the same man The strong man does not need armies to defend his liberty. He sacrifices his body to preserve his soul Such men are, however, few and far between It is the weak majority which needs protection The question is how to protect it by non-violent methods This is the real difficulty which every patriot feels when he thinks of adopting non-violence for the defence of his country

I hope you would be kind enough to throw light on this point in an early issue of *Harijan*."

The weak majority no doubt needs protection. If all were soldiers either of *ahimsa* or *himsa*, no such questions as call for discussion in these columns would arise. There is always a weak majority that would want protection against man's mischief. The orthodox method we know. Nazism is its logical outcome. It is an answer to a definite want A terrible wrong wantonly perpetrated against a whole nation cried out for redress. And Hitler arose to avenge it. Whatever the ultimate fortune of the war, Germany will not be humiliated again. Humanity will not stand a second outrage. But in seeking to avenge the wrong by the wrong method of violence brought to very near perfection, Hitler has brutalized not only Germans but a large part of humanity The end of it we have not yet reached. For Britain, so long as she holds to the orthodox method, has to copy the Nazi methods, if she is to put up a successful defence Thus the logical outcome of the violent method seems to be increasingly to brutalize man including "the weak majority". For it has to give its defenders the required measure of co-operation

Now imagine the same majority defended after the method of non-violence. As it admits of no grossness, no fraud, no malice, it must raise the moral tone of the defenders Hence there will be a corresponding rise in the moral tone of the "weak majority" to be defended. No doubt there will be difference in degree, but not in kind

But the snag comes in when we consider the ways and means of working the non-violent method In working the other, there is no difficulty in getting the human material Therefore that way seems easy. In getting non-violent defenders, we have to pick and choose. Money cannot buy them. The non-violent process is wholly different from the one commonly known. I can only say that my own experience in organizing non-violent action for half a century fills me with hope for the future. It has succeeded in a marked measure in protecting the "weak majority" But half a century is nothing in discovering the hidden possibilities of this force and working them out. Those, therefore, like the correspondent who are attracted to non-violence should, according to their ability and opportunity, join the experiment. It has entered upon a most interesting, though at the same time a most difficult, stage. I am myself sailing on

uncharted waters I have to take soundings every half-hour. The difficulty only braces me for the struggle.

Sevagram, 6-8-40
Harijan, 11-8-1940

126

IS NON-VIOLENCE IMPOSSIBLE?

"One may admit that in theory non-violence is an infallible weapon, and that no power on earth can be a match for the man who has achieved non-violence to the fullest extent. But is this possible? There may be a rare *yogi* who can tame wild beasts like lions and tigers and render them meek as lambs, but the average man must resort to a rifle or similar weapon to protect himself against such beasts You with your wonderful power may convert others by the sheer force of your thought, but the average man has to have recourse to worldly remedies like a law court, pleaders and so on. Even in the dim and distant past we rarely hear of men who practised *ahimsa* in daily affairs Lord Buddha tried for a time to lead people along the path of *ahimsa*, but what happened after him? Society went back to its old ways, forgetting Buddha's teaching The past, therefore, offers little promise, for the future, of society going along the lines of *ahimsa* any more than it has done before, and our sages, therefore, must have wisely left the world and resorted to the forest for practising truth and non-violence You may inspire a few persons to study *ahimsa* but society as a whole is not likely to take to it. The same argument applies to India as a nation. She must needs seek means other than those of *ahimsa* in order to win her liberty. It is idle to expect an infant learning his primer to understand a book like Tilak's Gita Even so is it idle to expect people steeped in worldly pleasures to understand the infallibility of *ahimsa*. Besides *ahimsa* is the final goal, attainment of which requires much greater preparation than is required in order to obtain a degree in medicine or engineering We shall have to have numerous colleges and universities for the teaching of the science and art of truth and non-violence. Today society directs its energies to creating new wants and satisfying them. How do you expect it to turn those energies in the direction of researches in *ahimsa*?"

The doubts and difficulties raised by this correspondent occur to others also, and I have on various occasions tried to solve them too But when the Working Committee of the Congress has been instrumental in making of *ahimsa* a live issue, it seems necessary to deal with these doubts and difficulties at some length

The correspondent doubts in substance the universal application of *ahimsa*, and asserts that society has made little progress towards it. Teachers like Buddha arose and made some effort with some little success perhaps in their lifetime, but society is just where it was in spite of them. *Ahimsa* may be good enough to be the duty of an individual; for society it is good for nothing, and India too will have to take to violence for her freedom.

The argument is, I think, fundamentally wrong The last statement is incorrect inasmuch as the Congress has adhered to non-violence as the means for the attainment of Swaraj It has indeed gone a step further The question having been raised as to whether non-violence continues to be the weapon against all internal disturbances, the A I C C clearly gave the answer in the affirmative. It is only for protection against outside aggression that the Congress has maintained that it would be necessary to have an army. And then even on this matter there was a considerable body of the members of the A.I.C C. who voted against the resolution. This dissent has got to be reckoned with when the question voted upon is one of principle. The Congress policy must always be decided by a majority vote, but it does not cancel the minority vote It stands. Where there is no principle involved and there is a programme to be carried out, the minority has got to follow the majority. But where there is a principle involved, the dissent stands, and it is bound to express itself in practice when the occasion arises That means that *ahimsa* for all occasions and all purposes has been recognized by a society, however small it may be, and that *ahimsa* as a remedy to be used by society has made fair strides. Whether it will make further strides or no is a different matter. The Working Committee's resolution, therefore, fails to lend any support to the correspondent's doubts. On the contrary it should, in a certain degree, dispel them.

Now for the argument that I am but a rare individual, and that what little society has done in the matter of *ahimsa* is due to

my influence, and that it is sure to disappear with me. This is not right. The Congress has a number of leaders who can think for themselves The Maulana is a great thinker of keen intellect and vast reading. Few can equal him in his Arabic and Persian scholarship. Experience has taught him that *ahimsa* alone can make India free It was he who insisted on the resolution accepting *ahimsa* as a weapon against internal disturbances. Pandit Jawaharlal is not a man to stand in awe of anyone His study of history and contemporary events is second to none. It is after mature thought that he has accepted *ahimsa* as a means for the attainment of Swaraj It is true that he has said that he would not hesitate to accept Swaraj if non-violence failed and it could be won by means of violence. But that is not relevant to the present issue There are not a few other big names in the Congress who believe in *ahimsa* as the only weapon at least for the attainment of Swaraj. To think that all of them will give up the way of *ahimsa* as soon as I am gone, is to insult them and to insult human nature. We must believe that everyone can think for himself. Mutual respect to that extent is essential for progress By crediting our companions with independent judgment we strengthen them and make it easy for them to be independent-minded even if they are proved to be weak.

I hope neither the correspondent nor anyone else believes that the Congress or many Congress leaders have bidden good-bye to *ahimsa* To the limited extent that I have pointed out faith in *ahimsa* has been reiterated and made clear beyond any doubt by the Congress I agree that the limit laid down by the Congress considerably narrows down the sphere of *ahimsa* and dims its splendour But the limited *ahimsa* of the Congress is good enough for the purpose of our present argument For I am trying to make out that the field of *ahimsa* is widening and the limited acceptance of *ahimsa* by the Congress sufficiently supports my position

If we turn our eyes to the time of which history has any record down to our own time, we shall find that man has been steadily progressing towards *ahimsa*. Our remote ancestors were cannibals. Then came a time when they were fed up with cannibalism and they began to live on chase Next came a stage when man was ashamed of leading the life of a wandering

hunter. He therefore took to agriculture and depended principally on mother earth for his food. Thus from being a nomad he settled down to civilized stable life, founded villages and towns, and from member of a family he became member of a community and a nation. All these are signs of progressive *ahimsa* and diminishing *himsa*. Had it been otherwise, the human species should have been extinct by now, even as many of the lower species have disappeared.

Prophets and *avatars* have also taught the lesson of *ahimsa* more or less. Not one of them has professed to teach *himsa*. And how should it be otherwise? *Himsa* does not need to be taught. Man as animal is violent, but as Spirit is non-violent. The moment he awakes to the Spirit within he cannot remain violent. Either he progresses towards *ahimsa* or rushes to his doom. That is why the prophets and *avatars* have taught the lessons of truth, harmony, brotherhood, justice, etc. — all attributes of *ahimsa*.

And yet violence seems to persist, even to the extent of thinking people like the correspondent regarding it as the final weapon. But, as I have shown, history and experience are against him.

If we believe that mankind has steadily progressed towards *ahimsa*, it follows that it has to progress towards it still further. Nothing in this world is static, everything is kinetic. If there is no progression, then there is inevitable retrogression. No one can remain without the eternal cycle, unless it be God Himself.

The present war is the saturation point in violence. It spells to my mind also its doom. Daily I have testimony of the fact that *ahimsa* was never before appreciated by mankind as it is today. All the testimony from the West that I continue to receive points in the same direction. The Congress has pledged itself to *ahimsa* however limited. I invite the correspondent and doubters like him to shed their doubts and plunge confidently into the sacred sacrificial fire of *ahimsa*. Then I have little doubt that the Congress will retrace its step. "It is always willing." Well has Pritam, our poet sung:

> Happiest are those that plunge in the fire,
> The lookers-on are all but scorched by flames.

Sevagram, 5-8-40
Harijan, 11-8-1940

THE LIVE ISSUE

Wait, Watch, and Pray

A few members of the A. I. C. C. who are out-and-out believers in non-violence visited Sevagram the other day Some of them had remained neutral on the Delhi resolution, some had opposed it What was the right attitude? What were they to do next? What was the programme before them? Should they not retire forthwith? These and other questions were troubling them, and they did not know what to do In response Gandhiji began to do some loud thinking

"Wait, watch, and pray," said he, and remained silent for a while. Then he said. "You must carefully follow what I am writing from week to week. You must be absolutely sure that you are out-and-out believers in non-violence. Can your *ahimsa* stand the test? Rehearse to yourself what you would do in case of a riot. Those who have differed from us are no cowards If they say that they cannot do without an army and police, they deserve a respectful hearing. I myself do not know what I should do in a difficult situation You know I have capitulated on the question of the desirability of maintaining a police force. But what I can say is that I shall hope to behave non-violently, should the occasion arise. I should not like to die before my death. I do not want to prepare India for military defence from today. We should never forget that we are not the whole of India. The Congress is without doubt a powerful organization, but the Congress is not the whole of India. The Congress may not have an army, but those who do not believe in non-violence will. And if the Congress too surrenders, there is no one to represent the no-army mentality. This was my argument in a nutshell. But I failed to carry conviction. Therefore I must find fault not with my comrades but with myself. There must be some weakness in my argument, and so I must prepare myself to carry conviction to those who differ from me."

"But," he added, "I have digressed. What you and I have to do is to show our *ahimsa* when there is rioting or similar disturbance. If every one of us, wherever he is, begins doing so there will come into being a non-violent army. Even the limited non-violence that all are subscribing to would not have come into being, if we had seen no successful demonstrations of it. So we have to hold on to our faith, even if when the time comes we may fail. There is no use arguing with the comrades who have differed from us, I hope, only temporarily. The question is one of demonstrating the truth in us, and we won't do it, unless we show that we have no malice, no bitterness, no inclination to find faults. We have to prepare ourselves for the terrible ordeal. The testing time may come sooner than we imagine. I am sailing on an uncharted sea. I have no cut and dried programme, and I am brooding from moment to moment. In the meanwhile follow my weekly writings and carry out the constructive programme. The time for resignation is not yet. We must not be misunderstood."

"But," said a worker, "you have asked us to retire immediately, and all of us are ready."

"Your readiness is good, and it is enough for me for the time being. If you conscientiously voted against the Delhi resolution, you did nothing wrong; if you remained neutral, you did nothing wrong. You would have been wrong, if you had joined hands with those who sought to defeat the resolution on any but the ground of non-violence. For having given your vote on the Wardha resolution and having lost, you could not vote against the Delhi resolution which was the natural corollary of the Wardha resolution."

"But it was by accident that those who voted against the Delhi resolution, did so."

"No, it was by design. They are no believers in non-violence to the extent you are, but they wanted to defeat the resolution for the sake of their policy."

"But then how long are we to remain in the Congress?"

"I cannot give a definite answer. I shall have a talk with the Maulana. We must not rush the Maulana and the Working Committee. If they find that the out-and-out non-violence people were the sustaining part of the Congress organization, they will retrace their step."

"Then how long shall we wait ?"

"Until I give you the word"

"But," said one of them, "I disliked the Wardha resolution and wanted to resign at once."

"You might have done so Then you would have been within your rights. But to do so now might savour of violence and self-righteousness. You must remember that I waited for more than a year before taking the final step, and then too I took it with the full approval of the friends of the Working Committee"

Sevagram, 7-8-40 M. D.

Harijan, 11-8-1940

128
MORAL SUPPORT

A friend writes as follows :

> "On the declaration of war you had advised giving moral support to Britain. Many persons never understood the implications of such support. You have never explained them either, so far as I know I am a regular reader of *Harijanbandhu*, but I have not seen a clear explanation there. Everyone puts his own interpretation on the words At the last sitting of the Gujarat Provincial Congress Committee the leaders said: 'Bapu was ready to give moral support to Britain. What else has the Congress done in its latest resolution? As a matter of fact, the Congress asks for more than it promises to give. Bapu was willing to give all for nothing' If war is itself a wrong act, how can it deserve moral support or blessings? In the Mahabharata, was the help that Lord Krishna gave to Arjuna moral, or was it more destructive than the deadliest weapons of war ?"

I did explain in *Harijan* what I meant by moral support. It is possible that the explanation did not appear in *Harijanbandhu* In my English writings things are often left to be understood The ellipses need, however, to be brought out in translations.

Broadly speaking, Britain could have had moral support from the Congress, if only she had acted justly towards India. There was no spirit of bargaining in my proposal because the help was not offered in exchange for anything.

Suppose my friend possesses moral strength which he has acquired through *tapasya*. And suppose I am in need of this

strength I shall not get it from him for the asking He may always be ready to give it to me, but if I have not the capacity within me to take it from him, how shall I ever obtain it? Moral support cannot really be given in the sense of giving It automatically comes to him who is qualified to take it And such a one can take it in abundance

The Congress has this moral reservoir. The acceptance of the creed of truth and non-violence has been its *tapasya* It has acquired world prestige through the acceptance of truth and non-violence for the attainment of its goal. If the Congress could have given its blessings to Britain, the world would have adjudged Britain's cause to be just The masses over whom the Congress holds sway would also have acknowledged justice to be on Britain's side. But in all this the Congress would have had nothing material to give. The British government would, by its own action, have acquired moral prestige or strength. Though the Congress would not give one man or one pice as material aid, its moral support and blessings would definitely have turned the scales in favour of Britain. This is my belief. That my belief may be groundless and that the Congress never had any moral prestige is quite possible. The determination of this question is unnecessary for my argument.

But the opportunity for rendering moral support now seems almost to have gone The Congress felt itself unable to adopt my course It cannot be taken mechanically It presupposes a living faith in truth and non-violence The greatest quality in the Congress is this that it has never claimed to have what it really does not possess. And therefore its resolutions are dignified and carry force with them.

The help that the Congress in its latest resolution promises to give is material and for a consideration, eminently just, no doubt, but it is not and cannot be unconditional. I do not suggest that this position is either untenable or morally wrong. The resolution has dignity because it is the considered opinion of the majority But by passing it the Congress has, in my opinion, surrendered the prestige it had or was supposed to have Many congressmen say that, while they firmly believed that they could attain Swaraj through non-violence, they had never meant it to be understood that they could retain it also through non-violence The entire outside world, however,

believed that the Congress was showing the golden way to the abolition of war. No one outside India ever dreamed that, if the Congress could wrest independence from a mighty power like Britain purely through non-violence, it would not be able to defend it also by the same means.

In my opinion Lord Krishna's help to Arjuna cannot be said to be moral, because he himself had an army and was an expert in the art of war. Duryodhana acted foolishly in that he asked for Krishna's army, while Arjuna got what he wanted in the person of the expert in the science of war. Therefore, if we interpret the Mahabharata literally, Lord Krishna's strength was certainly more destructive than that of his army. Because of his scientific skill Krishna was able, with an army of seven divisions, to destroy Duryodhana's army of eleven. But it is well known that I have never looked upon the Mahabharata as a mere record of earthly warfare In the garb of an epic the poet has described the eternal warfare within the individual as well as in society, between Truth and Untruth, Violence and Non-violence, Right and Wrong. Looking at the epic even superficially one can understand how the great Vyasa has demonstrated that in this war the victor was no better off than the vanquished. Out of that vast concourse of warriors only seven remained to tell the tale. And the poet gives a true picture of the woeful state of mind also of these seven. The author has shown clearly too that in armed warfare the contending parties are certain to stoop to meanness and trickery. When occasion arose even the great Yudhishthira had to resort to untruth to save the battle.

One more question of the writer remains to be answered. If war is itself a wrong act, how can it be worthy of moral support or blessings? I believe all war to be wholly wrong. But if we scrutinize the motives of two warring parties, we may find one to be in the right and the other in the wrong. For instance, if A wishes to seize B's country, B is obviously the wronged one. Both fight with arms. I do not believe in violent warfare, but all the same, B, whose cause is just, deserves my moral help and blessings.

Sevagram, 12-8-40
Harijan, 18-8-1940

WHAT SHOULD A BRITON DO AND NOT DO?

Q. In your appeal 'To Every Briton' you say: "You will invite Herr Hitler and Signor Mussolini to take what they want . . You will give all your 'earthly possessions' but never your souls or your minds..... You will refuse to own allegiance to them." Please explain clearly what a Briton should or should not do. I ask the question because your answer will have a bearing on the duty of every *satyagrahi*

A. Not to yield your soul to the conqueror means that you will refuse to do that which your conscience forbids you to do. Suppose the 'enemy' were to ask you to rub your nose on the ground or to pull your ears or to go through such humiliating performances, you will not submit to any of these humiliations. But if he robs you of your possessions, you will yield them because as a votary of *ahimsa* you have from the beginning decided that earthly possessions have nothing to do with your soul. That which you look upon as your own you may keep only so long as the world allows you to own it.

Not to yield your mind means that you will not give way to any temptation Man is oftentimes weak-minded enough to be caught in the snare of greed and honeyed words. We see this happening daily in our social life A weak-minded man can never be a *satyagrahi*. The latter's 'no' is invariably a 'no' and his 'yes' an eternal 'yes'. Such a man alone has the strength to be a devotee of truth and *ahimsa*. But here one must know the difference between steadfastness and obstinacy If after having said 'yes' or 'no' one finds out that the decision was wrong and in spite of that knowledge clings to it, that is obstinacy and folly. It is necessary to think things out carefully and thoroughly before coming to any decision.

The meaning of refusal to own allegiance is clear You will not bow to the supremacy of the victor, you will not help him to attain his object. Herr Hitler has never dreamt of possessing Britain. He wants the British to admit defeat The victor can then demand anything he likes from the vanquished, and the

latter has perforce to yield But if defeat is not admitted, the enemy will fight until he has killed his opponent A *satyagrahi*, however, is dead to his body even before the enemy attempts to kill him, i e. he is free from attachment to his body and only lives in the victory of the soul. Therefore, when he is already thus dead, why should he yearn to kill anyone ? To die in the act of killing is in essence to die defeated. Because, if the enemy is unable to get what he wants from you alive, he will decide to get it after killing you. If, on the other hand, he realizes that you have not the remotest thought in your mind of raising your hand against him even for the sake of your life, he will lack the zest to kill you. Every hunter has had this experience No one has ever heard of anyone hunting cows.

You may find that I have not answered the questions that you had in your mind. I have made a humble effort and dealt with your general question by giving you a few homely examples. I hope that from them you will be able to deduce answers to the questions left unanswered

Dignity of the soul and self-respect are interpreted differently by different persons I am aware that self-respect is often misinterpreted The over-sensitive man may see disrespect or hurt in almost everything. Such a man does not really understand what self-respect is. That has been my experience in many cases. But no harm accrues even if a non-violent man holds mistaken notions of self-respect He can die cheerfully for the sake of what he believes to be his dignity and self-respect. Only he has no right to injure or kill the supposed wrong-doer.

Sevagram, 13-8-40
Harijan, 18-8-1940

130

NAZISM IN ITS NAKEDNESS

A Dutch friend writes:

"You will perhaps be able to remember me having made a drawing of you at Romain Rolland's in 1931. Even before meeting you in person I followed the Indian movement with interest, in particular your leadership and your methods of combat. I am a Dutchman and lived for many years in Germany, where I had built up a living as an artist. Nazism, which gained hold in Germany seven years ago, caused me many conscientious doubts, especially with regard to the schooling of my three children. I often thought of writing to you to ask your advice, I knew, however, how fully your time was occupied and did not want to embarrass you with my difficulties. Eventually I managed to settle my affairs to my own satisfaction.

It is just one year ago since I left my house in Munich to spend some time in Holland. When war broke out in September we did not return to Germany but remained in Holland, since I had no wish to expose my children to the mental state of a country at war. On 10th May, by the use of every possible subtle trick, Holland was overpowered. After four days of the most ruthless bombing we fled to England and are now on our way to Java, the country of my birth, where I hope to find work, not as one of the group which exploits the colony but as a guest.

Europe has put its trust in violence, but for centuries past in the wars of soldier against soldier certain rules of chivalry were observed. Nazism has thrown that code overboard, and I can say with a clear conscience that no country has ever made use of such vicious treachery, villainy and cowardice in attaining its ends as modern Germany. The upbringing even of little children is based on the exercise of violence. Nazism systematically trains children to cunning and treachery even towards their own parents, to immorality and much more in the same spirit. When Indian students, who in the course of years have visited me at home, enthused about Germany, they were merely deceived by the gloss which Nazism knows so well how to lay on. A short explanation always convinced them of the tremendous difference between 'Indian National Socialism', as they often called your movement, and Hitler's Nazism.

Two books, *The Revolution of Destruction* and *Hitler Speaks*, have been written by Hermann Rauschning, a former intimate of Hitler These books give a very clear impression of the movement in Germany, and should be read by all thinking people Hitler aims at nothing less than the destruction of all moral values, and in the bulk of German youth he has already attained that end

Your article in *Harijan* about the Jewish problem in Germany particularly interested me since I had many Jewish friends there. You say in it that, if ever a war were justified, it is this one against Germany. In the same article, however, you write that, if you were a Jew, you would attempt to soften the hearts of the Nazis by non-violence. Recently you also advised Britain and the British people to surrender their beautiful island to the German invader, without resistance by force, and to conquer him afterwards by non-violence There is probably no man in the whole history who has a better knowledge of the practice of non-violence than yourself. Your views have awakened veneration and love for you in millions of hearts not only in India but in the outside world as well. Even your opponents are compelled eventually to honour you and admit the rightness of your aims and methods But your opponents are the British, a people that, though like every other it has its weak points, in spite of its business sense, retains a heart

Through Nazism, the German youth has lost all individuality of thought and feeling. The great mass of young people has lost its heart and is degraded to the level of a machine. The German conduct of the war is absolutely mechanical: machines are driven by robot men who have no qualms of conscience about crushing under their tanks the bodies of women and children, bombing open towns, killing hundreds of thousands of women and children, and on occasion using them as a screen for their advance, or distributing poisoned food These are all facts, the truth of which I can vouch for. I have spoken with many of your followers about the possibility of applying non-violence against Germany. A friend of mine, whose work it is to cross-examine German prisoners of war in England, was deeply shocked by the spiritual narrowness and heartlessness of these young men, and agreed with me that non-violence could not be applied with any success against such robots The most terrible side of the question is the fact that Hitler has been able, in the short space of seven years, to sink German youth to such depths of demoralization. A glance at either of the books mentioned above will convince you of that I know of no other example in history of a people sinking spiritually so low.

I hope that my letter has not trespassed too much upon your time On the return journey to Europe in a happier time which, if God wills, may soon be possible, I shall try to bring my wife and children for a short visit to you."

The friend has sent his name and address. But I withhold both for fear of harm coming to him through unnecessary publicity. The letter must be valued on its own intrinsic merits

What, however, concerns me is not so much his characterization of Nazism as his belief that non-violent action may have no effect on Hitler or the Germans whom he has turned into so many robots Non-violent action, if it is adequate, must influence Hitler and easily the duped Germans. No man can be turned into a permanent machine Immediately the dead weight of authority is lifted from his head, he begins to function normally To lay down any such general proposition as my friend has, betrays ignorance of the working of non-violence The British Government can take no risks, can make no experiments in which they have not even a workable faith. But if ever an opportunity could be given to me, I should not hesitate to try, in spite of my physical limitations, what would appear to be impossible. For in *ahimsa* it is not the votary who acts in his own strength Strength comes from God If, therefore, the way is opened for me to go, He will give me the physical endurance and clothe my word with the needed power. Anyway all through my life I have acted in that faith Never have I attributed any independent strength to myself. This may be considered by men who do not believe in a higher Power than themselves as a drawback and a helpless state I must admit that limitation of *ahimsa* if it be accounted as such.

Sevagram, 6-8-40
Harijan, 18-8-1940

131

AN INTERESTING DISCOURSE

I

The Scope of Ahimsa

"You see that I am answering every one of your questions straightaway without the slightest hesitation. That is because the great question underlying your questions possesses me and I have rehearsed to myself every one of the situations arising out of the various implications of *ahimsa*." These words were uttered by Gandhiji towards the close of a long and interesting discourse with a number of friends from Poona who had come for the sole purpose of clearing their doubts on *ahimsa* They were all out-and-out believers in non-violence, or if they were not, they had come all the way from Poona in order to be such believers, after their doubts were cleared. Balasaheb Kher, the ex-Prime Minister of Bombay, accompanied them. They had drawn up an exhaustive questionnaire, even at the risk of making some of their questions trite, for they did not want to leave any room for doubt.

The first question was about the limits and implications of *ahimsa* and the extent of its application. Should one stop with the human species or extend it to all creation?

Gandhiji said: "I was not prepared for this question For the Congress *ahimsa* is naturally confined to the political field and therefore only to the human species Hence out-and-out non-violence means for our purpose every variety of non-violence on the political field. In concrete terms it covers family relations, relations with constituted authority, internal disorders and external aggression. Put in another way it covers all human relations."

"Then what about meat-eating and egg-eating? Do they consist with non-violence?"

"They do. Otherwise we should have to exclude Mussalmans and Christians and a vast number of Hindus as possible

co-workers in *ahimsa*. I have known many meat-eaters to be far more non-violent than vegetarians "

"But what if we had to give them up for the sake of a principle ?"

"Oh yes, we would, if we had to compromise our principle Our principle is defined as I have shown already."

A Wrong Analogy

"If, as you have said, Polish resistance to the German invasion was almost non-violent, and you would thus seem to reconcile yourself with it, why do you object to the Wardha resolution of the Working Committee ?"

"Surely," said Gandhiji, "there is no analogy between the two cases If a man fights with his sword single-handed against a horde of dacoits armed to the teeth, I should say he is fighting almost non-violently. Haven't I said to our women that, if in defence of their honour they used their nails and teeth and even a dagger, I should regard their conduct non-violent ? She does not know the distinction between *himsa* and *ahimsa*. She acts spontaneously Supposing a mouse in fighting a cat tried to resist the cat with his sharp beak, would you call that mouse violent ? In the same way, for the Poles to stand valiantly against the German hordes vastly superior in numbers, military equipment and strength, was almost non-violence. I should not mind repeating that statement over and over again. You must give its full value to the word 'almost'. But we are 400 millions here. If we were to organize a big army and prepare ourselves to fight foreign aggression, how could we by any stretch of imagination call ourselves almost non-violent, let alone non-violent ? The Poles were unprepared for the way in which the enemy swooped down upon them When we talk of armed preparation, we contemplate preparation to meet any violent combination with our superior violence. If India ever prepared herself that way, she would constitute the greatest menace to world peace. For, if we take that path, we will also have to choose the path of exploitation like the European nations. That is why I still regret the moment when my words lacked the power of convincing the Sardar and Rajaji. By having passed that resolution we proclaimed to the world that the *ahimsa* we had subscribed to all these years was not really *ahimsa* but a form of *himsa* "

Administering Non-violently

Q. "How will you run your administration non-violently?"

A. "If you assume that we would have won independence by non-violent means, it means that the bulk of the country had been organized non-violently. Without the vast majority of people having become non-violent, we could not attain non-violent Swaraj. If, therefore, we attain Swaraj by purely non-violent means, it should not be difficult for us to carry on the administration without the military. The *goondas* too will then have come under our control. If, for instance, in Sevagram we have five or seven *goondas* in a population of seven hundred who are non-violently organized, the five or seven will either live under the discipline of the rest or leave the village.

"But you will see that I am answering the question with the utmost caution, and my truth makes me admit that we might have to maintain a police force. But the police will be after our pattern, and not the British pattern. As we shall have adult suffrage, the voice of even the youngest of us will count. That is why I have said that the ideally non-violent State will be an ordered anarchy. That State will be the best governed which is governed the least. The pity is that no one trusts me with the reins of government! Otherwise I would show how to govern non-violently. If I maintain a police force, it will be a body of reformers."

"But," someone retorted, "you had the power in the Congress?"

"That was a paper-boat," said Gandhiji. "And then you must not forget that I never spared the Congress ministries. Munshi and Pantji came in for a lot of strictures from me. As I have said in another connection even the dirty water from the gutter, when it mixes with the water of the Ganga, becomes as pure as the Ganga water; even so I had expected even the *goondas* would work under Congress discipline. But evidently our ministers had not attained the purifying potency of the fabled Ganga."

"But," said Shri Kher, intervening at this stage, "the Congress ministers had no non-violent power with them. Even if 500 *goondas* had run amok and had been allowed to go unchecked, they would have dealt untold havoc. I do not know how even you would have dealt with them."

"Surely, surely," said Gandhiji, "I had rehearsed such situations The ministers could on such occasions have gone out and allowed themselves to be done to death by the *goondas*. But let us face the fact that we had not the requisite *ahimsa*. We went in with our half-baked *ahimsa*. I do not mind it, inasmuch as we gave up power the moment we felt we should give it up I am sure that, if we had adhered to strictest non-violence during these two or three years, the Congress would have made a tremendous advance in the direction of *ahimsa* and also independence."

"But," said Balasaheb, "four or five years ago when there was a riot, and I appealed to the leaders to go and throw themselves into the conflagration, no one was ready"

"So you are supporting my argument. You agree that our loyalty to *ahimsa* was lip-loyalty and not heart-loyalty And if even the half-baked *ahimsa* carried us a long way, does it not follow that thorough *ahimsa* would have carried us very far indeed, even if it had not already brought us to the goal ?"

Non-violent Army

"But we cannot visualize how you will stand non-violently against a foreign invasion."

"I cannot draw the whole picture to you because we have no past experience to fall back upon and there is no reality facing us today. We have got the Government army manned by the Sikhs, Pathans and Gurkhas What I can conceive is this that with my non-violent army of, say, two thousand people I should put myself between the two contending armies But this, I know, is no answer. I can only say that we shall be able to reduce the invader's violence to a minimum The general of a non-violent army has got to have greater presence of mind than that of a violent army, and God would bless him with the necessary resourcefulness to meet situations as they arise"

Shri Kher now raised a philosophical question. "The world," he said, "is made up of pairs of opposites. Where there is fear, there is courage too. When we walk on the edge of a precipice we walk warily, for we have fear Fear is not a thing to despise. Will your non-violent army be above these pairs of opposites ?"

"No," said Gandhiji, replying in the same philosophical terminology. "No, for the simple reason that my army will represent one of the pair — *ahimsa* — out of the pair of *himsa* and

ahimsa. Neither I nor my army is above the pair of opposites. The state of *gunatita*, in the language of the Gita, rises above *himsa* and *ahimsa* both Fear has its use, but cowardice has none. I may not put my finger into the jaws of a snake, but the very sight of the snake need not strike terror into me The trouble is that we often die many times before death overtakes us.

"But let me explain what my army will be like They need not and will not have the resourcefulness or understanding of the general, but they will have a perfect sense of discipline to carry out faithfully his orders The general should have the quality which commands the unquestioning obedience of his army, and he will expect of them nothing more than this obedience. The Dandi March was entirely my conception. Pandit Motilalji first laughed at it, he thought it to be a quixotic adventure, and Jamnalalji suggested instead a march on the Viceroy's House! But I could not think of anything but the salt march as I had to think in terms of millions of our countrymen. It was a conception that God gave me. Pandit Motilalji argued for some time, and then he said he must not argue, as after all I was the general and he must have faith in me. Later when he saw me in Jambusar he was completely converted, for he saw with his own eyes the awakening that had come over the masses. And it was an almost magical awakening. Where in history shall we find parallels of the cool courage that our women displayed in such large numbers?

"And yet none of the thousands who took part in the movement were above the average They were erring, sinning mortals God has a way of making use of the most fragile instruments and remaining Himself untouched by everything Only He is *gunatita*

The Real Equipment

"And then what after all is the army that wins? You know Rama's reply to Vibhishana when the latter wondered how Rama would be able to conquer a foe like Ravana, when he had no chariot, no armour, nor any shoes to his feet? Rama says :*

> "The chariot, my dear Vibhishana, that wins the victory for Rama is of a different sort from the usual one. Manliness

* Gandhiji only made a reference to these verses from Tulsidas' Ramayan I translate them here fully for the benefit of the reader M D

and courage are its wheels, unflinching truth and character its banners and standards, strength, discrimination, self-restraint and benevolence its horses, with forgiveness, mercy, equanimity their reins, prayer to God is that conqueror's unerring charioteer, dispassion his shield, contentment his sword, charity his axe, intellect his spear, and perfect science his stout bow. His pure and unwavering mind stands for a quiver, his mental quietude and his practice of *yama* and *niyama* stand for the sheaf of arrows, and the homage he pays to Brahmans and his *guru* is his impenetrable armour There is no other equipment for victory comparable to this, and, my dear friend, there is no enemy who can conquer the man who takes his stand on the chariot of *dharma*. He who has a powerful chariot like this is a warrior who can conquer even that great and invincible enemy—the world Hearken unto me and fear not'

"That is the equipment," added Gandhiji, "that can lead us to victory I have not retired from the world, nor do I mean to. I am no recluse I am content to do what little work I can in Sevagram and give what guidance I can to those that come to me What we need is faith And what is there to be lost in following the right path? The worst that can happen to us is that we shall be crushed. Better to be crushed than to be vanquished.

"But if we had to equip ourselves violently, I should be at my wit's end. I cannot even think out an armament plan, much less work it. On the other hand my non-violent plan is incredibly simpler and easier, and with God as our Commander and Infallible Guide where is there cause for any fear?"

Sevagram, 21-8-40 M D

Harijan, 25-8-1940

132

AN INTERESTING DISCOURSE

II

A Sister's Questions

Now came questions of practical difficulty.

Q. "May a non-violent man possess wealth, and if he may, how can he keep it non-violently?"

A. "He may not own any wealth, though he may possess millions. Let him hold it in trust. If he lives among dacoits and thieves, he may possess very little, indeed little beyond a loin-cloth. And if he does this, he will convert them

"But you must not generalize. In a non-violent State there will be very few dacoits. For the individual the golden rule is that he will *own* nothing. If I decided to settle and work among the so-called criminal tribes, I should go to them without any belongings and depend on them for my food and shelter The moment they feel that I am in their midst in order to serve them, they will be my friends. In that attitude is true *ahimsa* But I have discussed this question at length in a recent article in *Harijan.*"

Q. "How is one to protect the honour of women?"

A "I am afraid you do not read *Harijan* regularly. I discussed this question years ago, and have discussed it often since The question may be discussed under two heads: (1) how is a woman to protect her own honour? and (2) how are her male relatives to protect it?

"As regards the first question, where there is a non-violent atmosphere, where there is the constant teaching of *ahimsa*, woman will not regard herself as dependent, weak or helpless She is not really helpless when she is really pure. Her purity makes her conscious of her strength I have always held that it is physically impossible to violate a woman against her will. The outrage takes place only when she gives way to fear or does not realize her moral strength. If she cannot meet the assailant's physical might, her purity will give her the strength

to die before he succeeds in violating her. Take the case of Sita. Physically she was a weakling before Ravana, but her purity was more than a match even for his giant might He tried to win her with all kinds of allurements but could not carnally touch her without her consent On the other hand, if a woman depends on her own physical strength or upon a weapon she possesses, she is sure to be discomfited whenever her strength is exhausted.

"The second question is easily answered. The brother or father or friend will stand between his protegee and her assailant He will then either dissuade the assailant from his wicked purpose or allow himself to be killed by him in preventing him In so laying down his life he will not only have done his duty, but given a new accession of strength to his protegee who will now know how to protect her honour."

"But," said one of the sisters from Poona, "there lies the rub How is a woman to lay down her life? Is it possible for her to do so?"

"Oh!" said Gandhiji, "any day more possible for her than for man I know that women are capable of throwing away their lives for a much lesser purpose. Only a few days ago a young girl of twenty burnt herself to death as she felt she was being persecuted for refusing to go in for ordinary studies And she perished with such cool courage and determination! She ignited her sari with an ordinary oil-light and did not so much as raise a cry, so that the people in the neighbouring room were unaware of the happening until all was over. I do not give these details to commend her example, but to show how easily woman can throw away her life. I at any rate am incapable of this courage But I agree that it is not the external light but the inner light that is needed."

The same sister wondered how one was to avoid anger and violence altogether in dealing with children. "You know our old adage," said Gandhiji laughing heartily, "play with him till he is five, hammer him for ten years, treat him as your friend when he is sixteen." "But," he added, "don't you worry If you have to be angry with your child on occasions, I shall call that anger non-violent anger. I am speaking of wise mothers, not the ignorant ones who do not deserve to be mothers."

Central Teaching of the Gita

The discussion again took a serious turn with a challenging question on the philosophy of the Gita: "Is the central teaching of the Gita selfless action or non-violence?"

"I have no doubt that it is *anasakti*—selfless action. Indeed I have called my little translation of the Gita *Anasakti-yoga* And *anasakti* transcends *ahimsa* He who would be *anasakta* (selfless) has necessarily to practise non-violence in order to attain the state of selflessness. *Ahimsa* is, therefore, a necessary preliminary, it is included in *anasakti*, it does not go beyond it"

"Then does the Gita teach *himsa* and *ahimsa* both?"

"I do not read that meaning in the Gita It is quite likely that the author did not write it to inculcate *ahimsa*, but as a commentator draws innumerable interpretations from a poetic text, even so I interpret the Gita to mean that, if its central theme is *anasakti*, it also teaches *ahimsa*. Whilst we are in the flesh and tread the solid earth, we have to practise *ahimsa* In the life beyond there is no *himsa* or *ahimsa*"

"But," said Balasaheb Kher, "Lord Krishna actually counters the doctrine of *ahimsa*. For Arjuna utters this pacifist resolve.

> Better I deem it, if my kinsmen strike,
> To face them weaponless, and bare my breast
> To shaft and spear, than answer blow with blow.

And Lord Krishna teaches him to answer blow with blow"

What to Do?

"There I join issue with you," said Gandhiji. "Those words of Arjuna were words of pretentious wisdom. 'Until yesterday,' says Krishna to him, 'you fought your kinsmen with deadly weapons without the slightest compunction. Even today you would strike if the enemy was a stranger and not your own kith and kin!' The question before him was not of non-violence, but whether he should slay his nearest and dearest"

Again the questioners came down to solid earth, and began to put questions about the Congress and the attitude of those who believed in complete non-violence. Gandhiji explained that they should refrain till he gave the word He wanted still to plead with the leaders who had passed the Poona resolution.

He expected to show them that the Congress would lose all its prestige if they adhered to the new policy. But the question had to be dealt with patiently On the other hand it did not matter even if the Congress resolution received no response. The resolution was as good as enforced, when it was deliberately passed, and their duty did not alter with refusal of the government to respond. "Besides," he added, "there is an inherent flaw in the Poona resolution It should be obvious to the meanest understanding that, if you think that you cannot do without arms in meeting foreign aggression, they would *a fortiori* be needed in dealing with daily disturbances — internecine feuds, dacoities and riots. For organized unarmed resistance against an organized invasion is any day easier than deliberate *ahimsa* in face of a dacoit who breaks into your house at night That calls for *ahimsa* of the highest type"

Sevagram, 26-8-40
Harijan, 1-9-1940 M. D

133

MY IDEA OF A POLICE FORCE

A friend writes as follows:

> "The English sister, whom you recently quoted, rightly says that efforts should always be made to stem external aggression by non-violent means, and that present circumstances offer a peculiarly suitable opportunity for demonstrating that it is possible to secure better results through non-violence than through armed force But *ahimsa* as a weapon to counter internal disorders presents difficulties. In our country there can be three types of internal disturbances, viz communal, industrial, and dacoities The root causes of these are mutual distrust, social injustice, and grinding poverty due to economic exploitation and unemployment. So long as these causes exist, the threefold disturbances will take place in spite of armed forces Your constructive programme is the only means of removing these root causes. But it will take time What is to be done in the meantime? Can non-violence solve the difficulties?
>
> "Can we envisage an order of society in which we will not have to resort to *himsa* in any form whatsoever? Suppose

for argument's sake that there exists a society where the majority does not possess goods that would excite envy and where everyone has the wherewithal to live contentedly Even then it does not seem possible that there will be no disputes over proprietorship of land, lending and borrowing of money, and other business dealings For these, therefore, we must provide the means of justice and see that the decisions of the courts or arbitration boards are carried out For this it will be essential to have a police force You have yielded this point But I should like to know what restrictions you would place on the police force If there were a non-violent government in power today, would it use the police force for quelling internal disorders? And are you willing to maintain it for all time or only temporarily? My mind refuses to go so far as to envisage a time when a police force will be a superfluity. There seems to me to be no escape from placing this limitation, as it were, on *ahimsa*"

The questions asked in this letter are of the utmost importance and deserve notice If true *ahimsa* had come into being within us, and if our so-called *satyagraha* movements had been truly non-violent, these questions would not have arisen because they would have been solved.

For one who has never seen the arctic regions an imaginary description of them, however elaborate, can convey but an inadequate idea of the reality. Even so is it with *ahimsa* If all Congressmen had been true to their creed, we would not be vacillating between violence and non-violence as we are today. The fruits of *ahimsa* would be in evidence everywhere. There would be communal harmony, the demon of untouchability would have been cast out, and, generally speaking, we should have evolved an ordered society. But the reverse is the case just now. There is even definite hostility to the Congress in certain quarters. The word of Congressmen is not always relied on. The Muslim League and most of the Princes have no faith in the Congress and are in fact inimical to it If Congressmen had true *ahimsa* in them, there would be none of this distrust. In fact the Congress would be the beloved of all

Therefore I can only place an imaginary picture before the votaries of *ahimsa*.

So long as we are not saturated with pure *ahimsa* we cannot possibly win Swaraj through non-violence We can come into power only when we are in a majority or, in other words, when

the large majority of people are willing to abide by the law of *ahimsa* When this happy state prevails the spirit of violence will have all but vanished and internal disorder will have come under control.

Nevertheless I have conceded that even in a non-violent State a police force may be necessary This, I admit, is a sign of my imperfect *ahimsa*. I have not the courage to declare that we can carry on without a police force as I have in respect of an army Of course I can and do envisage a State where the police will not be necessary , but whether we shall succeed in realizing it, the future alone will show.

The police of my conception will, however, be of a wholly different pattern from the present day force. Its ranks will be composed of believers in non-violence. They will be servants, not masters, of the people. The people will instinctively render them every help, and through mutual co-operation they will easily deal with the ever decreasing disturbances The police force will have some kind of arms, but they will be rarely used, if at all In fact the policemen will be reformers. Their police work will be confined primarily to robbers and dacoits. Quarrels between labour and capital and strikes will be few and far between in a non-violent State, because the influence of the non-violent majority will be so great as to command the respect of the principal elements in society. Similarly there will be no room for communal disturbances Then we must remember that when such a Congress government comes into power the large majority of men and women of 21 years and over will have been enfranchised. The rigid and cramped constitution of today has of course no place in this picture.

Sevagram, 20-8-40
Harijan, 1-9-1940

134

NON-VIOLENCE OF THE BRAVE

A correspondent writes :

> "You say non-violence is for the brave, not for cowards But, in my opinion, in India the brave are conspicuous by their absence. Even if we claim to be brave, how is the world to believe us when it knows that India has no arms and is therefore incapable of defending herself? What then should we do to cultivate non-violence of the brave?"

The correspondent is wrong in thinking that in India the brave are conspicuous by their absence. It is a matter for shame that because foreigners once labelled us as cowards we should accept the label. Man often becomes what he believes himself to be. If I keep on saying to myself that I *cannot* do a certain thing, it is possible that I may end by really becoming incapable of doing it. On the contrary, if I have the belief that I *can* do it, I shall surely acquire the capacity to do it even if I may not have it at the beginning. Again it is wrong to say that the world today believes us to be cowards. It has ceased to think so since the *satyagraha* campaign The Congress prestige has risen very high in the West during the past twenty years. The world is watching with astonished interest the fact that, although we have no arms, we are hoping to win Swaraj and have indeed come very near it. Moreover, it sees in our non-violent movement rays of hope for peace in the world and its salvation from the hell of carnage. The bulk of mankind has come to believe that, if ever the spirit of revenge is to vanish and bloody wars are to cease, the happy event can happen only through the policy of non-violence adopted by the Congress. The correspondent's fear and suspicion are, therefore, unfounded.

It will now be seen that the fact that India is unarmed is no obstacle in the path of *ahimsa*, The forcible disarmament of India by the British Government was indeed a grave wrong and a cruel injustice. But we can turn even injustice to our

advantage, if God be with us, or if you prefer, we have the skill to do so And such a thing has happened in India.

Arms are surely unnecessary for a training in *ahimsa*. In fact the arms, if any, have to be thrown away, as the Khansaheb did in the Frontier Province Those who hold that it is essential to learn violence before we can learn non-violence, would hold that only sinners can be saints.

Just as one must learn the art of killing in the training for violence, so one must learn the art of dying in the training for non-violence Violence does not mean emancipation from fear, but discovering the means of combating the cause for fear. Non-violence, on the other hand, has no cause for fear The votary of non-violence has to cultivate the capacity for sacrifice of the highest type in order to be free from fear. He recks not if he should lose his land, his wealth, his life. He who has not overcome all fear cannot practise *ahimsa* to perfection The votary of *ahimsa* has only one fear, that is of God He who seeks refuge in God ought to have a glimpse of the *Atman* that transcends the body; and the moment one has a glimpse of the Imperishable *Atman* one sheds the love of the perishable body. Training in non-violence is thus diametrically opposed to training in violence. Violence is needed for the protection of things external, non-violence is needed for the protection of the *Atman*, for the protection of one's honour.

This non-violence cannot be learnt by staying at home. It needs enterprise In order to test ourselves we should learn to dare danger and death, mortify the flesh, and acquire the capacity to endure all manner of hardships. He who trembles or takes to his heels the moment he sees two people fighting is not non-violent, but a coward. A non-violent person will lay down his life in preventing such quarrels The bravery of the non-violent is vastly superior to that of the violent. The badge of the violent is his weapon — spear, or sword, or rifle. God is the shield of the non-violent.

This is not a course of training for one intending to learn non-violence. But it is easy to evolve one from the principles I have laid down.

It will be evident from the foregoing that there is no comparison between the two types of bravery. The one is limited,

the other is limitless There is no such thing as out-daring or out-fighting non-violence Non-violence is invincible. There need be no doubt that this non-violence can be achieved. The history of the past twenty years should be enough to reassure us.

Sevagram, 27-8-40
Harijan, 1-9-1940

135

SINDH

The position of Congressmen in Sindh is by no means enviable. They have a most difficult time before them. Their non-violence, if they have it in them, has not benefited those who live in fear of their lives. It is true that no one else has helped them. I warned them at the very outset that they must learn the art of helping themselves as others do, or by non-violence as Congressmen are supposed or expected to do In some places they are organizing national guards Those who do, look up to Congressmen for help and guidance For the latter have been their helpers and guides hitherto. Some Congressmen feel that without any intention themselves of taking up arms they can put courage into the people, if they train them in the art of self-defence whether with or without arms. The question has attained importance and demands immediate answer in view of the unequivocal resolution of the A.I.C.C recently held at Poona. I am quite clear that no Congressman, so long as he is even a four anna member of the Congress, can take part in organizing or aiding self-defence groups without committing a breach of the Poona resolution. But I am equally clear that it is the duty of those Congressmen who feel the need for helping self-defence groups and have the capacity for doing so, to go to the rescue of the terror-stricken men. This they can do by resigning their membership of the Congress By doing so they will enhance the prestige of the Congress and

their own usefulness The fact that they feel the call to help is the decisive factor in determining their course of action.

Sevagram, 28-8-40
Harijan, 1-9-1940

136

ANOTHER DISCOURSE

Bharatanandji, whose acquaintance the reader will make in another column, demurred to the compliment given to his countrymen, the Poles, by Gandhiji. "You say that the Poles were 'almost non-violent' I do not think so There was black hatred in the breast of Poland, and I do not think the compliment is deserved."

"You must not take what I say, so terribly literally If ten soldiers resist a force of a thousand soldiers armed cap-a-pie, the former are almost non-violent, because there is no capacity for anything like proportionate violence in them. But the instance I have taken of the girl is more appropriate. A girl who attacks her assailant with her nails, if she has grown them, or with her teeth if she has them, is almost non-violent, because there is no premeditated violence in her. Her violence is the violence of the mouse against the cat"

"Well then, Bapuji, I will give you an instance. A young Russian girl was attacked by a soldier. She used her nails and teeth against him and tore him, so to say, to pieces. Was she almost non-violent?"

"How can it cease to be non-violence, if offered on the spur of the moment, simply because it was successful?" I interposed.

"No," said Gandhiji almost inadvertently

"Then I am really puzzled," said Bharatanandji. "You say there should be no premeditated violence and no capacity to offer proportionate violence. Here in this case she by her success proved that she had the capacity"

"I am sorry," said Gandhiji, "that I inadvertently said 'no' to Mahadev There was violence there. It was equally matched."

"But, then, is not intention ultimately the test? A surgeon uses his knife non-violently, or a keeper of the peace uses force against miscreants in order to protect society That too he does non-violently," said Bharatanandji

"Who is to judge the intention? Not we. And for us the deed in most cases is the test. We normally look at the action and not at the intention. God alone knows the intention"

"Then God alone knows what is *himsa* and what is *ahimsa*."

"Yes, God alone is the final judge. It is likely that what we believe to be an act of *ahimsa* is an act of *himsa* in the eyes of God But for us the path is chalked out And then you must know that a true practice of *ahimsa* means also in one who practises it the keenest intelligence and wide-awake conscience It is difficult for him to err When I used those words for Poland, and when I suggested to a girl believing herself to be helpless that she might use her nails and teeth without being guilty of violence, you must understand the meaning at the back of my mind. There is the refusal to bend before overwhelming might in the full knowledge that it means certain death The Poles knew that they would be crushed to atoms, and yet they resisted the German hordes. That was why I called it almost non-violence"

"But, Bapuji, I cannot somehow forget that it is God who is the judge, and God permits violence. There is a Puranic story I should like to tell you. God Shiva once suddenly disappeared as he and Parvati were in the midst of a conversation But soon he appeared again On being asked where he had been, he said he had gone to the rescue of a *bhakta* who had been attacked, but he had come back on finding that the *bhakta* had helped himself by striking his assailant with a stone."

"Well, well, no amount of argument can teach us *ahimsa*. And you must not forget that one cannot be sure of the purity of one's intention until one has gone through the whole course of spiritual training laid down by masters of *yoga* like Patanjali. Perfect *chittashuddhi* (purification of mind) cannot be achieved in any other way"

Here Bharatanandji seemed to be at one with Gandhiji, and he agreed that anyone might easily deceive himself. But he came with another poser "*Ahimsa*, *brahmacharya*, spinning are all *sadhanas*," he said, "and whereas one may suit one, it

may not suit another Why have you made of *ahimsa* a universal precept ? "

"When a means has been tried by a scientist and he has found it of infallible effect he puts it before all You know the maxim ' यथा पिण्डे तथा ब्रह्माण्डे.' What is true of the individual is true of the universe."

" But you lay down the same law for a saint and a robber ! "

" The law is the same , the way may be more difficult for the robber than for the saint. The law is the ideal, no matter how much individuals may fall short of the ideal."

" But you forget the reality before the ideal."

" No. The reality is always present before me, but my striving is always to reach the ideal. Euclid's straight line exists only in our conception, but we have always to postulate it. We have always to strive to draw a true line corresponding to Euclid's imaginary line "

As I listened to this I was put in mind of the exactly similar words of Carlyle. " Ideals can never be completely embodied in practice. And yet it is never to be forgotten that ideals do exist , that if they be not approximated to at all, the whole matter goes to wreck ! Infallibly No bricklayer builds a wall *perfectly* perpendicular ; mathematically this is not possible , a certain degree of perpendicularity suffices him, and he, like a good bricklayer who must have done with his job, leaves it so. And yet if he sways *too much* from the perpendicular , above all if he throws a planet and level quite away from him, and piles brick on brick heedless, just as it comes to hand — such a bricklayer is in a bad way He has forgotten himself ; but the law of gravitation does not forget to act on him , he and his wall rush down into confused welter of ruin."

Sevagram, 2-9-40 M. D.

Harijan, 8-9-1940

HOW TO QUENCH IT?

Elsewhere in these columns the reader will see "A Seeker's" letter * in which he has asked a question which must have occurred to everyone The beauty lies in the way in which he has introduced the question. He has depicted the present conflagration in such lurid colours that violence cannot but stink in our nostrils The reader is sure instinctively to exclaim: "Even if it were possible to win the kingdom of the world by means of such violence, I would not have it"

But this exclamation will be of no avail to quench the conflagration. No doubt it will some day quench itself, but it means mutual fratricidal slaughter like that of the Yadavas of

---

* "A SEEKER'S" QUESTION

The Editor, *Harijan*

Sir,

You must be reading in the papers how the war between Germany and England is being waged Aeroplanes filled with thousands of incendiary bombs do untold havoc, and newspapers and broadcasts describe with pleasure the amount of injury each side is able to inflict on the other The general public is consoled by being told that the damage done in the enemy country is greater than what the enemy has done in theirs It is said that military objectives are the sole targets of the raiders · but it is impossible to believe that, flying at great heights and often through smoke screens, the bombers can really take proper aim And then we hear from both sides of the 'successful' blockades, the object of which is to spread famine; and famine must necessarily, more than even bombs, hurt the civilian population

Is it impossible for these belligerents to think in terms of humanity and stop this carnage? How can any good ever come out of war? And must we not, therefore, declare ourselves unreservedly against war for or against anybody or any ideology?

There must be many godly people in the warring countries who think in this way but have not the strength to raise their voice in protest. May we not help them to do so and at the same time rouse the sleeping conscience of all thinking people?

Yours

A Seeker

old who destroyed themselves and relieved the earth of so much burden. And such a consummation would any day be preferable to a perpetual conflagration. But no one would wish for this. What one would devoutly wish for is some brave step to stop the conflagration before there is total destruction This can only be a non-violent step. How and when it can be taken has to be discovered The "Seeker" will be satisfied when the discovery is made In my opinion the discovery has already been made. If India can win Swaraj non-violently even while this conflagration is going on, the latter is bound to be extinguished by that one event That being my firm faith I fought the Wardha resolution tooth and nail, and secured my freedom — not indeed to tickle my vanity but for the success of the experiment And if I have to forego this freedom — as is quite likely — it will be for the same purpose

We read in our religious books that whenever, in the days of old, all ordinary means failed to secure release from an ordeal or a calamity, people resorted to *tapasya* (penance), i e actually burnt themselves. I do not regard these stories as legendary. *Tapasya* is of various kinds Misguided men can resort to it, as we find them doing today. The wise also can do it. It is worth while understanding the implication of *tapasya*. It was by dint of *tapasya* that Western scientists made their discoveries *Tapasya* does not simply consist of betaking oneself to the forest and sitting down there surrounded by blazing fires. That *tapasya* may even be the height of folly. We have, therefore, to discriminate

The question asked by "A Seeker" does not arise out of despair. It is intended to quicken the conscience of those who believe in *ahimsa* I have already shown the way It is the fulfilment of the thirteenfold constructive programme described in a recent article. Those who will carry it out in faith, in full knowledge, and without the slightest fuss, will have done their share in the *tapasya* to quench the conflagration. They will achieve two ends at the same time They will make India free, and will also quench the conflagration. It is likely that the number of such people is limited, so limited that it can have no effect. I have maintained that, even if there is one individual who is almost completely non-violent, he can put out the conflagration. But I have suggested a *tapasya* which can easily be performed by the

average individual In this age of democracy it is essential that desired results are achieved by the collective effort of the people. It will no doubt be good to achieve an objective through the effort of a supremely powerful individual, but it can never make the community conscious of its corporate strength. An individual's success will be like a millionaire doling free food to millions of starving people. We should, therefore, bend our energies to a fulfilment of the thirteenfold constructive programme. It may or may not bring Swaraj, but we shall surely have the satisfaction of having done our best

There is a warning in the "Seeker's" letter to which I should like to draw the reader's attention He says papers and broadcasts describe with devilish pleasure the amount of injury each is able to inflict upon the other, and suggests that such news should sicken people instead of providing pleasure, if they are to take part in the propagation of peace. I agree Such people will not be able to carry out even the constructive programme, for they will have no faith in it

However that may be, it is as clear as daylight that, if this conflagration is to be put out through non-violent effort, it will be done only by India.

Sevagram, 2-9-40
Harijan, 8-9-1940

CONGRESS AHIMSA

I am being inundated with letters complaining that by giving my definition of *ahimsa* before the Poona Congressmen, who saw me recently at Wardha, I have seriously circumscribed the scope of *ahimsa*. These friends forget that my remarks were confined to Congress *ahimsa* only. Personally I would not kill insects, scorpions or even snakes. Nor would I under any circumstance take meat. But I may not impose the creed of such *ahimsa* on the Congress. The Congress is not a religious institution, it is a political organization Its non-violence is limited to human beings. If it were to be further extended, only Hindus, and among even them only Vaishnavas and Jains, would be left to participate in it Millions of Hindus who eat fish and meat would be excluded My proposition, to my mind, is so simple and straight that I never thought that anybody could object to it

My critics should further understand that many Mussalmans have not accepted even the limited *ahimsa* of the Congress as their creed, and that the Congress itself has, by its Wardha and Poona resolutions, so cramped its scope as to render it almost meaningless It follows, therefore, that any attempt to widen its scope as to include the sub-human species will defeat its purpose Unlimited *ahimsa* will take time to be universalized. We will have ample cause to congratulate ourselves, if we learn to substitute the law of love in society for that of the jungle, and if, instead of harbouring ill-will and enmity in our bosoms against those whom we regard as our enemies, we learn to love them as actual and potential friends It should be remembered too that mere *jivadaya* (kindness to animals) does not enable us to overcome the 'six deadly enemies' within us, namely lust, anger, greed, infatuation, pride and falsehood Give me the man who has completely conquered self and is full of goodwill and love towards all, and is ruled by the law of love in all his actions, and I for one will offer him my respectful homage even though he be a meat-eater. On the other hand the *jivadaya* of a person who is steeped in anger and lust but daily

feeds the ants and insects and refrains from killing has hardly anything in it to recommend itself. It is a mechanical performance without any spiritual value. It may even be worse — a hypocritical screen for hiding the corruption within.

Sevagram, 9-9-40
Harijan, 15-9-1940

139

NON-VIOLENCE DURING RIOTS

A friend writes

> "How can non-violence be efficacious during riots? By self-immolation we can influence only those with whom we have already established living contacts But the hooligans who perpetrate violence during riots are, as a rule, hirelings imported from outside How can they have any scruples about hurting those whom they have never known before and for whom they can have no regard or consideration?"

The question deserves careful consideration. The friend who has put it is a valiant worker who nearly lost his life in trying to do his duty during a riot. I have often written on this question before The pity of it is that Congressmen have never seriously thought over the question of finding a non-violent way of quelling riots. Their non-violence was restricted to the sole purpose of offering civil resistance to the authorities. In my opinion the non-violence that goes so far and no further scarcely deserves the name '*ahimsa*'. You may, if you like, call it unarmed resistance So far as it is a device for embarrassing the Government it is a species of *himsa*. To quell riots non-violently, there must be true *ahimsa* in one's heart, an *ahimsa* that takes even the erring hooligan in its warm embrace. Such an attitude cannot be cultivated. It can only come as a prolonged and patient effort which must be made during peaceful times. The would-be member of a peace brigade should come into close touch and cultivate acquaintance with the so-called *goonda* element in his vicinity. He should know all and be known to all and win the hearts of all by his living and selfless service. No section should be regarded as too contemptible or mean to mix

with *Goondas* do not drop from the sky, nor do they spring from the earth like evil spirits They are the product of social disorganization, and society is therefore responsible for their existence In other words, they should be looked upon as a symptom of curruption in our body politic. To remove the disease we must first discover the underlying cause To find the remedy will then be a comparatively easy task So far we have not even attempted a proper beginning. But it is never too late to mend It is enough that we are at last alive to the necessity of it. We have now to follow it up with prompt action. Let everyone who is interested make a prompt beginning in his own neighbourhood. The difficulty mentioned by my correspondent will automatically resolve itself, if we proceed with our effort in the right spirit

Sevagram, 9-9-40
Harijan, 15-9-1940

140

SPEECH AT THE A. I C. C

[Here is the full text of Gandhiji's English speech at the A I. C. C meeting in Bombay on 15th September It is revised by Gandhiji and is thus an authorised version M D]

I know that you have listened to me with the greatest patience I am specially grateful to you today, for the simple reason that I have said many things which may have displeased you. But it was never my intention to displease those whom I want to harness for the great work that lies before you and before me. I have to speak to you at length because I have to shoulder this burden. I have not come with a prepared speech at all. The thoughts will come as I proceed

Let me begin with a thought which has been weighing with me for a considerable time When the war broke out and I went to Simla to see the Viceroy, I issued a statement the next day, not in a representative but in my individual capacity. A friend has now reminded me how good it would have been if I had simply hung on to that statement although I could not take the Congress with me; and on the eve of my shouldering this respon-

sibility, he prayed that I should be guided by God to take up that original position and retire. I have very great regard for him. I have not forgotten that statement, nor have I any regret or apology to offer. If such a thing occurs — and history often repeats itself—and I happen to go to another Viceroy, I should make the same statement.

Although I spoke only for myself, deep down in me there was the Congressman speaking. The Viceroy also did not send for me because I was M. K. Gandhi. M. K Gandhi has absolutely no place in his books The man who wields the sceptre can have no room for individuals. He sent for me because he thought I would represent the Congress view and I would be able to carry conviction to Congressmen.

I withdrew from that position, not as an individual but as a Congressman, and because I failed to carry conviction even to a single Congressman. Happily you have got on the Working Committee men with sterling honesty who had the courage to tell me that, although it was my statement, they did not feel like accepting it. They added that they had had bitter experience behind them, and that therefore they would not be able to take that position. Thus you had the resolution that was passed by the Congress immediately after the war. And I agreed with the resolution as a representative, although I said to them that, if I could carry conviction, my original position was the best possible one to take up. If I had pressed the members of the Working Committee to accept my position, they would have done so, but it would have been only mechanical. The statement was not made to deceive the Viceroy or, for the matter of that, a single soul. It came straight from the heart It was not a theatrical display It was the opening up of the secret recesses of the heart before the world, the Viceroy and the Congress If these words of mine could not find an echo in their hearts, they would have been of no use whatsoever to the Viceroy, to the great English nation or to India That still remains my sentiment If I could not convince the Congress of my attitude, it would not carry us further. It would have been a wrong step to take, and hence it was not taken. With that background I approach this resolution.

I have made repeated statements that I would not be guilty of embarrassing the British people or the British Government when their very existence hung in the balance, that I would not

be true to my *satyagraha*, would not be true to non-violence, would not be true to the truth which I hold dear if I did so, and therefore could not do so. That very man now stands before you to shoulder the burden of *satyagraha* Why ? There comes a time when a man in his weakness mistakes vice for virtue, and virtue itself, when taken away from its context and from the purpose for which it was dedicated, becomes vice I felt that, if I did not go to the assistance of the Congress and take the helm even if it be in fear and trembling, I would be untrue to myself

I feel that in taking the step that we are doing we are rendering a great service not only to the Congress but to the whole of India. And we are rendering a service not only to the whole of India History will record — and Englishmen will be able to grasp this statement some day — that we rendered help to the English nation, and they will find that we were true to our salt and had the same bravery and fearlessness of which the Englishman is proud and for which he is renowned I, who claim to be a fast friend of the British people, will be guilty of unfriendly conduct if, under a false sense of modesty, or because people may think otherwise about me, or because Englishmen themselves will be angry with me, I do not issue a warning that the virtue of self-restraint now becomes vice, because it will kill the Congress organization, and it will kill the very spirit which is exercising this restraint

When I say, I am speaking not only for the Congress, but for all who stand for national freedom — Muslims, Parsis, Christians and even those who are against the Congress, so long as they represent the aspiration of India, namely unadulterated independence. I should be untrue to all of them, if at this time I said, 'no embarrassment to the British' I must not repeat parrot-like 'no embarrassment' Then that repetition would be just as useful for my salvation or for the guarding of my virtue as the repetition by a parrot of God's name which cannot bring him salvation, because it is only a mechanical and vocal effort without any intelligence behind it. Therefore, if I exercise that suppression at this critical moment in the history of the nation, it would be useless. I should be perfectly untrue to myself if I hid myself in Sevagram and said, "No, I have told you, 'no embarrassment' "

The language of this resolution is in the main mine. It

appealed to Pandit Jawaharlal Nehru I used to be the Congress draftsman Now he has taken my place He saw it was inevitable, if we were to be true to non-violent resistance to the extent to which we wanted to go. The Working Committee has accepted this phraseology deliberately, well knowing its implications The result is this: If we can get a declaration from the British Government that the Congress can carry on anti-war propaganda, and preach non-co-operation with the Government in their war effort, we will not have civil disobedience.

I do not want England to be defeated or humiliated. It hurts me to find St. Paul's Cathedral damaged. It hurts me as much as I would be hurt if I heard that the Kashi Vishvanath temple or the Juma Masjid was damaged. I would like to defend both the Kashi Vishvanath temple and the Juma Musjid and even St. Paul's with my life, but would not take a single life for their defence. That is my fundamental difference with the British people. My sympathy is there with them nevertheless. Let there be no mistake on the part of Englishmen, Congressmen, or others whom my voice reaches, as to where my sympathy lies. It is not because I love the British nation and hate the German I do not think that the Germans as a nation are any worse than the English or the Italians are any worse We are all tarred with the same brush; we are all members of the vast human family. I decline to draw any distinctions. I cannot claim any superiority for Indians. We have the same virtues and the same vices. Humanity is not divided into watertight compartments so that we cannot go from one to another. They may occupy one thousand rooms, but they are all related to one another I would not say, 'India should be all in all, let the whole world perish.' That is not my message. India should be all in all, consistently with the well-being of other nations of the world I can keep India intact and its freedom also intact only if I have goodwill towards the whole of the human family and not merely for the human family which inhabits this little spot of the earth called India. It is big enough compared to other smaller nations, but what is India in the wide world or in the universe?

Let there be no mistake as to what I am about I want my individuality to remain unimpaired. If I lose it, I would be of no service to India, much less to the British people, still less

to humanity. My individual liberty is the same as the nation's, convertible with national liberty. I do not claim any greater liberty for myself. Hence my liberty is equal to the liberty of all of you and no greater. I feel that, if my liberty is at stake, yours is also at stake. I claim the liberty of going through the streets of Bombay and say that I shall have nothing to do with this war, because I do not believe in this war and in this fratricide that is going on in Europe. I admire the bravery. But what is the use of this bravery? I deplore the foolishness and the crass ignorance. These people do not know what they are fighting for. That is how I look at this war that is going on across the seas I cannot possibly take part in it. Nor do I want the Congress to do so

The part that I would like to take is the part of peacemaker. If the British people in their wisdom had recognized the independence not of the Congress but of all India, and if other parties in India had also co-operated with us, we would have taken the honourable place of peacemakers between these nations. Such is my ambition. But today I know that it is a day-dream. But sometimes a man lives in his day-dreams I live in mine, and picture the world as full of good human beings — not goody goody human beings In the Socialist's language, there will be a new structure of society, a new order of things. I am also aspiring after a new order of things that will astonish the world. If you try to dream these day-dreams, you will also feel exalted as I do.

And now I come to our 'tin-pot' Congress — tin-pot in the estimation of others, not my own If we do not take care, the Congress will disappear, and if the Congress disappears, the national spirit disappears. One after another Congressmen are being selected and jailed It is not *satyagraha* to watch people being taken away. It is much better for all of us to rush into the jaws of the opponent. After all, as the Maulana Saheb once said, India is a vast prison Let us get out of this slave-prison by breaking the prison bars. He said to the Sikhs at the time of the Nankana Saheb tragedy: "You may protect one *gurudwara*: but what about vast *gurudwara* that is India? We have to deliver it from bondage" Those words ring true even today in my ears. If this liberty of the nation or the movement for freedom is likely to be choked, then I say that the virtue of self-

restraint is going to become a vice. That virtue of restraint cannot be carried to the extent of the extinction of the national spirit wherever it may reside, whether among Congressmen or non-Congressmen.

I do not want to hurl civil disobedience or anything in the face of the Government without making my meaning clear, the meaning I attach to the sum total of Government actions—actions beginning with the declaration of the Viceroy, the statement of the Secretary of State for India, and the series of actions and the policy that the Government have pursued since. The sum total of all these has left an indelible impression on my mind that there is something wrong, some injustice being perpetrated against the whole nation, and that the voice of freedom is about to be stifled This is implied in the resolution, not in the exact language which I am using now, but you will see the meaning clear as daylight.

In order completely to clarify our position, I propose to approach the Viceroy with a request that he will be good enough to see me, and I have no doubt that he will I will place my difficulties before him, I will place the Congress difficulties before him I will approach him in your name. I will tell him that this is the position to which we have been reduced. We do not want to embarrass you and deflect you from your purpose in regard to war effort We go our way, and you go yours, undeterred, the common ground being non-violence. If we carry the people with us, there will be no war effort on the part of our people If, on the other hand, without your using any but moral pressure, you find that the people help the war effort, we can have no cause for grumbling If you get assistance from the Princes, from the Zamindars, from anybody high or low, you can have it; but let our voice also be heard. If you accept my proposal, it will be eminently honourable, it will certainly be a feather in your cap It will be honourable of you, although you are engaged in a life and death struggle, that you have given us this liberty. It will be honourable of you that you take this great step, although you have limitless powers to choke our voice; and that you give us the fullest possible freedom, consistently with the observance of non-violence, to tell the people of India not to join the war effort.

Let the people use any reasoning they like for refusal to help the war effort. My reasoning is the only one which will sit well on Congressmen's lips. But I do not expect all to restrict themselves to that reasoning. Those who have conscientious objection, as I have, will adopt my reasoning. Those who are tired of British imperialism will use that argument. There may be others who will have other arguments. All these should be covered under this freedom of speech, provided, however, that they all accept non-violence, provided also that what they say is said openly and not secretly. These are the implications of my generalship. If these do not satisfy you, you must reject this resolution summarily. So long as you can preach non-co-operation with war effort in men and money, there should be no civil disobedience. But if you have not that liberty, there is no Swaraj but perpetual bondage. I would like the British people and the Viceroy to be able to tell the world that they have given the leaders of the Indian people liberty to preach to their people what they like. The British can then say to the world: "Judge us by our conduct. Here in India we are playing the game."

I do not mind the British not responding to the Delhi resolution. They may say, "At the present moment you cannot interfere with the management of affairs as they stand. Deliverance will come to you in its own time. At this critical juncture do not worry us." I will understand that argument. I will sympathize with it. I will hold my hand so long as there is no fraud or falsity in what they say. It is impossible for them to give us freedom. If freedom has got to come, it must be obtained by our own internal strength, by our closing our ranks, by unity between all sections of the community. It cannot descend from heaven, nor can it be given as a gift from one nation to another. I do not know whether I am representing the feelings of the members of the Working Committee, because I have not discussed these things with them. But you have to take me with all my limitations, with the workings of my mind.

The Viceroy may say, "You are a visionary." I may fail in my mission, but we will not quarrel. If he says he is helpless, I will not feel helpless. I will make good my position. I cannot sit still when I see Ram Manohar Lohia and Jaiprakash Narain in jail, than whom I do not know braver or straighter men. They

have not preached violence, but simply carried out the behests of the Ramgarh resolution. It was a point of honour with them.

I have restrained myself, and will restrain myself I will not seek imprisonment. I do not want to offer civil disobedience. I will not place myself in peril. In this battle I will not expose myself to imprisonment. But if the Government chooses, it will not be difficult to take me away. I will not be able to seal my lips or restrain my pen It will be difficult for them to keep me in prison, not because India will rise in rebellion India will be wrong, if it does so. My own instinct is that they will not be able to keep me in jail.

I will place my argument before the Viceroy I may fail in my mission. But I have never approached a mission in despair. I may have approached it with the consciousness that I may be faced with a blind wall. But I have often penetrated blind walls I shall approach the Viceroy in the confidence and hope that he will understand the great reasonableness of the request of the Congress for full liberty to preach 'no war' in India. Every one should have perfect liberty to preach by pen and tongue: 'We cannot aid imperialism, we cannot help spoliation'

I shall strain every nerve to avoid *satyagraha* in your name. What shape it will take, when it comes, I do not know. But I know that there will be no mass civil disobedience, because mass civil disobedience is not required for this occasion. I have impenetrable darkness before me regarding the future course of action. I have no mysteries I do not know how I shall lead you, what action I shall put before you. I hope that any action that we may take will be worthy of the Congress traditions and of the occasion.

I have often said that I do not know the Congress mind as I have buried myself in Sevagram It is because of the Congress difficulty that I have dragged myself to Bombay, and immediately I am released from this duty you will find me in Sevagram. But I have got strength and resourcefulness enough to lead this battle, although I am buried in Sevagram I shall do better and clearer thinking in Sevagram than anywhere else, simply because I have built up there an atmosphere for my growth With the march of time my body must decay but, I hope, not my wisdom. I seem to see things more clearly with

the advance of age It may be self-deception, but there is no hypocrisy. Self-deception is good sometimes in that it helps one to remain cheerful and not to give way to despair It will be, therefore, wrong of you to drag me from Sevagram; and I promise that I shall give a good account of my stewardship

There are many parties in the Congress We are not all of the same opinion. There is indiscipline in the Congress. I know it is inevitable in a mass organization which is growing from day to day. If it is all indiscipline and no discipline, the organization is on the downward path. Let it not be said of you that you come to the Congress although you do not believe in non-violence. How can you possibly sign the Congress pledge with violence in your breasts? I want complete obedience to the policy of non-violence. While the policy lasts, it is the same as though it was a creed, for so long as it holds good it is as good as a creed. My creed holds me for life; yours so long as you hold it. Resign from the Congress, and you are free from it. Let us be clear regarding the language we use and the thoughts we nurture. For, what is language but the expression of thought? Let your thought be accurate and truthful, and you will hasten the advent of Swaraj even if the whole world is against you You will have won Swaraj without having to spend nine million pounds a day or without burning a single home. If you are true to your policy, I am sure that without doing any of these things you will build up the majestic edifice of freedom

Now for the violence party Do not mix up the methods, if you can help it. You have restrained yourselves for some years Restrain yourselves for some more years. Ours is not a small battle If you restrain yourselves, you will lose nothing.

Freedom of speech and pen is the foundation of Swaraj If the foundation-stone is in danger, you have to exert the whole of your might in order to defend that single stone. May God help you.

Harijan, 29-9-1940

141

I WAS UNJUST BECAUSE WEAK

I know Rajaji enough to understand that he is too brave to need any support from anybody, he is too philosophic to harbour an injury for many hours, if not minutes I know also that his fine sense of humour enables him to enjoy a joke at his expense. Therefore this confession must be taken as one for my own satisfaction only.

I have told the public that, had I not egged him on, Rajaji would never have brought forward his resolution at New Delhi. Having great regard for his judgment and his honesty, when he asserted with amazing assurance that I was wrong and he was right in the implications and application of non-violence, I allowed myself to doubt the correctness of my interpretation to the point of allowing and encouraging him to act on his. I showed weakness and became unjust to him. A weak man is just by accident. A strong but non-violent man is unjust by accident. I was unjust to Rajaji because I exposed him to ridicule and unkind attacks Though no ultimate harm has come to the Congress because what I still consider was an error has been rectified, it is not a good thing for a great leader to have his work undone all of a sudden, for I know that Rajaji still feels that he was right. If his view had prevailed, the resolution that now holds sway would not have taken the shape it has I would still have been out of the Congress. For I was out of it at Wardha before the Delhi resolution was taken as the natural outcome of Wardha.

If I was unjust to Rajaji, I was also unjust to the Working Committee For had I remained firm, the Wardha resolution too would not have been passed. I hold that, so long as I am accepted as the sole authority on *satyagraha* and its implications, they must not be a matter of vote. My colleagues may debate the pros and cons with me and try to convince me that their interpretation is right. If I cannot accept it, my judgement should prevail, because I am both the author of *satyagraha* and general in *satyagraha* action The only way the colleagues can

avoid my judgment is by absolving me from guidance. They did in so many words at Wardha. But it is plain that the absolution was not what the world means. They were most unwilling to give me absolution. It was given because I wrung it from them. My weakness began at Wardha. When a serious crisis arose, I should have raised the issue of jurisdiction. It was outside the Working Committee's jurisdiction to decide upon the meaning and application of a matter which belonged to their expert who was their interpreter and executive officer.

I am aware that all the members of the Working Committee do not accept my opinion as to jurisdiction. The matter has not come up for decision. But before the Committee and I came to the resolution now before the country, I had made the confession I have now published for the sake of an esteemed co-worker.

It is my conviction that, owing to a series of fortunate combination of acts of the members at the last Wardha meeting, the present resolution was conceived, and we have been saved from a national disaster We have come to a decision which, if Congressmen react to it as they should, must raise India to a position which it has never yet occupied, and brings it nearer to her goal as nothing else could have done.

Whether my estimate is right or wrong time alone can show. But this is merely by the way. Nor is the purpose of this confession to invite the reader to accept my judgment as to the jurisdiction of the Working Committee The mention of it was relevant to show the nature of my error It is unpardonable for a general to surrender his judgment to a fellow officer unless the conviction goes home to him that the latter *is* right — not *may be* right.

I hope I have given the public enough material to show that in all that Rajaji did he was throughout brave and correct The incorrectness was due to me.

And I wish to say the same thing about his 'sporting offer'. It is no part of this confession to defend it. But so far as I can see, the offer was truly sporting, if the correctness of the Poona resolution be accepted It should be remembered that the Muslim League is a great organization wielding influence upon the Muslims of India. The Congress has dealt with it before, and I have no doubt it will deal with it in future However mistaken Quaid-e-Azam may be in our estimation, let us give him the

same credit for honesty of purpose as we claim for ourselves When the war cloud is lifted and when India comes to her own, surely Congressmen would just as much welcome a Muslim, a Sikh, a Christian, or a Parsi as Premier, as they would a Hindu, and even a non-Congressman, no matter of what faith he may be. I am sure that Rajaji's 'sporting offer' meant no more and no less than this. When passions have died, critics will read his offer in its proper light. It is wrong to misjudge a public worker, and doubly so when he happens to be of Rajaji's calibre. He has lost nothing by the misjudgment. But a nation may easily harm itself by misjudging its true servants and denying itself their services Above all, when the Congress may have to embark upon a great non-violent struggle for fundamental liberty, it behoves Congressmen to guard against harsh, hasty and uncharitable judgments.

On the train to Wardha, 18-9-40
Harijan, 22-9-1940

142

SOME CRITICISM ANSWERED

The recent resolution of the A. I. C C. and my speeches before the meeting have come in for much criticism, if they have earned some praise. Of the criticism there are two items which I must attempt to answer For they are of permanent interest *The Times of India* of the 17th inst. in its temperate criticism takes me to task for my statement that 'the peoples of Europe do not know what they are fighting for'. It was bound to be resented. But truth, though seemingly harsh, has to be uttered when utterance becomes relevant, indeed imperative. I believe that the utterance had become overdue. I must say why I think that the warring nations do not know what they are fighting for. I had used the expression 'warring nations', not 'peoples of Europe'. This is not a distinction without a difference. I have distinguished between the nations and their leaders. The leaders of course know what they are fighting for. I make no admission that they are right. But neither the

English nor the Germans nor the Italians know what they are fighting for except that they trust their leaders and therefore follow them. I submit that this is not enough when the stake is so bloody and staggering as during the present war. It is perhaps common cause that Germans and Italians do not know why English children should be slaughtered in cold blood and beautiful English homes should be destroyed. But *The Times*' claim probably is that the British *people* know what they are fighting for When I asked the British soldiers in South Africa during the Boer War they could not tell me what they were fighting for. 'Theirs' was surely 'not to reason why'. They did not even know where they were being marched to The British people would not be able to give me a more satisfying answer, if I happened to be in London and asked them why their soldiers were working havoc in Berlin. If the Press accounts are to be relied upon, British skill and valour have wrought more havoc in Berlin than have the Germans in London What wrong have the German people done to the British people? Their leaders have. Hang them by all means, but why destroy German homes and German civilian life? What difference does it make to the dead, the orphans and the homeless, whether the mad destruction is wrought under the name of totalitarianism or the holy name of liberty or democracy? I assert in all humility, but with all the strength at my command, that liberty and democracy become unholy when their hands are dyed red with innocent blood. I hear the living Christ saying: "These so-called children of mine know not what they are doing They take my Father's name in vain, for they disobey the central command of my Father!" If my ears do not deceive, I have erred in good company, if I have erred at all

And why have I uttered the truth? Because I am confident that God has made me the instrument of showing the better way. If Britain seeks justice, she must appear before the imperial court of God with clean hands She will not defend liberty and democracy by following totalitarian methods so far as war is concerned. She will not be able to retrace her steps after out-Hitlering Hitler in war. The last war is a resounding lesson. Her victory, if attained, will be a snare and a delusion. I know mine is a voice in the wilderness. But it will some day ring true. If liberty and democracy are to be truly saved, they

will only be by non-violent resistance no less brave, no less glorious, than violent resistance. And it will be infinitely braver and more glorious because it will give life without taking any.

Now I come to *The Statesman's* article of 18th September. I am sorry to say it is intemperate and written in anger It is full of palpable mistakes (to use a mild word), no doubt unconscious. But I am not concerned with the intemperance of language. In the fierce heat of battle unknown before, the wonder is that the intemperance one sees sometimes is not much greater than it is.

Here is the cream of the heavy indictment .

> "We have often expressed our view about the fundamental immorality and contradictory character of the doctrine Non-co-operation is a method of war and not of peace . It carries with it a pretentious claim to spiritual value which involves sanctimonious insincerities and mass hypocrisy masking intensified hatreds. . . A nation which accepted this doctrine would doom itself to slavery."

This is all contrary to the history of our own times in India. I claim that there is nothing immoral in non-violent non-co-operation. Violent resistance is itself non-co-operation, and it is immoral because of its violence. It becomes moral when it is non-violent. Non-co-operation with evil is a sacred duty. It is essentially spiritual because of its non-violent character The adjectives used by the writer would be deserved, if it was non-violent in name only. For the present argument I must take the genuine article. Now for the facts. Non-violent non-co-operation, however imperfect it was, has redeemed India at least somewhat from the slavery under which she was groaning. It has raised India from the slough of despond, and has brought her prestige which nothing else could have. I make bold to say that, if the non-violence offered had been not adulterated, its effect would have been still more visible My greatest claim, however, is that it is this despised non-violent resistance which hitherto saved India from anarchy and red ruin. It is not yet entirely saved. If it is to be saved, it will only be by the non-violent method. I invite *The Statesman* writer to test the truth of my statement. He will have many infallible proofs in its

support. A dispassionate study will enable him to serve both Britain and India.

On the way to Simla, 25-9-40
Harijan, 29-9-1940

143

VICEROY-GANDHI CORRESPONDENCE

The correspondence that passed today between the Viceroy and Gandhiji on the Congress demand for freedom of speech has been released to the Press (Simla, September 30) .

Viceroy's Letter

Dear Mr Gandhi,

I think it will be convenient if I record very briefly in writing the origin of the talks we have had on the 27th and 30th September and their outcome.

As you will remember, you wrote to me on the 18th September to ask that I should grant you an interview, and you explained in your letter that you were anxious to discuss the situation covered by the recent resolution of the All-India Congress Committee, not only in your capacity as guide of the Congress but as a personal friend. I was, I need not say, most ready to talk things over with you, and we have now had the advantage of two conversations.

In the course of these conversations the situation has been exhaustively discussed, with particular reference to the question of free speech in time of war. On that matter, while professing yourself most anxious to avoid in any way embarrassing His Majesty's Government in the prosecution of the war, you made it clear to me that you regarded it as essential that the Indian National Congress and other members of the public should be in a position to give full expression to their views in relation to the war effort, provided only that such expression was fully non-violent.

I indicated to you the nature of the special treatment laid down by law in the United Kingdom for dealing with conscientious objector — which I may broadly describe as an arrangement

under which, while the conscientious objector is absolved from the duty of fighting and is allowed even to profess his faith in public, he is not permitted to carry his opposition to the length of endeavouring to persuade others, whether soldiers or munition workers, to abandon their allegiance or to discontinue their effort.

You made it clear to me that you would not regard treatment of that nature as adequate in the conditions of India, and that you regarded it as essential that in India, where in your judgment conditions were wholly different from those existing in Great Britain, the Indian objector, either to all war as such, or to the participation of India in the present war, should be untrammelled in the expression of his views.

It emerged further from our conversation that, while you would not yourself preach to workers engaged on war work at the actual works, in an endeavour there to dissuade them from working on war equipment, you would regard it as essential that it should be open to Congressmen and non-Congressmen alike to deliver addresses and otherwise to call upon people throughout the country to refrain from assisting India's war effort in any way which would involve India's participation in bloodshed

I listened with the utmost care and attention to your argument, and our examination of the situation has been full and close I felt bound, however, in the outcome, to make it clear to you that action such as you suggest would certainly amount not only to the inhibition of India's war effort, but to that embarrassment of Great Britain in the prosecution of the war which the Congress state that they are anxious to avoid, and that it would clearly not be possible in the interests of India herself, more particularly at this most critical juncture in the war, to acquiesce in the interference with the war effort which would be involved in freedom of speech so wide as that for which you had asked.

Gandhiji's Reply

Dear Lord Linlithgow,

I have your letter of even date It fairly sets forth the Congress position as I placed it before you. It is a matter of deep regret to me that the Government have not been able to

appreciate the Congress position, meant just to satisfy the bare requirements of the people, whether Congressmen or others, who felt a conscientious objection to helping a war to which they were never invited and which they regard, so far as they are concerned, as one for saving Imperialism, of which India is the greatest victim. Their objection is just as conscientious as mine as a war resister I cannot claim greater freedom for my conscience than for that of those I have named

As I made it plain in the course of our talks, the Congress is as much opposed to victory for Nazism as any Britisher can be But their objection cannot be carried to the extent of their participation in the war And since you and the Secretary of State for India have declared that the whole of India is voluntarily helping the war effort, it becomes necessary to make clear that the vast majority of the people of India are not interested in it They make no distinction between Nazism and the double autocracy that rules India. Had His Majesty's Government recognized the freedom required in the special condition of India, they would have justified the claim that they were receiving from India only such effort as they could voluntarily. The war party and the no-war party would have been placed on an equal footing so far each worked fully non-violently.

As to the last paragraph of your letter, I wish to remind you that it was never contemplated to carry non-embarrassment to the point of self-extinction or, in other words, stopping all national activities which were designed to make India peace-minded and show that India's participation could not benefit anyone, not excluding Great Britain. Indeed I hold that, if India were left free to make her choice which freedom of speech implied, India would probably have turned the scales in favour of Britain and true liberty by the moral prestige which Britain would have then gained.

I must, therefore, repeat that the Congress does still want to refrain from embarrassing the British Government in their war effort But it is impossible for the Congress to make of the policy a fetish by denying its creed at this critical period in the history of mankind. If the Congress has to die, it should do so in the act of proclaiming its faith. It is unfortunate that we have not been able to arrive at an agreement on the single issue of freedom of speech But I shall hug the hope that it will be

possible for the Government to work out their policy in the spirit of the Congress position

I should like to touch upon the other points I raised in our talks. But for fear of burdening this reply, I refrain I shall hope to make a public statement on them as early as I can

In conclusion, let me thank you publicly for the great courtesy and patience with which you listened to my very long statement and argument. And though our ways seem to diverge for the moment, our personal friendship will, as you have kindly said at the time of saying farewell, bear the strain of divergence.

As arranged I am handing our correspondence to the Press for publication

Harijan, 6-10-1940

144

SINDH HINDUS

Shri Shamlal Gidwani writes a letter on the situation in Sindh from which I quote the following:

> "Most of us Sindh Hindus have not been able to follow the advice you thought fit to tender regarding non-violence. We find that this advice is in direct conflict with the teachings of Lord Shri Krishna We have also been advised by you to migrate somewhere else if we are not able to protect ourselves This advice again we are unable to follow for practical reasons
>
> You know several of the reasons, and you have yourself mentioned some of them in your recent article in *Harijan* which we all greatly appreciate You also know that Hindus in Sindh, though only 27 per cent of the population, own fifty per cent of the land We are scattered all over the province, and our population in some of the villages is between two to ten per cent We pay nearly the whole of the income-tax revenue in the province; we contribute fifty per cent of land revenue and nearly the whole of the customs revenue If we follow your advice, we shall be not only cowards but become landless and countryless There is also a danger in our migrating to other provinces, for it will mean a public confession of the failure of a community to protect itself merely because an aggressive community thought fit to tyrannize over us and the protecting arms of law and order were not made available for us That

would again be a confession of the failure of democracy. On the other hand, what we feel is that, if you had advised the people of Sindh — particularly the Hindus — to train themselves in the use of arms, they would have been able to better protect themselves and would have been saved all the series of murders that had been wantonly committed on the Hindus in recent months"

At the same time that I received this letter, I saw in the papers that five Hindus were shot dead openly while they were pursuing their normal business. As usual the murderers have not been traced Is this a plan of terrorism to drive the Hindus out of Sindh, or is it something else? Someone in Sindh ought to be able to answer the question

Shri Gidwani does not subscribe to non-violence He thinks that my advice is contrary to the teachings of Lord Krishna He thinks, for (for himself) very good reasons, that the Hindus cannot act non-violently For equally good reasons he thinks they cannot migrate. But he would like me to advise them to defend themselves by arms. This is like asking a nature cure physician to prescribe allopathic drugs Can he be trusted to prescribe the right drugs? And what will my advice be worth when my own hands are incapable of wielding arms? Shri Gidwani should go to a physician who knows the business and will, on due occasion, run to the rescue and always be ready to give the necessary training. I have said that for those who do not believe in non-violence armed defence is the only remedy. But if I am asked to advise how it can be done, I can only say, "I do not know"

But Shri Gidwani is trifling with the crisis when he lazily looks up to me to guide Sindh Hindus on impossible terms If he sincerely believes in the solution he has proposed, he must himself take the training at once and lead the terrified Hindus of Sindh along the path of armed defence It is wrong for the leaders of Sindh to look for outside help. They should cease to write. They should seriously think out a plan of action, violent or non-violent, and follow it up firmly and bravely.

I venture also to suggest to the responsible Muslims of Sindh that their reputation is at stake. If they cannot stop those senseless murders of innocent people, history will find them guilty They will never persuade anybody that they are helpless to deal with the mischief. Such acts are impossible

without the silent sympathy of the society to which the perpetrators of murders belong

Simla 28-9-40
Harijan, 6-10-1940

145

A BRITISH ENDORSEMENT

Mr. Stephen J Thorne, Secretary to the Society of Friends, sends me the following letter

"We, a group of Friends, want to send you our deep gratitude for your impressive appeal to every Briton to adopt the way of non-violence and bring about a cessation of hostilities We drew together immediately on receiving the short version of your statement, and were later able to obtain a fuller text.

You have received through the Viceroy the response of the British Government to your proffer of help, a response that, from your knowledge of the situation, you will easily understand To most Britons the call to throw down their arms in a moment of supreme national peril and in face of imminent aggression is indistinguishable from cowardice and treason; it is an invitation to save their lives by the surrender of principles of liberty and justice that are dearer than life itself But to those of us who in the peace movement share your faith in non-violence, you present a tremendous challenge. Few of us are fully prepared to stand up to it; and we are conscious of our own failure to convince our fellow countrymen of the value and practicability of this 'more excellent Way'. The technique of non-violent resistance as you have practised it is little understood in the West; especially in its positive and reconciling sense, and for that reason, apart from any other, your appeal has not yet met with any wide response.

Though the British Government's reply must have been disappointing, we are anxious to make full use of your offered help. Your appeal gave expression to a spirit that is moving, we are certain, in the hearts of men all over the world. We believe that it would be a step forward if you were willing to address a further message to certain of the spiritual leaders of the world, urging them to act together in taking hold of the situation before the winter brings fresh disaster.

We hope you will maintain the closest contact with us, sending direct the full text of anything you may say in the days ahead.

Your friends sincerely,

Horace G Alexander
Anna Bidder
Robert Davis
Carl Heath
James H Hudson
J Cuthbert Wigham
Percy W Bartlett
W Maude Brayshaw
A. Ruth Fry
Elizabeth Fox Howard
Francis E Pollard
Alexander C Wilson

P S The Council of Christian Pacifist Groups associates itself with this letter, and looks forward to an early opportunity of discussing the deep issues involved"

I am thankful to the signatories for their support of my appeal. I can assure them that immediately I see the psychological moment I shall act up to the signatories' expectation that I should address the spiritual leaders of the world It is quite evident that the very thoroughness of the Nazi method makes them think that nothing but counter-violence can check the terror I have suggested that counter-violence can only result in further brutalization of human nature Drastic diseases require drastic remedies In this instance nothing but non-violence can cure Nazi violence.

Sevagram, 16-10-40
Harijan, 20-10-1940

PEACE ORGANIZATION

If the Congress were an organization with a military bias, there is no doubt that today it would be a full-fledged military unit, every member becoming trained to be an efficient soldier. Fortunately for India and humanity, the Congress is not such an organization No other purely national organization is or can be in the India of today Fortunately again for India and humanity, the Congress has pledged itself since 1920 to win India's freedom through non-violent means But up to now it has been largely a debating society, offering civil disobedience at intervals and all the time only playing with its vital programme of construction. At one time every Congressman was expected to create something for the nation. He or she was to spin for the nation. Congressmen would not respond, and the clause about spinning was dropped. There were other items too which every Congressman was to work But he has not done so to the extent expected The moment has now come for him to make a definite choice. The only programme before him is to become a servant or soldier of peace A soldier of peace, unlike the one of the sword, has to give all his spare time to the promotion of peace alike in war time as in peace time His work in peace time is both a measure of prevention of, as also that of preparation for, war time

If then I was a Congressman with a vote, I would vote, as an emergency measure, for requiring every Congressman now on the Congress register or to come hereafter to possess the minimum qualifications for working the constructive programme It would be wrong to remind me that the Congress should retain its democratic character. It will not lose it because, of its own motion, it becomes an efficient working body which anybody undertaking to obey its discipline and conditions of membership may join. The Congress will cease to be popular, if it cannot deserve popularity in times of stress. If it cannot provide work for the workless and hungry, if it cannot protect the people from depredations or teach them how to face them,

if it cannot help them in the face of danger, it will lose its prestige and popularity. No person or corporation can live long on his or its capital The latter has to circulate and multiply itself.

The Congress has become popular because it has been foremost in fighting imperialism Today the old way is of no avail. Nobody thinks of mass revolt at the present moment The best, quickest, and most efficient way is to build up from the bottom. The psychological moment has come. "Back to the villages!" has become a necessity from every point of view. Now is the time to decentralize production and distribution Every village has to become a self-sufficient republic. This does not require brave resolutions It requires brave, corporate, intelligent work As far as I know at the present moment this is common ground between the rulers and the people

Let every Congressman answer for himself whether he will be a soldier or servant of peace or whether he will become a nonentity unwilling to take his place in building up Swaraj.

On train Bardoli-Wardha, 9-1-42
Harijan, 18-1-1942

REAL WAR EFFORT

The greatest need of the immediate present is to feed the hungry and clothe the naked There is already scarcity in the land both of food and clothing As the war progresses, both the scarcities must increase. There are no imports from outside, either of food-stuff or of cloth The well-to-do may not feel the pinch as yet or at all, but the poor are feeling it now. The well-to-do live on the poor. There is no other way What is then their duty? He who saves gains as much, that is to say he produces as much Hence those who feel for the poor, those who would be one with them must curtail their wants There are many ways I shall only mention some here There is much, too much food eaten and wasted by the well-to-do.

Use one grain at a time. Chapati, rice, and pulses, milk, ghee, *gud*, and oil are used in ordinary house-holds besides vegetables and fruit. I regard this as an unhealthy combination Those who get animal protein in the shape of milk, cheese, eggs or meat need not use pulses at all The poor people get only vegetable protein. If the well-to-do give up pulses and oils, they set free those two essentials for the poor who get neither animal protein nor animal fat. Then the grain eaten should not be sloppy. Half the quantity suffices when it is eaten dry and not dipped in any gravy It is well to eat it with raw salads such as onion, carrot, radish, salad leaves, tomatoes An ounce or two of salads serves the purpose of eight ounces of cooked vegetables Chapatis or bread should not be eaten with milk. To begin with, one meal may be raw vegetables and chapati or bread, and the other cooked vegetables with milk or curds.

Sweet dishes should be eliminated altogether. Instead *gud* or sugar in small quantities may be taken with milk or bread or by itself.

Fresh fruit is good to eat, but only a little is necessary to give tone to the system. It is an expensive article, and an over-indulgence by the well-to-do has deprived the poor and the ailing of an article which they need much more than the well-to-do.

Any medical man who has studied the science of dietetics will certify that what I have suggested can do no harm to the body, on the contrary it must conduce to better health.

This is only one way of saving food-stuff. It is obvious. But by itself it cannot produce much visible effect.

Grain-dealers have to shed their greed and the habit of making as much profit as possible. They must be satisfied with as little as possible. They run the risk of being looted, if they do not gain the credit of being keepers of grain for the sake of the poor. They should be in touch with the people in their neighbourhood. Congressmen have to visit grain-dealers within their beat and give them the message of the time.

By far the most important part of the work consists in educating the villagers to keep what they have and to induce cultivation of fresh crops wherever water is available. This requires widespread and intelligent propaganda. It is not generally known that bananas, potatoes, beetroot, yam and *suran* and in a measure pumpkin are a food crop easily grown. They can take the place of bread in time of need.

There is too scarcity of money. There may be grain available but no money to buy it with. There is no money because there is no employment. This has to be found. Spinning is the readiest and the handiest. But local needs may supply other sources of labour Every available source has to be tapped so that there is no want of employment. Only the lazy ones need and must starve. Patient handling will induce even this class to shed their laziness.

The problem of clothing is much easier than feeding, if it is handled well in time The mills may not be relied on in these times. There is ample cotton to be had in India. It is a problem for cotton cultivators how to dispose of their stock. The outside market is closed to them. Our mills cannot absorb the whole of the crop. It can be utilized, if the nation takes to spinning not for wages but for the sake of clothing the naked. Of course those who need employment will spin for profit. This number must be limited. They need organizing. Much money will be needed for the purpose. But national spinning does not need so much organizing Profit motive being eliminated and willingness being assumed, organization is reduced to simplest terms.

This is no time for multiplying wheels They take time to manufacture. Raw material is daily becoming dearer. Wheels cannot be manufactured everywhere. Places where they are can be counted by the fingers of one hand.

Therefore I suggest the plying of the *dhanush takli* and even the simple *takli*. The former should be manufactured locally. Indeed it is difficult to manufacture the simple *takli* at once in lakhs. The *dhanush takli* is the only thing which can be the easiest manufactured Slivers cannot be supplied to spinners Each one should get some cotton for himself or herself, and card it as well as may be with the hand or with a home-made small bow such as the children in the Bihar basic schools have All this can be done because no one is expected to manufacture a large quantity of yarn If every one of our available millions span for one hour daily, there would be enough yarn to keep every handloom going. The reader should know that there are lakhs of hand-loom weavers in the land. There is danger of their starving for want of yarn.

Here is a great task for every Congressman to undertake He has to become a good spinner and carder and know how to manufacture the *dhanush takli*. Let every Congressman begin with himself and his family and neighbours, and he will find that the life-giving contagion spreads like wild fire which envelops you before you hardly know what you are witnessing

Any organization that tackles these two problems successfully will command the love and confidence of the people I hope that all will join in this real war effort It is none the less effective because it is peaceful and constructive

Will the Princes let their people do this work without let or hindrance ? Will Quaid-e-Azam Jinnah allow the members of the Muslim League to co-operate with the Congress workers in this truly national but non-political work which is also humanitarian ? There are 23,000 Muslim spinners, carders and weavers earning their daily bread through the A. I. S. A.

On the way to Kashi, 19-1-42
Harijan, 25-1-1942

148
A DEPLORABLE INCIDENT

As Sardar Vallabhbhai was leaving Sevagram the other day he told me of a dacoity in a home in Kheda District. Armed dacoits entered the house, belaboured the inmates, and escaped with the loot. The story was heart-rending. What should I do under similar circumstances, I thought to myself. What should Congressmen do in the circumstances was the next thought; and since then the train of thought arising from the dacoity has taken possession of me. The Congress has been working continuously since 1920 under the policy of non-violence The province of Gujarat has also had the advantage of a leader of the Sardar's calibre And yet daring dacoities can take place. How far then can Congress influence be said to have penetrated? People imagine that, if the British Government were to cease to function today, it would be the non-violent Congressmen who would automatically take over. But it is not so. I have been working to this end for the last twenty years, but my dream has not materialized. For the Congress has not had a living faith in the very means which it adopted in 1920 Therefore the non-violence of the Congress has really been non-violence of the weak. But governments can only be run by the strong. And a non-violent government can only be run by those who believe that non-violence is the mightiest force on earth. If we had had this strength, there would be no Hindu-Muslim riots, there would be no robbers or dacoits. Some might say that for such strength you need either a Jesus or a Buddha. But this is not so. Neither Jesus nor Buddha tried non-violence in the political sphere, or it would be truer to say that the present day type of politics did not exist in their day The Congress experiment is, therefore, a new one. The tragedy is that Congressmen have not tried it with full faith, full understanding and sincerity. If they had had these three essential qualities, the Congress would today have been far taller than it is. But I may not cry over spilt milk I refer to the past only in order to guide us in the present. Even if we wake up now, the game is ours; if we do not, we shall surely lose. Power invariably

elects to go into the hands of the strong. That strength may be physical or of the heart, or, if we do not fight shy of the word, of the spirit. Strength of the heart connotes soul force. If today we decide that we should try to get power by force of arms, we shall have to undo all the work of twenty years among the masses. We shall have to spend a considerable time in giving people a contrary training. We cannot afford to give the required time at this critical juncture. It is certain that today whoever has any strength of any kind will use it for seizing power It is my firm conviction that, if Congressmen are to get power, it should only be through non-violence or soul force.

We have neither time nor material to do new work even in this line. When we have so far employed non-violence as a weapon of the weak, how can we all of a sudden expect to convert it into a weapon of the strong? But in spite of this I feel that at the present moment this experiment alone is feasible and proper for us. There is no risk involved in it. Even failure in it takes the form of success because, even if the people are not able to go the whole length in the experiment, they cannot possibly be led into a ditch. By following the way of physical force they may not only be proved cowards, but in attempting to follow an untrodden path thousands may also be destroyed.

It is then the duty of Congressmen to seek out dacoits and robbers. They should try to understand and convert them Such workers cannot be had for the asking; but Congressmen should know that this work is just as important as it is fraught with risk, and a certain number of them have to devote themselves to it.

The second thing requisite is that we should prepare such workers as would, under difficult circumstances, stand up to dacoits and, whilst trying to check or convert them from their evil ways, be prepared to suffer hurt or even death. Perhaps few workers will be forthcoming for this task too, but peace brigades throughout the country are a definite necessity. Or else in times of chaos Congressmen will lose all the reputation they have so far gained.

Thirdly, the rich should ponder well as to what is their duty today. They who employ mercenaries to guard their wealth may find those very guardians turning on them. The moneyed classes have got to learn how to fight either with arms

or with the weapon of non-violence. For those who wish to follow the latter way the best and most effective *mantram* is: तेन त्यक्तेन भुंजीथा: (Enjoy thy wealth by renouncing it). Expanded it means: "Earn your crores by all means. But understand that your wealth is not yours; it belongs to the people. Take what you require for your legitimate needs, and use the remainder for society." This truth has hitherto not been acted upon; but, if the moneyed classes do not even act on it in these times of stress, they will remain the slaves of their riches and passions and consequently of those who overpower them

But I have visions that the end of this war will mean also the end of the rule of capital. I see coming the day of the rule of the poor, whether that rule be through force of arms or of non-violence. Let it be remembered that physical force is transitory even as the body is transitory. But the power of the spirit is permanent, even as the spirit is everlasting.

Sevagram, 25-1-42
Harijan, 1-2-1942

149

"SUPPOSE GERMANY WINS"

> "Suppose Germany wins with India not having entered the war, would Hitler leave India alone? Certainly not, my dear Mr Gandhi, he will have a greater say in India than what Britain has now The difference is this You can fight the Englishman, but you cannot fight the German once he puts his foot on India's soil Civil Disobedience is the terror of the Englishman, it is the daily bread of the Nazi"

This is a question extracted from a very long and earnest letter from an English correspondent from South Africa. The first fallacy is that India is assumed not to have entered the war when to all intents and purposes she is in the war in spite of the powerful protest of the Congress. She is so much in the war that Great Britain is effectively using all the available fighting material which her generals have brought into being and trained, and is draining all the money she can. Politically-minded Indians have never been trained except for doing the

rulers' clerical work. They are certainly holding themselves aloof until certain obviously necessary conditions are fulfilled I do not see how they can be blamed for demanding the very liberty in defence of which the Allied Powers are said to be fighting What Indians can do even if their demand is accepted is to give their moral weight to the struggle. This the rulers evidently do not care for. It cannot, in their opinion, turn the scales in their favour. Moral values do not count when each party swears by its material and physical resources. The Congress, with all the will in the world to defeat Nazism, cannot thrust its help on Great Britain which evidently it does not want or about which it is at least indifferent. If, therefore, Great Britain suffers defeat, it will not be for want of Congress co-operation but for causes over which the Congress can have no control.

If the Nazis come to India, the Congress will give them the same fight that it has given Great Britain. I do not underrate the power of *satyagraha* as the questioner does. But that is pure speculation. Imperialism has kept its grip on India for more than 150 years. If it is overthrown by a worse type of rule, the Congress can have the negative satisfaction of knowing that no other 'ism' can possibly last beyond a few years even if it establishes a foothold in India That is as I read the Congress mind Personally I think the end of this giant war will be what happened in the fabled Mahabharata War. The Mahabharata has been aptly described by a Travancorin as the Permanent History of Man. What is described in that great epic is happening today before our very eyes. The warring nations are destroying themselves with such fury and ferocity that the end will be mutual exhaustion. The victor will share the fate that awaited the surviving Pandavas. The mighty warrior Arjuna was looted in broad daylight by a petty robber. And out of this holocaust must arise a new order for which the exploited millions of toilers have so long thirsted The prayers of peace-lovers cannot go in vain. *Satyagraha* is itself an unmistakable mute prayer of an agonized soul.

Sevagram, 10-2-42
Harijan, 15-2-1942

150

PLEA FOR CALMNESS

The recent British reverses ought not to create panic in the land. In all the wars that Britain has fought or in which she has been engaged there have been reverses some of which may be considered disastrous. But the British have a knack of surviving them and turning them into stepping-stones to success. Hence the saying peculiar to them that they blunder through to success Failures do not dismay or demoralize them. They take them with calmness and in a sportsmanlike spirit. Wars are for them a national game like football The defeated team heartily congratulates the successful one almost as if it was a joint victory, and drowns the sorrow of defeat in an exchange of glasses of whisky If we have learnt nothing worth from the contact with the British, let us at least learn their calmness in the face of misfortunes

And is there the slightest cause for alarm? Certainly not for those who believe in non-violence For fear and distrust of self are no part of their composition, nor are they part of a panoplied soldier The attribute of non-violence is perhaps only a copy book maxim We do not see it in actual practice in any measure But this war is abundant proof that neither party though steeped in violence betrays any fear or distrust I am filled with amazement and admiration at the reckless bravery displayed by combatants on either side This war is demonstration of the unthinkable nerve that human beings are capable of possessing Looked at from either standpoint, therefore, we should be ashamed of fear, distrust and nervelessness in the face of danger. It is, therefore, the sacred duty of every worker to steel himself against cowardly panic and prevent its spread as far as he can "Cowards die many times before their death." Let this not be proved of us.

The true danger exists only for cities. It may be very near due to the fall of Singapore and probable loss of Burma One of the best precautions consists in those people who are not wanted in the cities or those who want to shun danger, migrat-

ing to the villages in an orderly manner. There should be no panicky rush. Those who must remain in the cities whether for business or otherwise should carry out instructions that may be issued by the authorities from time to time Those who will not, for any reason whatsoever, should clear out in good time. If this simple precaution is taken, we may face the future without perturbation. More I cannot say, for we are a house divided against itself and there is no living bond between the rulers and the ruled It is tragic but it is true. The tragedy is deepened by the knowledge that all parties feel so helpless

On the train to Calcutta, 17-2-42
Harijan, 22-2-1942

151

CRIMINAL ASSAULTS

The whole world is on trial today No one can escape from the war. Whilst the Ramayana and the Mahabharata are products of poets' imagination, their authors were not mere rhymsters They were seers. What they depicted is happening before our very eyes today. Ravanas are warring with each other. They are showing matchless strength They throw their deadly weapons from the air No deed of bravery in the battlefield is beyond their capacity or imagination.

Man would not fight in this manner, certainly not the gods Only brutes can. Soldiers drunk with the pride of physical strength loot shops and are not even ashamed to take liberties with women. The administration is powerless in war time to prevent such happenings. The army fulfils their primary need, and they wink the eye at their misdeeds. Where a whole nation is militarized the way of military life becomes part and parcel of its civilization Therefore a soldier's taking such liberties is not a matter for condemnation. But it would take generations for India to become so.

Hence arise questions like the following which a sister sends me .

> " (1) If a soldier commits an assault on a woman, can she be said to have lost her virtue ?

(2) Is such a woman to be condemned and ostracized by society ?

(3) What should women and the public do under such circumstances ? "

Whilst the woman has in point of fact lost her virtue, the loss cannot in any way render her liable to be condemned or treated as an outcast She is entitled to our sympathy for she has been cruelly injured, and we should tend her wounds as we would those of any injured person.

A woman is worthy of condemnation only when she is a willing party to her dishonour In no case are adultery and criminal assault synonymous terms. If we were to view the matter in this light, we would not hide such instances as has thus far been our wont. Public opinion against such conduct on the part of men towards women would then be created and freely exercised.

If the Press carried on a sustained agitation, soldiers White or Brown would probably cease to behave in this manner. Their officers would be compelled to prevent such misbehaviour.

My advice to women is that they should leave the cities and migrate to the villages where a wide field of service awaits them. There is comparatively little risk of their being assaulted in villages. They must, however, live simple lives and make themselves one with the poor If they will display their wealth by dressing in silks and satins and wearing jewellery, they will, in running away from one danger, expose themselves to a double. Naturally the advice cannot refer to those whom duty compels to live in cities

The main thing, however, is for women to know how to be fearless. It is my firm conviction that a fearless woman who knows that her purity is her best shield can never be dishonoured. However beastly the man, he will bow in shame before the flame of her dazzling purity. There are examples even in modern times of women who have thus defended themselves. I can, as I write, recall two such instances I therefore recommend women who read this article to try to cultivate this courage. They will become wholly fearless, if they can and cease to tremble as they do today at the mere thought of assaults It is not, however, necessary for a woman to go through a bitter experience for the sake of passing a test of courage.

These experiences mercifully do not come in the way of lakhs or even thousands. Every soldier is not a beast. It is a minority that loses all sense of decency. Only twenty per cent of snakes are poisonous, and out of these a few only bite. They do not attack unless trodden on. But this knowledge does not help those who are full of fear and tremble at the sight of a snake. Parents and husbands should, therefore, instruct women in the art of becoming fearless. It can best be learnt from a living faith in God. Though He is invisible, He is one's unfailing protector. He who has this faith is the most fearless of all.

But such faith or courage cannot be acquired in a day. Meantime we must try to explore other means. When a woman is assaulted she may not stop to think in terms of *himsa* or *ahimsa*. Her primary duty is self-protection. She is at liberty to employ every method or means that come to her mind in order to defend her honour. God has given her nails and teeth. She must use them with all her strength and, if need be, die in the effort. The man or woman who has shed all fear of death will be able not only to protect himself or herself but others also through laying down his life. In truth we fear death most, and hence we ultimately submit to superior physical force. Some will bend the knee to the invader, some will resort to bribery, some will crawl on their bellies or submit to other forms of humiliation, and some women will even give their bodies rather than die. I have not written this in a carping spirit. I am only illustrating human nature. Whether we crawl on our bellies or whether a woman yields to the lust of man is symbolic of that same love of life which makes us stoop to anything. Therefore only he who loses his life shall save it: तेन त्यक्तेन भुञ्जीथाः. Every reader should commit this matchless *shloka* to memory. But mere lip loyalty to it will be of no avail. It must penetrate deep down to the innermost recesses of his heart. To enjoy life one should give up the lure of life. That should be part of our nature.

So much for what a woman should do. But what about a man who is witness to such crimes? The answer is implied in the foregoing. He must not be a passive onlooker. He must protect the woman. He must not run for police help; he must not rest satisfied by pulling the alarm chain in the train. If he is able to practise non-violence, he will die in doing so and thus

save the woman in jeopardy. If he does not believe in non-violence or cannot practise it, he must try to save her by using all the force he may have In either way there must be readiness on his part to lay down his life.

If old, decrepit and toothless, as I am, I were to plead non-violence and be a helpless witness of assault on the honour of a sister, my so-called Mahatmaship would be ridiculed, dishonoured and lost If I or those like me were to intervene and lay down our lives whether violently or non-violently, we would surely save the prey and at any rate we would not remain living witnesses to her dishonour.

So much about the witnesses. But if the courageous spirit pervades the entire atmosphere of our country and it is known that no Indian will stand women being assaulted, I venture to say that no soldier will dare to touch them. That such a spirit does not exist is a matter of shame for us. But it will be something, if persons ready to wipe out this blot are forthcoming

Those who have influence with the Government will try to get the authorities to take the necessary action. But self-help is best help In the present circumstances we may rely only on our own strength and God's help

On the train to Wardha, 19-2-42
Harijan, 1-3-1942

"I am a pacifist still in one sense; that is to say, I realize that Christians should be able to meet material force with spiritual power. It is horrifying to reflect that after nineteen hundred years, we are still unable to do it except in individual cases and on a small scale. But to me it seems merely 'wishful thinking' to act as though we had a power which in fact we have not and for which we have neither trained nor disciplined ourselves in the past Such power does not come to those who have not disciplined themselves, at the last moment, in the hour of need It has not come to us. I would rather, therefore, do what I can in defence of principles which I believe to be both right in themselves and of enormous importance to the future of the human race, than stand aside and do nothing It is *doing nothing* that is the worst expedient of all

When, therefore, my pacifist friends ask me whether I can imagine Jesus Christ dropping a bomb or firing a gun I am entitled to reply · 'No, I cannot, but neither can I imagine him standing aside and doing nothing at all'

I am compelled to echo the words of a very dear relative of mine who, loathing war as much as any pacifist that ever breathed, said to me at the beginning of the last war (in which he lost his life): 'If you can stop war with spiritual power, do it. If you can't, let me do what I can, and if you are right in thinking that war is so damnable that anyone who takes part in it is damned, then I would rather be damned than let these things go on without doing all I can to stop them, even at the cost of my own life.'

Is this not very close to the meaning of our Lord when he said · 'He that loseth his life shall save it'?"

The foregoing is the concluding portion of a touchingly sorrowful article contributed to *The Survey Graphic* of December 1941 by the celebrated Dr. Maude Royden of the Guildhouse, London. She is one of the foremost pacifists of the West Like many she has felt compelled to revise her position and is now most reluctantly but fully ranged on the side of the defenders of the British Isles.

The article demands a considered reply. I have been in constant touch with the Western pacifists. In my opinion

Dr. Royden has surrendered her position in the portion I have quoted. If individuals have lived up to the Christian teaching (i.e. on non-violence) and that on a small scale, one would think practice should make such a life possible for many people and on a large scale. It is undoubtedly wrong and foolish "to act as though one had the power which in fact one has not." "But," says the worthy writer, "such power does not come to those who have not disciplined themselves, at the last moment, in the hour of need."

I suggest that with the knowledge of the defect no time should be lost in seeking to remove it. That by itself is doing not only something but the right thing. To deny one's faith by contrary practice is surely the worst thing one can do.

And I am not sure that "doing nothing is the worst expedient of all" In septic treatment, for instance, doing nothing is not only expedient, it is obligatory.

There is no cause whatsoever for despondency, much less for denial of one's faith at the crucial moment. Why should not British pacifists stand aside and remodel their life in its entirety? They might be unable to bring about peace outright, but they would lay a solid foundation for it and give the surest test of their faith. When, in the face of an upheaval such as we are witnessing, there are only a few individuals of immovable faith, they have to live up to their faith even though they may produce no visible effect on the course of events. They should believe that their action will produce tangible results in due course. Their staunchness is bound to attract sceptics. I would also suggest that individuals like Dr Maude Royden are not mere camp followers. They are leaders. Therefore, they have to live their lives in strict accord with the Sermon on the Mount, and they will find immediately that there is much to give up and much to remodel The greatest thing that they have to deny themselves is the fruit of imperialism. The present complicated life of the Londoner and his high living is possible only because of the hoards brought from Asia, Africa and other parts of the world. In spite of the fierce criticism which has been levelled against my letter 'To Every Briton', I adhere to every word of it, and I am convinced that posterity will adopt the remedy suggested therein against violence however organized and fierce. And now that the enemy is at the gates of India I am advising

my countrymen the same course of action I advised the British people. My advice may or may not be accepted by my countrymen I would remain unmoved. Their non-acceptance will be no test of failure of non-violence. I would subscribe to the charge of my imperfection. But a *satyagrahi* does not wait for perfection before he invites others to experiment with him, provided always that his faith is immovable like a mountain. The advice that Dr. Royden's relative gave her and which she quotes approvingly is altogether wrong. If the war is damnable, how can he stop the things that go on by taking part in it, even though it may be on the defensive side and at the cost of his own life? For the defence has to resort to all the damnable things that the enemy does, and that with greater vigour if it has to succeed Such a giving of life is not only not saving it but a mere waste.

I have attended the Doctor's services in her Church where a living belief in the efficacy of prayer is much in vogue. When the impenetrable gloom surrounded her, why did she not find strength and consolation and real action in heart-prayer? It is never too late to mend. She and her fellow-pacifists, many of whom I have the privilege of knowing, should take heart and, like Peter, repent of the momentary loss of faith and return to the old faith in non-violence with renewed vigour Their return will mean no material loss to the war effort but will mean a great deal to the anti-war effort which is bound to succeed sooner rather than later, if man is to live as man and not become a two-footed brute

Sevagram, 8-3-42
Harijan, 15-3-1942

153

DESIRABILITY OF EXODUS

A correspondent asks for my detailed views on the exodus that I have advised from the cities of all who are not wanted there and all who are unfit or unwilling to stay there. No one is obliged to stay in against his will In the event of bombardment, it is clear that non-combatants can only be a burden in every way. Successful defence against a powerful enemy requires exclusive concentration on holding the enemy at bay. The defendants' attention must not be divided. This is from the military point of view.

But we have war resisters too, either humanitarian or political. They may not stay unless their object is merely to cause embarrassment for the sake of it. I hope there are none such They should, therefore, be out of the cities Then there are those who do not know what to do in the event of bombardment. They should all evacuate. As the reader will see, my opinion has little to do with my war resistance For in this case and up to a point military necessity and duty of war resisters demand the same action.

If I could convert any city or all cities wholly, including the combatants of yesterday, I should welcome the invading host and try to convert even them or challenge them to do their worst, without offering retaliation. But no such good luck awaits me If the cities were converted, all India including the rulers would be converted and there would be peace in India and peace in the world. But that must remain a daydream yet awhile. Only I won't be moved from my position by being told that the Jap or the Nazi is not the same man as the Englishman I draw no such fundamental distinction between man and man. But I must not detain the reader on the speculative side of the matter-of-fact question that faces us

Assuming then that all who should or a part of them have evacuated the cities and have gone to the villages or are about to go, what should they do ? They must go with the village mind to live the village life as much as possible. They may not re-

produce city conditions and build temporary palaces. They should go to the villages in a spirit of service, study their economic and other conditions, and ameliorate them not by giving alms but by giving the villagers work of a permanent nature. In other words, they should work the constructive programme among the villagers. Thus they will identify themselves with the villagers and become a kind of co-operative society with an ordered programme of economic, social, hygienic and political reconstruction

The greatest problem the new-comers will have to tackle will be to deal with loot and dacoities. It will tax their resources to the utmost. The non-violent way is there. If that is not clear to them, with the co-operation of the villagers they should organize themselves for armed defence against robbers and dacoits. We have too long looked to the Government to do this elementary work for us, not excluding even the reclamation of castes called criminal tribes. The Government cannot do much, if anything at all, at this critical time. The work has perforce to be done by the evacuees violently, non-violently, or both ways.

Sevagram, 10-3-42
Harijan, 15-3-1942

154

EVACUATION

Q. You have advised evacuation from the cities of those who are not wanted for service or other reasons But what are those poor people to do who have no homes to go to and who would be unwelcome wherever they go ?

A. This is a real difficulty They must be provided for by the people of the provinces to which they belong If we are one nation, we should have no difficulty in providing for every contingency that may arise. If we are to establish a new order of society, we can act from now I can only speak from the non-violent angle and no other. If the national mind is working in that direction, consciously or unconsciously individuals and institutions will, without fuss, be absorbing all such persons as you mention I know that the process is going on, but not on a scale large enough to be impressive. No able-bodied person should be put on charity, he should be given work enough to feed him properly This shifting of the population, if it is wisely done, must result in a silent reorganization of villages.

Sevagram, 26-4-42
Harijan, 3-5-1942

Q You advise evacuation of cities likely to be bombed and migration into the villages Do you imagine the villages to be safe ? As a matter of fact the villagers are more panic-stricken than the city-dwellers There is inadequate police protection, and villagers live in hourly terror of dacoits and robbers. Is it not a matter of jumping from the frying pan into the fire ?

A. I have not suggested migration to the villages for the soft life they will provide Fright was no ingredient of my plan It was and still is good even from a military point of view as has now been made abundantly clear. All the danger you present is undoubtedly bound up with the migration to the villages But that to my mind is an additional reason for it Who will put heart into the villagers and dispel panic, if it is not the right type of experienced city people ? They will cover not only the

aged and the infirm who may migrate to the villages, but they will also help and serve the villagers in the many ways I have pointed out in these columns Courage is indispensable in these times for every true act.

Sevagram, 4-5-42
Harijan, 10-5-1942

155

VILLAGE SWARAJ

Q. In view of the situation that may arise at any moment in India, would you give an outline or skeleton of a Village Swaraj Committee, which could function in all village matters in the absence of, and without relying upon an over-head Government or other organization ? In particular, how would you ensure that the Committee should be fully representative and that it would act impartially, efficiently and without favour or fear ? What should be the scope of authority and the machinery to enforce its commands ? And what should be the manner in which a committee or an individual member of it could be removed for corruption, inefficiency or other unfitness ?

A. My idea of village Swaraj is that it is a complete republic, independent of its neighbours for its own vital wants, and yet interdependent for many others in which dependence is a necessity. Thus every village's first concern will be to grow its own food crops and cotton for its cloth. It should have a reserve for its cattle, recreation and playground for adults and children Then if there is more land available, it will grow *useful* money crops, thus excluding *ganja*, tobacco, opium and the like The village will maintain a village theatre, school and public hall. It will have its own waterworks ensuring clean water supply This can be done through controlled wells or tanks. Education will be compulsory up to the final basic course As far as possible every activity will be conducted on the co-operative basis. There will be no caste such as we have today with their graded untouchability. Non-violence with its technique of *satyagraha* and non-co-operation will be the sanction

of the village community. There will be a compulsory service of village guards who will be selected by rotation from the register maintained by the village. The Government of the village will be conducted by the Panchayat of five persons annually elected by the adult villagers, male and female, possessing minimum prescribed qualifications These will have all the authority and jurisdiction required. Since there will be no system of punishments in the accepted sense, this Panchayat will be the legislature, judiciary and executive combined to operate for its year of office. Any village can become such a republic today without much interference, even from the present Government whose sole effective connection with the villages is the exaction of the village revenue. I have not examined here the question of relations with the neighbouring villages and the centre if any. My purpose is to present an outline of village government. Here there is perfect democracy based upon individual freedom. The individual is the architect of his own government The law of non-violence rules him and his government He and his village are able to defy the might of the world For the law governing every villager is that he will suffer death in the defence of his and his village's honour

The reader may well ask me, as I am asking myself while penning these lines, as to why I have not been able to model Sevagram after the picture here drawn. My answer is, I am making the attempt. I can see dim traces of success though I can show nothing visible But there is nothing inherently impossible in the picture drawn here. To model such a village may be the work of a life time. Any lover of true democracy and village life can take up a village, treat it as his world and sole work, and he will find good results. He begins by being the village scavenger, spinner, watchman, medicine man and schoolmaster all at once. If nobody comes near him, he will be satisfied with scavenging and spinning

Sevagram, 18-7-42
Harijan, 26-7-1942

156

'SCORCHED EARTH'

The Russian technique of scorched earth has staggered humanity, but humanity has been powerless to do anything except applaud the amazing sacrifice and bravery that counted no cost too great to circumvent the enemy I have shared the amazement with the admirers but not their admiration

We like to imitate what we admire Now that the prospect faces us, are we able to contemplate with equanimity, or feel the glow of bravery and sacrifice at, the prospect of India's earth being scorched and everything destroyed in order that the enemy's march may be hampered ?

As a war resister my answer can only be one I see neither bravery nor sacrifice in destroying life or property for offence or defence I would far rather leave, if I must, my crops and homestead for the enemy to use than destroy them for the sake of preventing their use by him There is reason, sacrifice and even bravery in so leaving my homestead and crops, if I do so not out of fear but because I refuse to regard anyone as my enemy — that is, out of a humanitarian motive.

But in India's case there is, too, a practical consideration. Unlike Russia's, India's masses have no national instinct developed in the sense that Russia's have India is not fighting Her conquerors are. Supposing that the conquerors are worsted and the Japanese come, the inarticulate masses will not even notice the change for the time being or for a long time. The intelligentsia are divided on the issue of the war The motive here is irrelevant India's soldiers are in no sense a national army. They are soldiers because it is their profession They will as soon fight under the Japanese or any other provided they are paid for fighting. In these circumstances the policy of scorched earth would be a wholly indefensible act

It is therefore a matter for satisfaction that Indian opinion is being expressed against the policy of scorching I know nothing of the requirements of the military, but they can never be allowed to supersede national or humanitarian considerations

which the nation may have accepted The military must thus be an arm of the dominant civil power, not its substitute. The Government of India will considerably ease the situation and allay anxiety by declaring in unequivocal terms that they will not apply, if the occasion ever arise, the scorched earth policy to India, especial regard being had to her peculiar position

Sevagram, 16-3-42
Harijan, 22-3-1942

157

SCORCHED EARTH

Thus writes a correspondent on my article 'Scorched Earth' in *Harijan* .

"In your article headed 'Scorched Earth' appearing in *Harijan* of the 22nd March you say as follows:

'As a war resister my answer can only be one I see neither bravery nor sacrifice in destroying life or property for offence or defence I would far rather leave, if I must, my crops and homestead for the enemy to use than destroy them for the sake of preventing their use by him There is reason, sacrifice and even bravery in so leaving my homestead and crops, if I do so not out of fear but because I refuse to regard anyone as my enemy — that is, out of a humanitarian motive.'

Firstly, although I do not approve of the violence which characterized Russia's resistance, I am of the view that there is great bravery and sacrifice in the scorched earth policy which they are adopting to resist the invader I cannot, therefore, understand your saying that there is neither bravery nor sacrifice in destroying property for defence Secondly, although you ask people to resist the invader, you would prefer them to leave their crops and homestead for the invader to use, not out of fear but out of a humanitarian motive. I cannot understand how this can be reconciled with your teaching of resistance to evil. I think that non-violent resistance to the invader demands it of me that I should prevent anything which will be of use to him, such as crops or homestead etc, from falling into his hands even if this means sacrificing my life May I request you to clarify this subject because it is of vital

importance that people should know how they should offer non-violent resistance to the invader?"

Surely the meaning is plain. There is no bravery in my poisoning my well or filling it in so that my brother who is at war with me may not use the water Let us assume that I am fighting him in the orthodox manner. Nor is there sacrifice in it, for it does not purify me, and sacrifice, as its root meaning implies, presupposes purity. Such destruction may be likened to cutting one's nose to spite one's face. Warriors of old had wholesome laws of war. Among the excluded things were poisoning wells and destroying food crops But I do claim that there are bravery and sacrifice in my leaving my wells, crops and homestead intact, bravery in that I deliberately run the risk of the enemy feeding himself at my expense and pursuing me, and sacrifice in that the sentiment of leaving something for the enemy purifies and ennobles me

My questioner has missed the conditional expression "if I must". I have imagined a state of things in which I am not prepared just now to die and therefore I want to retreat in an orderly manner in the hope of resisting under other and better auspices The thing to consider here is not resistance but non-destruction of food crops and the like. Resistance, violent or non-violent, has to be well thought out. Thoughtless resistance will be regarded as bravado in military parlance, and violence or folly in the language of non-violence. Retreat itself is often a plan of resistance and may be a precursor of great bravery and sacrifice. Every retreat is not cowardice which implies fear to die. Of course a brave man would more often die in violently or non-violently resisting the aggressor in the latter's attempt to oust him from his property, but he will be no less brave if wisdom dictates present retreat.

Sevagram, 7-4-42
Harijan, 12-4-1942

158

INHUMAN IF TRUE

The Honorary Secretary of the Social Service Department of the Marwari Relief Society writes:

"I have to place before you a very brief review of the activities of the Marwari Relief Society, Calcutta, in connection with rendering relief to evacuees from Burma and Malaya, absolutely irrespective of caste, creed and colour, and also to humbly seek your invaluable advice on a very grave matter The Society has undertaken to provide food, medical aid, and facilities for repatriation to thousands of helpless refugees who are arriving in Calcutta daily by rail, road and sea Several emergent cases of delivery have also been attended to The Society is also trying to secure suitable jobs for unemployed evacuees with the kind co-operation of respectable local firms

In this connection I beg to report a certain very regrettable incident to you, and shall be grateful if you kindly advise me as to my duties in the matter.

On the night of the 14th March, shortly after the arrival of the Chittagong Mail, as I, in company with a number of volunteers, was attending to the wants of the evacuees, a British tommy got hold of a small child belonging to one of the poor evacuees and threw it under the train Although I am a humble follower of your noble creed of non-violence, it was with the greatest difficulty that I restrained myself and my volunteers from punishing the soldier bodily for his brutal act I reported the matter to the station military authorities, but their attitude was anything but sympathetic. I later approached Mr K C Sen, I C. S over the matter, and though he promised to duly enquire into the matter, nothing has been done as yet to rectify it There are still large numbers of soldiers loitering about the platforms every night, and a violent clash between these soldiers and relief volunteers and the public is a possibility which has to be tackled in no time I have already placed the matter before the Bengal Congress Civil Protection Committee.

I should be thankful, if you kindly advise me on the following points:

1 Should I start an agitation in the press over the matter?

2 Supposing a soldier behaves indecently towards a helpless female evacuee, are we to put up with it silently, or should the soldier be forcibly dealt with?

> It would help us very greatly, if you kindly issued a statement in *Harijan* in this connection. I am prepared to accept all responsibility regarding the truthfulness of the above incident"

I have suppressed many letters giving me authentic details about the misbehaviour of soldiers. I have published them when it would have been wrong, if not cowardly, to suppress them The letter in question demands, in my opinion, the widest publicity, not merely for the safety of the public but also for the sake of the soldiers and the Government. The Marwari Relief Society is a big philanthropic institution of twentyfive years' standing having an all-India reputation It has funds and seasoned workers Its prestige should have been enough security for the good behaviour of the soldiers in the presence of its workers The soldier must have run amuck or been under the influence of drink to have behaved as he is reported to have done. I trust that the Marwari Relief Society will not leave the matter till it is thoroughly thrashed out, and I trust too that the authorities will not wish to hush up the matter but will make ample amends, if the case is proved as reported by my correspondent

So much for the case itself. The correspondent desires my guidance about similar cases in future The action of *himsa* or *ahimsa* would have been identical. The volunteers should have, if they could, bodily prevented the soldier from touching the child or snatched the child from him, even if the soldier had been hurt in the act of preventing or snatching The proceedings after the delivery of the child or the failure of the attempt would vary according as the deliverers were actuated violently or non-violently Non-violent behaviour would dictate generous and gentle behaviour towards the culprit. But generosity and gentleness would have to be thoughtful and reasoned. It is difficult to lay down in advance the rule of conduct applicable in all cases. I can say this much that a truly generous act demands sincere recognition on the part of the culprit. I have known instances of Africans in South Africa insulted at railway stations saying to the rude White men, "My brother, God will forgive you for your rudeness," and the White men giggling, if not adding injury to insult. In similar circumstances I have myself remained silent and suffered the insult. I am quite clear that the Africans' so-called generosity was a mere mechanical act justly evoking

derision. Mine was timidity. I did not wish to evoke further insult I certainly did not want to take legal proceedings I was trying then to shape my non-violent conduct If I had had the real courage, I would have expostulated with the insulters and risked the worst

I have interpolated an examination of so-called non-violent conduct in cases of personal insult or injury What about the child injured or the injury imagined by my correspondent? I think non-violent conduct would not, should not, be different. The distinction that is often drawn between personal injury and injury done to wards is unjustified, if not wrong A man is not expected to do more for his wards than he would for himself He would no doubt sacrifice himself for his ward's honour, but he would be expected to do likewise for his own If he did otherwise, he would be voted a coward and is not likely to protect his ward's honour, if he is not able to protect his own. But I own that correct non-violent conduct does not come through mere reasoning Reason is a necessary preliminary But correctness of conduct will come only through repeated practice, maybe even repeated failures

What violent conduct should be surely needs no examination.

Sevagram, 23-3-42
Harijan, 29-3-1942

QUESTION BOX

Weakening Non-violence ?

Q. In the instructions issued by the Working Committee of the Congress in the matter of organizing Congress volunteers it has been clearly stated that the organization should be based on "strictly non-violent basis". In the pledge forms prepared in this behalf by some Congress committees, however, it is stated that volunteers when on duty only should observe non-violence The Karnatak P. C C has prepared volunteers' pledge in this form. The Chief Organizer of volunteers in the Province appointed by the K. P C. C. declared in a public meeting held for the purpose of enrolling volunteers that a Congress volunteer even on duty might exercise the right of private defence by resorting to violence in an emergency, and further that such an action on his part did not contravene the instructions of the Working Committee All this is creating confusion If the instructions of the Working Committee are not to be strictly carried out, it would be better that the condition of non-violence were dropped altogether rather than were allowed to be diluted to suit individual ideas What is your opinion in the matter ?

A. My answers must not be taken as authentic in questions the answers to which require the *imprimatur* of the Congress as this one does My personal opinion is decisive Violence in self-defence has no place in any corps organized by or in the name of the Congress There can be no laxity in the enforcement of this rule without risking a break-down of the whole non-violent structure in the Congress Use of violence in private self-defence is said to be permitted by the Congress because the Congress does not and cannot regulate the personal and private life of individual Congressmen. The individual in his private life is unfettered by the rules of the Congress He is dominated by his own ethical code, if any

Full Faith in Non-violence

Q There are some Congressmen, though their number is very small, who have full faith in non-violence and who desire

to organize on that basis. Should not Congress committees organize such men ? Or should not Congress committees allow such men to form their corps under the auspices of the Congress ?

A Congress committees cannot organize sections The Congress can have only one policy Today it is pure non-violence so far as internal affairs are concerned Therefore I see no reason for separate peace committees Purists, if they are humble and not self-opinionated, will act as a leaven in bodies which may contain men and women even of doubtful faith, as there must be in democratic organizations

Bewildering Conflict

Q. There is a bewildering conflict of opinion among Congress leaders Sardar speaks with one voice, Rajaji with another, Maulana with a third, and Jawaharlalji with a fourth — not to speak of the lesser lights Whom is one to follow, who is to be regarded as the sole authority to interpret the Congress policy and resolutions ?

A Legally and constitutionally the President is the sole authority If there is a conflict of opinion between the majority of the Working Committee and the President, as was once the case in the early stages, the majority view would supersede the President's But on critical occasions legal opinion is not of much value People have their favourite heroes, and they will follow the heroes even blindly. My advice, therefore, is that in the ticklish question of *ahimsa* each one should be his own authority not on the law but on interpretation If all the four distinguished leaders whom you have mentioned were to sit together, they could probably give the same interpretation, but in the course of their speeches each would put his special emphasis on one aspect or another of the same matter.

On the way to Delhi, 26-3-42
Harijan, 5-4-1942

Expedience

Q Several years ago I once had the temerity to ask whether the fact that you had allowed non-violence to come into the Congress as an expedient rather than as a creed would not be conducive to its breakdown at the critical time You said you did not think so. But do you still feel the same ? Would

you not today have had an organized band of believers in non-violence whom you could have sent in groups all over the country? It almost seems as if we had lost time and are found unprepared, as it were, to shoulder responsibility.

A Yes, I adhere to my opinion that I did well to present to the Congress non-violence as an expedient I could not have done otherwise, if I was to introduce it into politics In South Africa too I introduced it as an expedient. It was successful there because resisters were a small number in a compact area and therefore easily controlled. Here we had numberless persons scattered over a huge country The result was that they could not be easily controlled or trained And yet it is a marvel the way they have responded They might have responded much better and shown far better results But I have no sense of disappointment in me over the results obtained If I had started with men who accepted non-violence as a creed, I might have ended with myself. Imperfect as I am, I started with imperfect men and women and sailed on an uncharted ocean. Thank God that, though the boat has not reached its haven, it has proved fairly stormproof

Sevagram, 7-4-42
Harijan, 12-4-1942

Japan is knocking at our gates What are we to do in a non-violent way? If we were a free country, things could be done non-violently to prevent the Japanese from entering the country As it is, non-violent resistance could commence the moment they effected a landing Thus non-violent resisters would refuse them any help, even water. For it is no part of their duty to help anyone to steal their country. But if a Japanese had missed his way and was dying of thirst and sought help as a human being, a non-violent resister, who may not regard anyone as his enemy, would give water to the thirsty one Suppose the Japanese compel resisters to give them water, the resisters must die in the act of resistance It is conceivable that they will exterminate all resisters The underlying belief in such non-violent resistance is that the aggressor will, in time, be mentally and even physically tired of killing non-violent resisters He will begin to search what this new (for him) force is which refuses co-operation without seeking to hurt, and will probably desist from further slaughter But the resisters may find that the Japanese are utterly heartless and that they do not care how many they kill The non-violent resisters will have won the day inasmuch as they will have preferred extermination to submission.

But things will not happen quite so simply as I have put them There are at least four parties in the country First, the British and the army they have brought into being. The Japanese declare that they have no designs upon India Their quarrel is only with the British In this they are assisted by some Indians who are in Japan. It is difficult to guess how many, but there must be a fairly large number who believe in the declaration of the Japanese and think that they will deliver the country from the British yoke and retire Even if the worst happens. their fatigue of the British yoke is so great that they would even welcome the Japanese yoke for a change. This is the second party. The third are the neutrals, who though not non-violent will help neither the British nor the Japanese.

The fourth and last are non-violent resisters If they are only a few, their resistance will be ineffective except as an example for the future. Such resisters will calmly die wherever they are but will not bend the knee before the aggressor. They will not be deceived by promises. They do not seek deliverance from the British yoke through the help of a third party. They believe implicitly in their own way of fighting and no other. Their fight is on behalf of the dumb millions who do not perhaps know that there is such a thing as deliverance They have neither hatred for the British nor love for the Japanese. They wish well to both as to all others. They would like both to do what is right. They believe that non-violence alone will lead men to do right under all circumstances. Therefore, if for want of enough companions non-violent resisters cannot reach the goal, they will not give up their way but pursue it to death

The task before the votaries of non-violence is very difficult. But no difficulty can baffle men who have faith in their mission.

This is going to be a long drawn out agony Let non-violent resisters not make impossible attempts Their powers are limited. A resister in Kerala is not physically responsible for the defence of Assam which is just now in imminent danger If Assam is non-violently inclined, it is well able to take care of itself. If it is not, no party of non-violent resisters from Kerala can help it or any other province. Kerala can help Assam etc. by demonstrating its non-violence in Kerala itself The Japanese army, if it gets a foothold in India, will not stop at Assam. In order to defeat the British, it has to overrun the whole country. The British will fight every inch of the ground Loss of India will probably be admission of complete defeat for them. But whether it is so or not, it is quite clear that Japan will not rest till India is wholly in her hands. Hence non-violent resisters must remain at their posts wherever they are.

One thing has to be made clear. Where the British army is actually engaging the 'enemy', it would be perhaps improper for direct resistance to function. It will not be non-violent resistance when it is mixed with, or allies itself to, violence.

Let me therefore reiterate what I have said so often The best preparation for, and even the expression of, non-violence

lies in the determined pursuit of the constructive programme. Anyone who believes that without the backing of the constructive programme he will show non-violent strength when the testing time comes will fail miserably It will be, like the attempt of a starving unarmed man to match his physical strength against a fully fed and panoplied soldier, foredoomed to failure. He who has no belief in the constructive programme has, in my opinion, no concrete feeling for the starved millions. He who is devoid of that feeling cannot fight non-violently. In actual practice the expansion of my non-violence has kept exact pace with that of my identification with starved humanity. I am still far from the non-violence of my conception, for am I not still far away from the identification of my conception with dumb humanity ?

On the train to Wardha, 5-4-42
Harijan, 12-4-1942

161

TO EVERY BRITON

When I had just begun my public career in South Africa I wrote "An Open Letter to Every Briton in South Africa". It had its effect. I feel that I should repeat the example at this critical juncture in the history of the world This time my appeal must be to every Briton in the world. He may be nobody in the counsels of his nation. But in the empire of non-violence every true thought counts, every true voice has its full value *Vox populi vox dei* is not a copy-book maxim. It is an expression of the solid experience of mankind. But it has one qualification. Its truth is confined to the field of non-violence Violence can for the moment completely frustrate a people's voice But since I work on the field of non-violence only, every true thought expressed or unexpressed counts for me.

I ask every Briton to support me in my appeal to the British at this very hour to retire from every Asiatic and African possession and at least from India. That step is essential for the safety of the world and for the destruction of Nazism and Fascism. In this I include Japan's 'ism' also. It is a good copy of the two. Acceptance of my appeal will confound all the

military plans of all the Axis Powers and even of the military advisers of Great Britain.

If my appeal goes home, I am sure the cost of British interests in India and Africa would be nothing compared to the present ever-growing cost of the war to Britain. And when one puts morals in the scales, there is nothing but gain to Britain, India and the world.

Though I ask for their withdrawal from Asia and Africa, let me confine myself for the moment to India. British statesmen talk glibly of India's participation in the war. Now India was never even formally consulted on the declaration of war. Why should it be? India does not belong to Indians It belongs to the British. It has been even called a British possession. The British practically do with it as they like. They make me — an all-war resister — pay a war tax in a variety of ways Thus I pay two pice as war tax on every letter I post, one pice on every postcard, and two annas on every wire I send This is the lightest side of the dismal picture But it shows British ingenuity. If I was a student of economics, I could produce startling figures as to what India has been made to pay towards the war apart from what are miscalled voluntary contributions No contribution made to a conqueror can be truly described as voluntary. What a conqueror the Briton makes! He is well saddled in his seat I do not exaggerate when I say that a whisper of his wish is promptly answered in India Britain may, therefore, be said to be at perpetual war with India which she holds by right of conquest and through an army of occupation. How does India profit by this enforced participation in Britain's war? The bravery of Indian soldiers profits India nothing

Before the Japanese menace overtakes India, India's homesteads are being occupied by British troops — Indian and non-Indian. The dwellers are summarily ejected and expected to shift for themselves. They are paid a paltry vacating expense which carries them nowhere Their occupation is gone They have to build their cottages and search for their livelihood These people do not vacate out of a spirit of patriotism When this incident was referred to me a few days ago, I wrote in these columns that the dispossessed people should be asked to bear their lot with resignation But my co-workers protested

and invited me to go to the evacuees and console them myself or send someone to perform the impossible task. They were right These poor people should never have been treated as they were They should have been lodged suitably at the same time that they were asked to vacate.

People in East Bengal may almost be regarded as amphibious. They live partly on land and partly on the waters of the rivers. They have light canoes which enable them to go from place to place. For fear of the Japanese using the canoes the people have been called upon to surrender them For a Bengali to part with his canoe, is almost like parting with his life So those who take away his canoe he regards as his enemy.

Great Britain has to win the war. Need she do so at India's expense ? Should she do so ?

But I have something more to add to this sad chapter The falsity that envelops Indian life is suffocating Almost every Indian you meet is discontented But he will not own it publicly. The Government employees high and low are no exception. I am not giving hearsay evidence. Many British officials know this. But they have evolved the art of taking work from such elements This all-pervading distrust and falsity make life worthless unless one resists it with one's whole soul.

You may refuse to believe all I say. Of course I shall be contradicted. I shall survive the contradictions

I have stated what I believe to be the truth, the whole truth and nothing but the truth.

My people may or may not approve of this loud thinking. I have consulted nobody. This appeal is being written during my silence day I am just now concerned with Britain's action. When slavery was abolished in America many slaves protested, some even wept But protests and tears notwithstanding, slavery was abolished in law But the abolition was the result of a bloody war between the South and the North ; and so though the Negro's lot is considerably better than before, he still remains the outcast of high society I am asking for something much higher I ask for a bloodless end of an unnatural domination and for a new era, even though there may be protests and wailing from some of us

Bombay, 11-5-42
Harijan, 17-5-1942

162

TWO ACTIONS

My proposal for the withdrawal of the British power involves two actions One is to deal with the present emergency, and the other to secure freedom from British supremacy. The second admits of delay. There is a lot of confusion about its implications. I am trying to the best of my ability to deal with the questions as they arise from time to time.

The first admits of no delay and demands specific action irrespective of the proposal for British withdrawal This is in connection with (1) the behaviour of troops, (2) the impending salt famine, (3) control of food grains, (4) evacuation for the sake of the military, (5) discrimination between Europeans and Anglo-Indians and Anglo-Burmans on the one hand and Indians on the other.

On the first item the people have the law and public opinion wholly on their side. The Government machinery is always slow to move, more so now, when it is all premortgaged for military preparations People must everywhere learn to defend themselves against misbehaving individuals, no matter who they are. The question of non-violence and violence does not arise No doubt the non-violent way is always the best, but where that does not come naturally the violent way is both necessary and honourable Inaction here is rank cowardice and unmanly. It must be shunned at all cost Pandit Nehru told me that at the stations in the north, platform hawkers have banded themselves for self-defence, so the troops are careful at those stations.

As to salt famine, the law is not quite on the people's side, but right is wholly on their side. I am hoping that the Government will put the widest construction on the clause referring to salt in Gandhi-Irwin Pact and allow people to manufacture salt wherever they can And I would advise them to manufacture salt even at the risk of prosecution. Necessity knows no law. A starving man will help himself to food wherever he finds it. Rishi Vishwamitra did so

Number three is difficult to deal with. But the same rule applies as to the second Food cannot be manufactured as easily as salt. It is up to the merchants to band themselves to do what they can and force the hands of the Government to do the right thing by suggesting wise rules for the supply of food to the poor people at fixed prices. If this is not done in time looting shops is sure to be a daily event.

As to four, I have no doubt that the authorities may not ask people to vacate except where they are ready to offer equivalent land and buildings and cart the people and their belongings to the places prepared for them and pay them a living wage till they find suitable occupation. The people, if they have nowhere to move to, should simply refuse to vacate and suffer the consequences.

As to the fifth, the people should refuse to submit to discrimination and it will break down. Most of these difficulties take place because we have cultivated the habit of submitting to them. In the words of the late Lord Willingdon, we must learn resolutely to say 'no', when that is the real answer possible, and take the consequence.

Sevagram, 22-6-42
Harijan, 28-6-1942

163

LETTER TO THE GENERALISSIMO

(*The following letter had been addressed to Generalissimo Chiang Kai-Shek by Gandhiji.* —Ed.)

Sevagram,
June 14, 1942

Dear Generalissimo,

I can never forget the five hours' close contact I had with you and your noble wife in Calcutta. I had always felt drawn towards you in your fight for freedom, and that contact and our conversation brought China and her problem still nearer to me. Long ago, between 1905 and 1913, when I was in South Africa, I was in constant touch with the small Chinese colony in Johannesburg. I knew them first as clients and then as comrades in the Indian passive resistance struggle in South Africa. I came in touch with them in Mauritius also. I learnt then to admire their thrift, industry, resourcefulness and internal unity. Later in India I had a very fine Chinese friend living with me for a few years and we all learnt to like him.

I have thus felt greatly attracted towards your great country and, in common with my countrymen, our sympathy has gone out to you in your terrible struggle. Our mutual friend, Jawaharlal Nehru, whose love of China is only excelled, if at all by his love of his own country, has kept us in intimate touch with the developments of the Chinese struggle.

Because of this feeling I have towards China and my earnest desire that our two great countries should come closer to one another and co-operate to their mutual advantage, I am anxious to explain to you that my appeal to the British Power to withdraw from India is not meant in any shape or form to weaken India's defence against the Japanese or embarrass you in your struggle. India must not submit to any aggressor or invader and must resist him. I would not be guilty of purchasing the freedom of my country at the cost of your country's freedom. That problem does not arise before me as I am clear that India cannot gain her freedom in this way, and a Japanese

domination of either India or China would be equally injurious to the other country and to world peace. That domination must therefore be prevented and I should like India to play her natural and rightful part in this.

I feel India cannot do so while she is in bondage India has been a helpless witness of the withdrawals from Malaya, Singapore and Burma. We must learn the lesson from these tragic events and prevent by all means at our disposal a repetition of what befell these unfortunate countries. But unless we are free we can do nothing to prevent it, and the same process might well occur again, crippling India and China disastrously I do not want a repetition of this tragic tale of woe.

Our proffered help has repeatedly been rejected by the British Government and the recent failure of the Cripps mission has left a deep wound which is still running Out of that anguish has come the cry for immediate withdrawal of British Power so that India can look after herself and help China to the best of her ability.

I have told you of my faith in non-violence and my belief in the effectiveness of this method if the whole nation could turn to it That faith in it is as firm as ever. But I realize that India today as a whole has not that faith and belief, and the Government in free India would be formed from the various elements composing the nation.

Today the whole of India is impotent and feels frustrated The Indian army consists largely of people who have joined up because of economic pressure They have no feeling of a cause to fight for, and in no sense are they a national army Those of us who would fight for a cause, for India and China, with armed forces or with non-violence, cannot, under the foreign heel, function as they want to. And yet our people know for certain that India free can play even a decisive part not only on her own behalf, but also on behalf of China and world peace. Many like me feel that it is not proper or manly to remain in this helpless state and allow events to overwhelm us when a way to effective action can be opened to us. They feel, therefore, that every possible effort should be made to ensure independence and that freedom of action which is so urgently needed This is the origin of my appeal to the British Power to end

immediately the unnatural connection between Britain and India

Unless we make the effort there is grave danger of public feeling in India going into wrong and harmful channels There is every likelihood of subterranean sympathy for Japan growing simply in order to weaken and oust British authority in India. This feeling may take the place of robust confidence in our ability never to look to outsiders for help in winning our freedom. We have to learn self-reliance and develop the strength to work out our own salvation. This is only possible if we make a determined effort to free ourselves from bondage That freedom has become a present necessity to enable us to take our due place among the free nations of the world.

To make it perfectly clear that we want to prevent in every way Japanese aggression, I would personally agree, that the Allied Powers might, under treaty with us, keep their armed forces in India and use the country as a base for operation against the threatened Japanese attack.

I need hardly give you my assurance that, as the author of the new move in India, I shall take no hasty action And whatever action I may recommend will be governed by the consideration that it should not injure China, or encourage Japanese aggression in India or China. I am trying to enlist world opinion in favour of a proposition which to me appears self-proved and which must lead to the strengthening of India's and China's defence I am also educating public opinion in India and conferring with my colleagues Needless to say, any movement against the British Government with which I may be connected will be essentially non-violent. I am straining every nerve to avoid a conflict with British authority. But if in the vindication of the freedom which has become an immediate desideratum, this becomes inevitable, I shall not hesitate to run any risk however great.

Very soon you will have completed five years of war against Japanese aggression and invasion and all the sorrow and misery that these have brought to China My heart goes out to the people of China in deep sympathy and in admiration for their heroic struggle and endless sacrifices in the cause of their country's freedom and integrity against tremendous odds. I am convinced that this heroism and sacrifice cannot be in vain ; they

must bear fruit. To you, to Madame Chiang and to the great people of China, I send my earnest and sincere wishes for your success. I look forward to the day when a free India and a free China will co-operate together in friendship and brotherhood for their own good and for the good of Asia and the world.

In anticipation of your permission, I am taking liberty of publishing this letter in *Harijan*

Yours sincerely,
Sd M K. GANDHI

164

TO EVERY JAPANESE

I must confess at the outset that, though I have no ill-will against you, I intensely dislike your attack upon China. From your lofty height you have descended to imperial ambition You will fail to realize that ambition and may become the authors of the dismemberment of Asia, thus unwittingly preventing world federation and brotherhood without which there can be no hope for humanity.

Ever since I was a lad of eighteen studying in London over fifty years ago, I learnt, through the writings of the late Sir Edwin Arnold, to prize the many excellent qualities of your nation I was thrilled when in South Africa I learnt of your brilliant victory over Russian arms After my return to India from South Africa in 1915, I came in close touch with Japanese monks who lived as members of our Ashram from time to time One of them became a valuable member of the Ashram in Sevagram, and his application to duty, his dignified bearing, his unfailing devotion to daily worship, affability, unruffledness under varying circumstances, and his natural smile which was positive evidence of his inner peace had endeared him to all of us And now that owing to your declaration of war against Great Britain he has been taken away from us, we miss him as a dear co-worker. He has left behind him as a memory his daily prayer and his little drum, to the accompaniment of which we open our morning and evening prayers

In the background of these pleasant recollections I grieve deeply as I contemplate what appears to me to be your unprovoked attack against China and, if reports are to be believed, your merciless devastation of that great and ancient land

It was a worthy ambition of yours to take equal rank with the Great Powers of the world. Your aggression against China and your alliance with the Axis Powers was surely an unwarranted excess of that ambition.

I should have thought that you would be proud of the fact that that great and ancient people, whose old classical literature you have adopted as your own, are your neighbours Your understanding of one another's history, tradition, literature should bind you as friends rather than make you the enemies you are today

If I was a free man, and if you allowed me to come to your country, frail though I am, I would not mind risking my health, maybe my life, to come to your country to plead with you to desist from the wrong you are doing to China and the world and therefore to yourself

But I enjoy no such freedom And we are in the unique position of having to resist an imperialism that we detest no less than yours and Nazism. Our resistance to it does not mean harm to the British people. We seek to convert them Ours is an unarmed revolt against British rule An important party in the country is engaged in a deadly but friendly quarrel with the foreign rulers.

But in this they need no aid from foreign Powers You have been gravely misinformed, as I know you are, that we have chosen this particular moment to embarrass the Allies when your attack against India is imminent If we wanted to turn Britain's difficulty into our opportunity we should have done it as soon as the war broke out nearly three years ago.

Our movement demanding the withdrawal of the British Power from India should in no way be misunderstood In fact, if we are to believe your reported anxiety for the independence of India, a recognition of that independence by Britain should leave you no excuse for any attack on India Moreover the reported profession sorts ill with your ruthless aggression against China

I would ask you to make no mistake about the fact that you will be sadly disillusioned if you believe that you will receive a willing welcome from India The end and aim of the movement for British withdrawal is to prepare India, by making her free for resisting all militarist and imperialist ambition, whether it is called British Imperialism, German Nazism, or your pattern. If we do not, we shall have been ignoble spectators of the militarization of the world in spite of our belief that in non-violence we have the only solvent of the militarist spirit and ambition. Personally I fear that without declaring the independence of India the Allied Powers will not be able to beat the Axis combination which has raised violence to the dignity of a religion The Allies cannot beat you and your partners unless they beat you in your ruthless and skilled warfare If they copy it their declaration that they will save the world for democracy and individual freedom must come to naught I feel that they can only gain strength to avoid copying your ruthlessness by declaring and recognizing *now* the freedom of India, and turning sullen India's forced co-operation into freed India's voluntary co-operation.

To Britain and the Allies we have appealed in the name of justice, in proof of their professions, and in their own self-interest. To you I appeal in the name of humanity It is a marvel to me that you do not see that ruthless warfare is nobody's monopoly. If not the Allies some other Power will certainly improve upon your method and beat you with your own weapon Even if you win you will leave no legacy to your people of which they would feel proud They cannot take pride in a recital of cruel deeds however skilfully achieved.

Even if you win it will not prove that you were in the right, it will only prove that your power of destruction was greater This applies obviously to the Allies too, unless they perform *now* the just and righteous act of freeing India as an earnest and promise of similarly freeing all other subject peoples in Asia and Africa.

Our appeal to Britain is coupled with the offer of Free India's willingness to let the Allies retain their troops in India The offer is made in order to prove that we do not in any way mean to harm the Allied cause, and in order to prevent you from being misled into feeling that you have but to step into

the country that Britain has vacated. Needless to repeat that if you cherish any such idea and will carry it out, we will not fail in resisting you with all the might that our country can muster. I address this appeal to you in the hope that our movement may even influence you and your partners in the right direction and deflect you and them from the course which is bound to end in your moral ruin and the reduction of human beings to robots.

The hope of your response to my appeal is much fainter than that of response from Britain I know that the British are not devoid of a sense of justice and they know me I do not know you enough to be able to judge. All I have read tells me that you listen to no appeal but to the sword. How I wish that you are cruelly misrepresented and that I shall touch the right chord in your heart! Any way I have an undying faith in the responsiveness of human nature On the strength of that faith I have conceived the impending movement in India, and it is that faith which has prompted this appeal to you

I am

Your friend and well-wisher,

M. K. GANDHI

Sevagram, 18-7-42

Harijan, 26-7-1942

FASTING IN NON-VIOLENT ACTION

If the struggle which we are seeking to avoid with all our might has to come, and if it is to remain non-violent as it must in order to succeed, fasting is likely to play an important part in it. It has its place in the tussle with authority and with our own people in the event of wanton acts of violence and obstinate riots for instance.

There is a natural prejudice against it as part of a political struggle It has a recognized place in religious practice. But it is considered a vulgar interpolation in politics by the ordinary politician though it has always been resorted to by prisoners in a haphazard way with more or less success. By fasting, however, they have always succeeded in drawing public attention and disturbing the peace of jail authorities

My own fasts have always, as I hold, been strictly according to the law of *satyagraha*. Fellow *satyagrahis* too in South Africa fasted partially or wholly My fasts have been varied. There was the Hindu-Muslim unity fast of 21 days in 1924 started under the late Maulana Mahomed Ali's roof in Delhi. The indeterminate fast against the MacDonald Award was taken in the Yeravda Prison in 1932. The 21 days' purificatory fast was begun in the Yeravda Prison and was finished at Lady Thakersey's, as the Government would not take the burden of my being in the prison in that condition. Then followed another fast in the Yeravda Prison in 1933 against the Government refusal to let me carry on anti-untouchability work through *Harijan* (issued from prison) on the same basis as facilities had been allowed me four months before. They would not yield, but they discharged me when their medical advisers thought I could not live many days if the fast was not given up Then followed the ill-fated Rajkot fast in 1939. A false step taken by me thoughtlessly during that fast thwarted the brilliant result that would otherwise certainly have been achieved. In spite of all these fasts, fasting has not been accepted as a recognized part of *satyagraha*. It has only been tolerated by

the politicians I have however been driven to the conclusion that fasting unto death is an integral part of *satyagraha* programme, and it is the greatest and most effective weapon in its armoury under given circumstances. Not every one is qualified for undertaking it without a proper course of training

I may not burden this note with an examination of the circumstances under which fasting may be resorted to and the training required for it Non-violence in its positive aspect as benevolence (I do not use the word love as it has fallen into disrepute) is the greatest force because of the limitless scope it affords for self-suffering without causing or intending any physical or material injury to the wrong-doer. The object always is to evoke the best in him Self-suffering is an appeal to his better nature, as retaliation is to his baser Fasting under proper circumstances is such an appeal par excellence If the politician does not perceive its propriety in political matters, it is because it is a novel use of this very fine weapon

To practise non-violence in mundane matters is to know its true value It is to bring heaven upon earth. There is no such thing as the other world All worlds are one There is no 'here' and no 'there' As Jeans has demonstrated, the whole universe including the most distant stars, invisible even through the most powerful telescope in the world, is compressed in an atom. I hold it therefore to be wrong to limit the use of non-violence to cave dwellers and for acquiring merit for a favoured position in the other world. All virtue ceases to have use if it serves no purpose in every walk of life I would therefore plead with the purely political-minded people to study non-violence and fasting as its extreme manifestation with sympathy and understanding.

Sevagram, 20-7-42
Harijan, 26-7-1942

166

TO AMERICAN FRIENDS

Dear Friends,

As I am supposed to be the spirit behind the much discussed and equally well abused resolution of the Working Committee of the Indian National Congress on Independence, it has become necessary for me to explain my position For I am not unknown to you. I have in America perhaps the largest number of friends in the West — not even excepting Great Britain British friends knowing me personally are more discerning than the American. In America I suffer from the well-known malady called hero-worship Good Dr Holmes, until recently of the Unity Church of New York, without knowing me personally became my advertising agent. Some of the nice things he said about me I never knew myself. So I receive often embarrassing letters from America expecting me to perform miracles Dr Holmes was followed much later by the late Bishop Fisher who knew me personally in India He very nearly dragged me to America but fates had ordained otherwise and I could not visit your vast and great country with its wonderful people.

Moreover, you have given me a teacher in Thoreau, who furnished me through his essay on the 'Duty of Civil Disobedience' scientific confirmation of what I was doing in South Africa. Great Britain gave me Ruskin, whose 'Unto This Last' transformed me overnight from a lawyer and city-dweller into a rustic living away from Durban on a farm, three miles from the nearest railway station, and Russia gave me in Tolstoy a teacher who furnished a reasoned basis for my non-violence. He blessed my movement in South Africa when it was still in its infancy and of whose wonderful possibilities I had yet to learn It was he who had prophesied in his letter to me that I was leading a movement which was destined to bring a message of hope to the down-trodden people of the earth. So you will see that I have not approached the present task in any spirit of enmity to Great Britain and the West After having imbibed and assimilated the message of 'Unto This Last', I

could not be guilty of approving of Fascism or Nazism, whose cult is suppression of the individual and his liberty.

I invite you to read my formula of withdrawal or as it has been popularly called 'Quit India' with this background. You may not read into it more than the context warrants.

I claim to be a votary of truth from my childhood. It was the most natural thing to me. My prayerful search gave me the revealing maxim 'Truth is God' instead of the usual one 'God is Truth'. That maxim enables me to see God face to face as it were. I feel Him pervade every fibre of my being. With this Truth as witness between you and me, I assert that I would not have asked my country to invite Great Britain to withdraw her rule over India, irrespective of any demand to the contrary, if I had not seen at once that, for the sake of Great Britain and the Allied cause, it was necessary for Britain boldly to perform the duty of freeing India from bondage. Without this essential act of tardy justice, Britain could not justify her position before the unmurmuring World Conscience, which is there nevertheless. Singapore, Malaya and Burma taught me that the disaster must not be repeated in India. I make bold to say that it cannot be averted unless Britain trusts the people of India to use their liberty in favour of the Allied cause. By that supreme act of justice Britain would have taken away all cause for the seething discontent of India. She will turn the growing ill-will into active goodwill. I submit that it is worth all the battleships and airships that your wonder-working engineers and financial resources can produce.

I know that interested propaganda has filled your ears and eyes with distorted versions of the Congress position. I have been painted as a hypocrite and enemy of Britain under disguise. My demonstrable spirit of accommodation has been described as my inconsistency, proving me to be an utterly unreliable man. I am not going to burden this letter with proof in support of my assertions. If the credit I have enjoyed in America will not stand me in good stead, nothing I may argue in self-defence will carry conviction against the formidable but false propaganda that has poisoned American ears.

You have made common cause with Great Britain. You cannot therefore disown responsibility for anything that her representatives do in India. You will do a grievous wrong to

the Allied cause, if you do not sift the truth from the chaff whilst there is yet time. Just think of it. Is there anything wrong in the Congress demanding unconditional recognition of India's independence? It is being said, 'But this is not the time.' We say, 'This is the psychological moment for that recognition. For then and then only can there be irresistible opposition to Japanese aggression It is of immense value to the Allied cause if it is also of equal value to India The Congress has anticipated and provided for every possible difficulty in the way of recognition I want you to look upon the immediate recognition of India's independence as a war measure of first class magnitude

I am,
Your Friend,
M K GANDHI

On the way to Bombay, 3-8-42
Harijan, 9-8-1942

Appendices

I

WAR AGAINST WAR

(By B. de Ligt)

[Rev B de Ligt of Oned, Geneva, it will be remembered, wrote an open letter to me last year on my attitude towards war especially in view of my participation in the Boer War, the Zulu Rebellion in Natal and the late War The open letter was published in the European Press, and I satisfied myself with merely publishing my reply* in these pages without publishing the former The reverend gentleman has now sent me a rejoinder which he calls second open letter, and would have me to publish his letter. Although it is too long for these pages I may not resist the writer's request He has taken great pains over his composition, and I appreciate the interest peace lovers in the West are taking in my views and conduct I am publishing the letter in two parts And after the conclusion of the second part I hope to write out a brief reply † M K G]

I

It is in the name of all those who, throughout the world, are fighting against the terrible mania for violence which is ravaging the universe, that I thank you for having kindly replied, so frankly and precisely, to my open letter of May, 1928

Your reply cuts both ways. It is reassuring on the one side, but disappointing on the other From the point of view of immediate opposition to war, it is of great importance that you should have declared openly that you no longer wish to participate in any combat whatsoever on behalf of England. If the masses who are with you are ready to oppose, at critical moments, any war movement on the part of the Government in London, no longer desiring to give either their gold or their blood, they become a real factor for peace.

As I already wrote you, movements are being born today throughout the whole world, and spreading continuously, to

---

* Chapter 80, p. 73, in this book.

† Chapter 36, p. 86, in this book.

oppose war You may be sure that your courageous declaration is received in those quarters with joy

Moreover, you have recognized without circumlocution that the work of the Red Cross forms part of the gigantic machinery of war This declaration was confirmed by the impressive statement of the American nursing sister, who sent back to the French Government her Croix de Guerre because she had finally become convinced that all her so-called humanitarian work had been, on the whole, only a refined sort of war work By a happy coincidence, this statement was published in the same number of *The World Tomorrow* (November 1928) as the report of our correspondence This statement was further confirmed by the letter of Albert de Jong, Secretary of the International Anti-militarist Bureau, to the XIIIth Conference of the Red Cross at the Hague, showing how this institution, willynilly, is today forced even to collaborate in the perfecting of war gases In the same manner, you also are helping us to tear the veil from the face of false charities, under cover of which the militarists of all countries are trying to arouse at least a little sympathy for 'national defence' in good-hearted men and women

On the other hand, however, your reply has caused in Western countries profound disappointment. For, in the main, you have accepted rather than rejected war, if not on your own behalf, at least, in principle, for your people At the present time you are, happily, against India's participation in any war whatsoever undertaken by the British Empire. But, until when? If, for instance, in a short time, a Macdonald Government is formed in England and it should be tactician enough to make advances to you with regard to Home Rule and to give you, at least formally, the reward which you vainly hoped to obtain by participating in the war of 1914-1918, what would you do, if that friendly Government let itself be driven into a new war? That is a burning question Do not say that such a war is impossible. Everywhere one is preparing for it, even more than before 1914 Politicians are already speaking of an eventual war between the United States and England; strategists and technicians are studying it And like any other socialist Government, an English socialist Government would continue the tradition of 'national defence', it would in any case be

ready to go to war on behalf of the League of Nations, that trust of modern imperialism whose fatal character I have made manifest in my book *Contre la Guerre Nouvelle* which I have sent to you Macdonald, who was formerly among the conscientious objectors, has just declared that as a practical man it is for him impossible to renounce the force of arms

That is why your reply, however frank and courageous it may be, can reassure us, alas, but very relatively

As I have already written you, I also do not hold to a dogmatic point of view of non-violence. I recognize impartially the right of any oppressed class or race to liberate itself by means of arms. I grant that from a moral point of view a people which defends itself militarily does better than if it did not defend itself at all because of cowardice or lack of character—although I can quite well imagine a people which, urged by worthy humanitarian sentiments, renounces war methods, even while still incapable of liberating itself by higher means But today the international situation has changed so much that such an affirmation can only have quite an abstract sense. Modern warfare has become a technical-industrial system, so complicated and so refined that for many years to come coloured races will not be able to employ it against the hypercivilized barbarians who have been unconsciously preparing themselves for it for a few centuries past, and have been consciously adapting themselves to it for the last hundred years or so The surest arm, especially for coloured races, is non-violent resistance. Their objective right of armed resistance is becoming, from the practical point of view, something more and more academic By the force of things, oppressed races are even obliged to make, so to speak, a virtue of necessity. For that their minds are, fortunately, very favourably constituted, as was shown by your experiences in Africa and in India and by the success of the general strikes and the boycott in China

All that, moreover, is taking place at a time when the Western nations are beginning to recognize that they must renounce war, the latter becoming more and more a fatal peril for victors as well as for vanquished The man of the West risks becoming the victim of his destructive machines: he can no longer control them. "Our civilization," writes Hans Prager, in a little book about yourself, "hides behind a very virile mask

our moral weakness, our fear. We are no longer warriors, but mere servants of lifeless machines. Heroic war has turned away from Europe Soon it will no longer be the men who will take up arms, but the machines which will take up men. That which makes the pride of man, his inventive genius, will become his shame for having created machinery which prevents his defending himself any more by his own strength." This double fact — that while the coloured races are not masters of the means of modern warfare, the modern nations are themselves mastered by these means — constitutes what seems to me the right starting point in order to try to win over all peoples of all races to a united and powerful direct action against war.

I am, therefore, not an absolutist, bestriding some obsolete hobby. Although detesting all vulgar opportunism, I understand perfectly your statement as to the necessity for compromise in order to be able to live and to act I also feel for you profoundly in your moral difficulties concerning the need of defending your crops by force against the monkeys As a vegetarian who has lived the greater part of his life in the country, I know by bitter experience that *ahimsa* can only be applied in quite a relative manner. The interesting publications on plants of your eminent compatriot, J. C. Bose, render this question still more complicated In any case, as regards animals, you hope, with good reason, to reach one day a more satisfactory solution You will understand, therefore, how I hope still more to reach, as regards men, a better solution than this false solution of international questions by means of war, especially the 'total war' of modern times And I reckon upon attaining my object in this field more quickly than in others, because our political and social enemies are neither beasts nor plants, but living beings endowed with intelligence and human conscience That, indeed, is the reason why your example concerning the monkeys has scarcely convinced me as an explanation of your conduct towards the Zulus, the Boers, and the Central Powers.

Just while preparing this letter I heard that your intimate friend, the Rev C. F Andrews, had arrived at Geneva. In the course of a conversation I had with him, you and your work, you may be sure, were the principle topic of discussion, the more so as he was engaged in the composition of a book on your conception of the world and your ideas concerning life and morality,

from the point of view of Swaraj During those unforgettable hours, he described to me your life, spoke to me of your devotion to India, a work which seemed to be extending more and more until finally its influence should embrace the universe. You, who desired only to be a simple servant of India, have become, little by little, one of the moral leaders of the world

You feel it yourself Only a short time ago you sent to the White peoples, through an American press bureau, a statement published even in the *Journal de Geneva*, saying that you reject official Christianity in the name of the message of Jesus as you understand it in his Sermon on the Mount. Since the memorable letter which Tolstoy sent to you in 1909, we Western peoples recognize more and more our need of you, of your traditions, of your nation. As I wrote you previously, we owe to you some experiences decisive for the world's future

What more gladsome news than to learn from the mouth of your own friend that it is possible, not to say certain, that you may come to Europe soon? You will understand how much I am hoping to see you, in order that we may be able to discuss thoroughly our respective points of view In the meantime, I shall continue to study your ideas as well as the important book of Mr. Andrews, who is trying, for the first time, to make Western nations understand how your religious point of view dominates your ideas and your acts.

Young India, 2-5-1929

II

WAR AGAINST WAR

(By B. de Ligt)

Let me venture, for the moment, to explain to you how the greater number of Western war-resisters conceive their attitude towards their Governments, what impression your reply has made, and how we fear, as its consequence, an eventual militarization of India.

In the first place we all know that modern governments are only functionaries of capitalism and imperialism. We likewise all — including libertarians and anarchists — recognize that the present governments from time to time, perhaps even as a rule, do good more or less. But that can never be for us a sufficient motive for collaborating unreservedly with them in all their enterprises. I am supposing, for instance, that some one — or some government — does me a great service. Am I then obliged, from the moral point of view, to come to his assistance even when he acts badly, offends and kills, and forms schemes which are in flagrant opposition to any religious or humanitarian conceptions? No, quite the contrary. The more grateful I feel towards him, the less can I collaborate with him in evil work. Quite apart from my own conduct, is it not my duty to hold him back as far as possible from wrong-doing? In the anti-militarist movement of the Netherlands, we have always said to ourselves: Our first duty is to prevent the Dutch Government — that Government which is, besides, rather our enemy than our friend — from committing the great crime of taking part in collective murder. Moreover, if we do not oppose military measures on the part of our Government, we ourselves shall be obliged to take part in acts unworthy of man, the Government compelling us even to attack and kill other men who are not our enemies, but comrades in misfortune on the other side of the frontier, ill-treated by their authorities who, like our own, are serving above all capitalist and imperialist interests whose defence by

violence is in conflict with the essential needs of all peoples. Even if we suppose for an instant that foreign armies should really be our enemies, well, also in this extreme case we would like to defend ourselves in a different manner, in a manner which would at any rate gain for us a moral victory Doubtless this mode of action might fail from a practical point of view, but the other might fail also; and the first alone assures the existence in the world of that which is truly humane. In short, we wish to strengthen in all countries the power of direct action against war, in such a way that no Government would have the courage to resort to war, that it would even be impossible for it to do so for lack of men, arms and munitions.

During the world war, the greater number of the radical conscientious objectors, English, American, Australian, Canadian, German, Austrian, etc. were convinced that it was their duty, as good citizens, not to participate in the great crime Indeed, what better could they do from their point of view, than remain during that senseless time beings truly conscious of their humane ideal, representing the conscience of their nations waging war without scruple? What better could they do than put humanity before their inhuman fatherland, so as to gain over to this cause not only their compatriots but, in the long run, their official enemies also? Romain Rolland has expressed this point of view when he says that no one has the right to betray his conscience for love of his country, his race or his nation. In doing so, does one not betray in oneself that which is best in one's nation, in one's race, in one's country? We have always been told that if Christ had acted in this sense, he would have become the ally of Judas, he would not have been crucified and would have become the Messiah of those of his compatriots who did not wish to renounce national defence. Tolstoy has told us several times that one should have the courage to risk the sacrifice of one's country in favour of humanity and the universal conscience

You will understand therefore why your participation in three wars of the British Empire against peoples, who were not even your enemies, is for us something very painful. For in doing that you forsook humanity in favour of merciless and unscrupulous imperialism You accept, in principle, all the consequences of those enterprises of violence · the death of 26,370

women and children in the concentration camps in South Africa, the death and sufferings of hundreds of thousands of little children of the Central Powers by the monstrous blockade, the unemployment without end to which the British working classes are condemned, etc., etc., all the horror and shame, physical and moral, consequent upon the Great War We are convinced that you did not foresee such an ocean of misery and decadence. We willingly believe you when you declare that you never wished to act otherwise than in the sense of *ahimsa* But we ask you whether the world-war has not shown sufficiently that if one desires above all *ahimsa* for men, one must at once break with any system of national defence based on violence

Do not imagine that we refuse to defend our rights and liberties. We wish, indeed, to defend them in the most sublime manner, by remaining faithful to the noblest traditions of secular Christianity and modern socialism Even if we considered to be just the cause for which our nation would be fighting in a supposed war, we would only come to its aid in our own fashion, because over and above our country, as well as for the sake of that country, we would have to uphold universal humanity.

These arguments hold good still more today when, as everyone knows, the chief question is this · In what manner will humanity free itself as quickly as possible from that unworthy expedient, war, an expedient which is an unparalleled menace for all, vanquished and victors alike? We must inevitably raise international relations to a higher level, else humanity will be lacerated to such an extent, physically, morally and intellectually, that it will perhaps never recover

Now, just when the courageous women of the American section of the International Women's League for Peace and Liberty are asserting with reason that all war is today civil war, because of the economic and intellectual interdependence of all nations, just when Western nations, after a frightful experience, are beginning to recognize the meaning of your mission and the practical character of the methods of passive resistance, the efficacy of which is proved by your Asiatic peoples, you write that it is possible that, once India is liberated, you might vote for the military training of the Indian people This declaration coincides with the publication of the Nehru plan, according to

which India not only demands Home Rule, but also asks to be allowed to assure her own national defence. The articles published in *Foreign Affairs* of April 1928 by Sir Sivaswami Aiyar, conversations I have had with Swami Satya Deva, Dr. Datta, and other compatriots of yours, have led me to fear that India also may one day let herself be carried away by the fatal current of armaments. It is for that reason that a few months ago, at the International Conference against War, at Sonntagsberg, I warned those present of the possibility of a militarization of the Asiatic nations, favoured in a very imprudent manner by Moscow and the III International, accompanied by the menace of a similar militarization of Africa. Making almost desperate efforts to rid the world of the monstrosity of modern war, and just when our action is beginning to be taken seriously, we all at once perceive on the Eastern horizon a new military danger which serves as a pretext to our militarists to say: "But we must arm in order to be able to defend ourselves against the awakening East!" Thus, the vicious circle, on the point of being broken, seems to be closing mercilessly

You know Afghanistan is becoming militarized Chiang Kai-Shek has already declared that China will have, in about 15 years, a fleet of war and a powerful army A Chinese friend, formerly an enemy of all war, whom I saw again in Europe a few months ago, assured me that not only is militarism in China — until now totally unknown — growing steadily, but that the world is threatened by something unheard of in history, a Chinese imperialism! He, who had formerly awakened in me a love for the great anti-war traditions of his country and a profound sympathy for the pacifism of its immortal sages, acknowledged that he too had broken with his anti-militarist past. He spoke almost like you, Gandhi, who now say essentially what has always been preached by the clergy of the West: "We are for love and against all violence, in which personally we shall never participate. But if it is necessary, we shall support military training and national defence, since nations ought to be able to defend their rights, to gain their liberty, to be themselves, whilst the masses have not the spiritual strength of a Jesus, a St Paul or a Francois d'Assisi"

Perhaps in bygone days this conception had some meaning. But today? The Japanese professor, Inazo Nitobe who under-

stands and loves so intensely the heroic tradition of the *bushido* of his warlike country, writes in his book on the Soul of Japan, in which he recognizes in such an admirable manner the relative right of the fighting instinct which slumbers in each individual "If one is to believe history, the State, built up on warlike virtues — whether it be a city like Sparta or an empire like Rome — will never be able to build upon earth 'a durable city' . . Life has widened out extraordinarily in the present era Missions nobler and greater than that of armies are today soliciting our attention Men have become more than subjects, being raised to the state of citizens, what am I saying? *They are more than citizens*, they are men . The history of the world confirms the prophecy that the meek shall inherit the earth" And he seeks sublime forces for a nobler fight in the most profound traditions of Christianity, of Buddhism, of Chinese philosophy, and in the religious and moral traditions of his own country Thus expresses himself a man, in no wise a revolutionary, who, in our opinion, has too much confidence in present-day political methods. But he is right in principle

It is necessary, in fact, finally to break with this system of scientific ferocity which characterizes modern war, as much from the practical as from the moral point of view We are, therefore, puzzled and uneasy on hearing you declare yourself ready eventually to vote for the military training of the Indians, you who seemed to us the appointed missionary to awaken such moral forces in your compatriots that they would have less and less need of means of defence as barbarous as hypercivilized

We wonder whether you, Gandhi, always so distrustful with regard to the industrialism of the West, are not, in fact, now inclined to accept, along with a military training for your people, the most pernicious consequence of this industrialism, the new war. The Nehru Committee, has it not already proposed an army, a naval fleet and an air fleet for India? That is to say, by your last declaration you encourage the introduction into your country of industrialized war, of war industries, of chemical, electro-technical and even bacteriological war. Once launched upon this path, there is no stopping. It is a question of all or nothing

Why not keep to the means the most efficacious for Orientals, that is passive resistance, boycott and general strikes?

Even if these means fail, their consequence will never be serious as those of modern war. For if a modern war were successful, it would smite fatally the masses of the people in the conquering States as in the conquered. The militarist system is no longer a means of defence worthy of confidence But confidence in the deepest forces of man himself is an inexhaustible source of energy which is proving itself more and more efficacious in enabling one to stand one's ground, as a man, a nation or a race. And if this energy is not yet sufficiently developed, what better can one do than arouse and strengthen it everywhere ?

I am profoundly sorry to feel myself again obliged to write you a letter concerning the question of war I understand how much your time and your energy are taken up with the work which you devote to the education of your people. But you have also become a world leader. The most bigoted nationalists, the Bolshevists, the Fascists even, are already using your reply to my letter against those in all countries who frankly object to the so-called national defence. You are at the same time sowing a pernicious hesitation in the hearts of thousands of men and women, at a moment when a single affirmative word on your part might perhaps have gained them over definitely to our cause.

It is of the greatest importance that, for the time being, we should be able to count upon you for our war against war. But that, however, is not enough The day you would vote for the military training of your compatriots, you would be setting yourself against those Indians who are in agreement with us who, in the spirit of Garrison, of Ballóu, of Keir Hardie and of Tolstoy, according to the ethics of true Christianity and the methods of truly modern socialism, disapprove of all national defence and condemn all military preparation, against those who are endeavouring to relieve humanity of a burden under which it is in danger of succumbing.

In *Die Emporung Asiens* Colin Rosz tells us how you fear, from the humanitarian point of view, the threatening militarization of China. I share your anxiety, as is shown by my statement on China in *Gewalt und Gewaltlosigkeit*, the German manual of the W R I. Can you not also share our anxiety regarding an eventual militarization of China and of India, of Asia and of Africa ? The consequences of such militarization

would be a world-madness of war, a universal return to barbarism. Help us to shatter the vicious circle that holds the world in thrall rather than fortify it to the detriment of your own people and all the other peoples of the globe

It is particularly in the name of Tolstoy, the centenary of whose birth we have commemorated this year, that I appeal to you to meditate on this question, in the name of Tolstoy who wrote in 1906 to Kou Hou Ming:

"Whilst European nations have long ago chosen the deceptive path in which liberation from human violence is extraordinarily difficult, the Oriental nations have only arrived at the crossroads"

Your Asiatic nations can still choose the right road Seeing the misery of the Western peoples, the Oriental peoples should, according to Tolstoy, renounce any attempt to free themselves by political means and endeavour to remain faithful to the only true law which renders impossible the submission of man to violence.

You have not published my former letter in *Young India* I should be very grateful to you, if you would kindly publish this one, as I, on my side, publish what you write to me in the European and American press. For, in appealing to you, I appeal to those who are with you and who will be with us, I hope, in hatred of war and love of humanity.

I would like those who are interested in the question of the liberation of classes and races to realize that war, as a means of resistance, is morally wrong and practically harmful, and that our conscience and our reasoning condemn it as well as any form of preparation, including military training.

Always ready to collaborate with you against war and for the liberation of oppressed races and classes, and assuring you of my profound sympathy.

Young India, 9-5-1929

III

'CAT AND MOUSE'

Most Venerated Gandhi,

I agree entirely with you in recognizing that indeed the inhuman rivalry, which goes on throughout the whole earth to exploit all the so-called inferior races, is one of the principal causes of modern war. This rivalry was even one of the essential causes of the world war Moreover, those other two wars also, on the occasion of which we exchanged views, presented an imperialist character. But alas! the explanation of your participation in those wars convinces me less and less

In any case, the fact that you declare yourself unable "in any conceivable circumstance" to participate in British wars, is a real step forward As I have already noted, you and the millions who are with you could become (unless some quite unforeseen circumstances should arise) a factor for world peace of the highest importance

But on the other hand, your assertion, almost diplomatic, that you will never take part in no matter what "war of exploitation" entered upon by an India eventually free, does not seem to offer sufficient security for the future. This assertion, does it perhaps proceed from the fact that you were thinking of an India which, as a dominion, would be obliged in certain circumstances to take part in armed measures of the Empire and in eventual sanctions of the League of Nations?

I repeat, my objections do not concern your relative appreciation in regard to the violent struggle for freedom. But they are aimed solely at the fact that you, who personally uphold a more sublime form of struggle and who from several points of view have carried out this struggle within the frontiers of the British Empire, are at the same time morally encouraging the militarization of your own country rather than opposing it, and that at a time when any national armament begins to be a menace for the whole of humanity. My objections, likewise, are directed against your collaboration in the preparation of a national Indian State, organized on the same lines as the Western

States, in which, according to the supplementary report of the enlarged Nehru Committee, published in the Indian *Forward* of the 21st December 1928, there would function a Committee of Defence formed by the Prime Minister, the Minister of Defence, the Minister of Foreign Affairs, the Commander-in-Chief of the Army, the Commander of the Air Forces, the Commander of the Naval Forces and the Chief of the General Staff, whilst the Head of the Government, in the event of India being attacked, or if he should consider it as the victim of an aggression, will have the right to raise what money he will esteem necessary for the security of India or of any part of it.*

It seems to us, that in acting in quite a different manner from Tolstoy, you put too much confidence in measures of bourgeois policy, and that you participate in the organization of a form of government which not only will oppress the great masses of your people, but at the same time might become a danger for the evolution of world history That status of dominion, to the realization of which you are today devoting all your strength, is a political institution which will inevitably be used against the great masses of the population of your own country by the dominant Indian classes, which will become more and more allied with the dominant foreign classes And the military, aero-chemical and naval measures, for which your country is going also to decide, risk promoting still further the world competition in armaments

The statements made in the Frankfort Conference on Modern War Methods and the Protection of Civil Populations have just been published. More clearly than ever they show that all technique and science are today being directed in the first place towards collective destruction and murder This Conference has shown that war has become a method of struggle operating in such a fatal manner, from the moral as well as from the practical point of view, that it must be abolished, not only for national defence but also for the realization of the social revolution. Not to enlarge further on this point, I beg

* Committee of *Defence*, Minister of *Defence*, then in paragraph 77 in the event of foreign aggression on India *or upon his being satisfied that there is a reasonable apprehension of such aggression* As one can see, all that is borrowed from Western States, where, in order to tranquillize public opinion, one is beginning today to speak of "Ministry of Defence", and where one declares the intention of only defending oneself against aggression or against that which is considered as aggression

you to refer to the speech of Arthur Muller-Lehning and to my own, delivered at the World Congress against Imperialism in Frankfort. We have now reached a decisive moment in history when the question is to find in all continents powerful groups of men and women who can declare conscientiously. "As for us, we refuse, in all cases, to prepare or to employ any engines of war, and we seek to reduce more and more their preparation and their employment Strictly speaking, we prefer even to lose our national independence — independence which, moreover, is today becoming more and more fictitious — rather than maintain it by such means"

Could national independence become a fiction? It might indeed, as we are traversing an epoch of growing international interdependence. As I have stated elsewhere, national units are no longer in a position to dispose of themselves in a really free fashion from the political and economic point of view, neither within the boundaries of Soviet Russia which contains so many different peoples, nor outside those boundaries. If your India attains the Dominion Status, it will be, from several points of view, allied to the British Empire. It will be subjected, as part of that Empire, and also as a member of the League of Nations, to all sorts of obligations which it will not be able to escape, however little they may interest India directly. As soon as your country begins to arm, it becomes immediately dependent upon international capital for munitions, and upon the great foreign banks, as soon as it begins to develop its industry, your ruling class immediately calls upon financial powers outside the country, which inevitably will place heavy chains about the neck of your own people Modern capitalism, which today embraces almost the whole earth, is tending unremittingly towards a universal dictatorship. This can only be weakened and eventually crushed, if we create against it a united international front, formed of all races and all peoples, which would fight, not for out-of-date nationalist conceptions, but for the realization of a universal and supernational community. Today all nationalism, considered purely as such, is old-fashioned, not only from the historical but likewise from the moral point of view, although from the ideological point of view it still flourishes and works everywhere. In fact national war is becoming more and more transformed into a struggle of

classes and races, embracing the whole earth. It is for us now to fight in the most humane and the most universal manner for our own liberation and for that of all classes and of all oppressed races.

It seems, however, that you, venerated Gandhi, have first of all concentrated your attention in too one-sided a manner upon India instead of taking into consideration, in the first place, the whole of humanity of which, nevertheless, your people also form part, and that with an impatience only too comprehensible you sought, as principal object, to attain 'tangible results'. It is this attitude which risks limiting your horizon and causing your tactics to swerve from their universal tendency Of course we sympathize with you in your impatience, from several points of view, because we ourselves are fighting against our own Governments for the liberation of the coloured races Since even we of the Occident cannot bear the injustice endured by the coloured peoples, how much more must it be unbearable to you, son of an oppressed Oriental nation? But when your impatience, so noble in itself, leads you to make use of methods which have a most dangerous tendency, we must set ourselves against it. And when, moreover, you even try to make it believed that India has hitherto been "helplessly driven into Britain's wars of exploitation", in spite of herself, we can only reply: No. Things have not happened thus, for you yourself are one of those who have consciously induced India to participate in such wars, and because of that you also must bear the full responsibility for it.

On your side, you state that those who set themselves against Western wars pay, nevertheless, taxes which are used by the State for war and the oppression of the coloured peoples That is quite true In fact our anti-militarist struggle also is as yet only something very relative, and it must go on extending But in any case, we have fixed clear and inflexible bounds. we refuse absolutely all direct personal participation in war and in its social and moral preparation. But several of us employ still other means of fighting against it I refer, for instance, to the Dutch movement of the Manifesto refusing direct and indirect military service, which cost several of its signatories, both men and women, the loss of their social position and created for others all sorts of difficulties, to the Ponsonby Action, begun

in England and spreading widely throughout Germany — and there are still others. Moreover, a few of us have already decided individually to refuse to pay any taxes, whilst the organization of which I am a member has already several times been the propagandist of collective refusal of taxation. But whereas refusal, even on a very restricted scale, to do military service has been morally and socially efficacious, the refusal to pay taxes by a restricted number of citizens only has so far had very little result, as the authorities, in confiscating property and inflicting fines, take possession of sums much larger than a direct payment of taxes would have brought them From this point of view, your compatriots have already given some impressive examples of collective refusal, although they also were not able to avoid the regular exactions of the Government

However it may be, in the struggle against the oppression of the coloured races, we are *at one* with you. I even belong to an organization which has participated in this struggle already since the beginning of this century (1904). We are endeavouring to do away as soon as possible with the relation of " cat and mouse " existing between the different races of humanity without, however, wishing to replace them by the relations of cat and dog. That is why we not only object to violent tendencies amongst our own people and race, but we also exhort other peoples and races, who are not defenceless mice but moral beings, not to let themselves be seduced by violence, but to adopt those higher forms of combat which, for centuries past, have been recommended on moral grounds by the most sublime representatives of humanity, and the practical significance of which you yourself in particular have taught us.

As for your expectation of a Great Britain reborn from a moral point of view, it seems to me that you would favour this regeneration more by siding with the most radical of the British war resisters, instead of hoping for salvation from a so-called socialist Government which is inevitably condemned to play a fatal political role in the international imperialistic development You will already have noticed how, under Macdonald likewise, the persecution of your compatriots who are fighting for their rights and liberty continues However amiable may be the manner in which you speak to the British rulers and however benevolent may perhaps be the tone of those who

answer you, they will, nevertheless, only try to satisfy your people by an apparent and fictitious solution

I have esteemed you too highly, venerated Gandhi, to content myself with merely taking note of your "unconscious weakness" as you so kindly propose that I should do, but what I tried up to now to do is to reply to your arguments by other arguments. It is in the desire that you may be able to revise your attitude in the past and the present that I have written to you, and that I do so again today The life of the world has become such a unity, that national interests cannot be really understood nor served except from the point of view of the universal interests of all humanity. This, moreover, is the summing up of everything that I have laid before you.

Onex, Geneva, 5, XII, 1929

Young India, 30-1-1930

IV

NON-VIOLENCE THE ONLY WAY

(*By Richard B. Gregg*)

[I share with the reader the following letter from Mr. Gregg, the author of several books on *khadi* and non-violence. He is an industrious and accurate student of world events. The reader may know that Mr Gregg was in India long enough to study things Indian M K G]

Those two articles of yours in *Harijan* on the Czechoslovak situation and threatened war in Europe * have my entire agreement and approval. I am sorry that there are still members of the Working Committee who think that freedom of any group or nation could be won by bargaining to permit violence. Can they not see that the hour when the British Empire goes to war, fascism becomes established in every corner of it by virtue of the necessities of modern totalitarian war, and that dictatorship is never voluntarily abandoned, no matter what the promises may be at the start? In view of the myriad of violated promises dating from the beginning of the World War, how

* Chapters 60 and 61 in this book.

can they believe that a promise glibly given by a system in danger of its life can possibly be fulfilled when its fulfilment means suicide? The kind of change of heart you seek by *satyagraha* could hardly take place in such an atmosphere of fear and hatred

The spectacle of what is happening in Europe does not in the least upset my faith in non-violence. All that has happened in the past twenty years proves up to the hilt that violence does not win anything worth while, and that whatever it creates is most unstable. Who now has won the World War — the Allies or Germany? Who believes that the conditions now being established in Germany and the rest of Central Europe by means of violence will be permanent? Nor can Japan create stable conditions in China by her present tactics.

Modern war is very different from even that of 1914-18 The use of airplane bombing with high explosives, incendiary and poison gas, plus anti-aircraft guns, means that the bombers have to fly so high that they cannot aim accurately. Hence, modern war is not only vastly more destructive, it is also vastly more indiscriminate in its attack. In former wars the women, children and aged were mostly unharmed and at home, and able to carry on most of the habits of normal life. Now they are even more defenceless than the soldiers. There are no longer any front line trenches Everyone is, in effect, in the trenches. A wise general always kept certain troops in reserve, and never threw them all into battle But modern militarists are compelled to throw the entire nation into danger of destruction There may be no reserve to carry on and re-establish civilization after a modern war.

A modern thermite bomb cannot be quenched by water It burns at a temperature of about 2,000 degrees and melts its way through steel beams and eight inches of solid concrete cement. Not even Spain has shown the full horrors of modern war. Incendiary bombs have been used there only twice; probably as an experiment, by the Germans, and gas bombs not at all. The Germans developed a new and terrible form of attack from the air, in which the planes rose to a great height of about 20,000 feet where they were invisible from the ground, and at a distance of about 75 miles from the city they were aiming at, they shut off their engines, thus becoming entirely silent and

incapable of detection by even the most delicate sound recorders They coasted that 75 miles and dropped high explosive bombs upon the city of Barcelona without the slightest warning. I have seen photographs of the results. Defensive measures against that form of attack are utterly futile Modern war is not men fighting against men; it is men, women and children exposed to the limitless violence of machines and chemicals. To pretend to fight that is not bravery but utter folly. I do not mean to pile on the horrors. I mean to point out that *satyagraha* is the only weapon left Science and machinery have really ended militarism Some stupid people may not yet see it and may try another European war. It will only result in destruction such that only the humble dark-skinned races and nations will be left to carry on the torch of human culture. *Nothing* worth while can be defended or maintained or promoted by modern war.

I pray that you may be able to keep India out of entering any war whatsoever. In order possibly to help you with some unconvinced friends, I am sending you a little book by a distinguished British thinker, Bertrand Russell. It is called *Which Way to Peace?* In it he first sets forth the nature of modern war, mostly by quotations from the experts. Then he examines in turn each of the proposed ways out — collective security, isolationism, alliances, the policy of expedients, wars of principle — that is, wars for democracy, to end war, to end fascism, to promote any great political principle. His analysis is clear and cogent. Against his will he was driven to the conclusion that complete pacifism is the only possible practical policy He does not develop the possibilities of *satyagraha* at all, just leaves the matter there And he is not a religious man at all, and is far more pessimistic about human nature than you or I. I commend it to you as a brilliant and thorough piece of analysis, and likely to be helpful to you, if you are going to discuss this topic further.

I have also read and agree with your article 'What Are Basic Assumptions?' in *Harijan* of October 22 Let me tell you how modern scientific researches have supplied knowledge which backs up the faith of a *satyagrahi*

A great English investigator named G. Elliott Smith and his followers and collaborators have deeply studied the evolution

of the human brain and nervous system, and compared it diligently with the corresponding evolution of the nervous system of animals. It has been proved beyond doubt that man is the only animal in whom the nerves of sensation do not end in the lower nerve centres along the spine, nor in the hinder portion of the brain called the cerebellum which controls all involuntary movements, nor even in a further advanced part of the brain called the corpus striatum, but go on through and past these to cerebrum, the front part of the brain where we do our thinking Also, in connection with this fact, discriminatory action, as distinguished from involuntary instinctive movements, is initiated also in the cerebrum, the part where we do our thinking. This second fact is proved not only by the anatomy of the nerves, but also by the fact that the human baby, as distinguished from the young of most animals, cannot walk at birth but has to watch its parents and elders until it acquires an intellectual pattern of walking After we learn how to walk we do not delegate that activity to our lower nerve centres. It still continues an activity of the cerebrum, even though habit makes its control unconscious most of the time This is proved by the fact that when that part of the cerebrum is organically and physically injured by an accident to the head or by degenerative disease there, then such a person can no longer walk, even though all the rest of his brain and nervous system is intact

From this I deduce the conclusion that, by his very anatomy, by the structure of his nervous system (in which all men of all races and nations are alike), man is compelled to seek the truth, to find what will work in his life here on earth with other men, to learn all the subtle forces as well as the obvious and readily observed forces that play upon and control our total life. No dictator can change human anatomy It means of course that, since man is thus controlled by his thinking, if he makes an intellectual error, it may cause appalling losses and suffering But it also means that, no matter what disasters may come upon him, man has to pick himself up and search on again for a more complete and accurate vision of the truth. Man's spiritual nature and his emotional nature are also a part of the truth. Nazis and Fascists have the same nervous systems as democrats or communists. To me this gives a further strong

basis for optimism, and for faith in *satyagraha* I submitted this deduction of mine to a distinguished physician who is in close touch with all the modern research in this and other medical fields, and he said he thought my conclusion was right, although he preferred to state it in terms of a very high degree of probability, just as cautious scientists are now stating the law of gravity only as a very high degree of probability.

By way of historical examples of the *satyagrahi's* faith that non-violence will be effective even against people trained in modern totalitarian ideas, it is interesting to know that the few Quakers in Germany have been interfered with less than any other religious group, apparently because the German people remember still with gratitude the relief work of the Quakers towards the Germans during the starvation days of the Allied blockade after the war. Also I have read in the actual history of a prisoner in a Nazi prison, who later escaped, that a great service rendered by one of the prisoners to the Storm Trooper guards of the prison resulted later in their secretly saving his life when he was being tortured by the officers in charge of the prison.

While it has not yet been tested for a whole nation, I believe that disciplined *satyagraha* by a considerable number of people in a nation can win its freedom, and with that method can go on to build better civilization for mankind to emulate and thereby save the world

It seems to me that the Western nations are mistaken in condemning Germany with such moral indignation, for Germany's deeds now are the natural result of the way the Allies treated Germany during and after the war, by such methods as the nine months' naval blockade of Germany, which caused the death by starvation of nearly a million German children and women, and similar starvation in Austria, and the impossible financial indemnities levied as pretended punishment on Germany, and all the self-righteous accusations that Germany alone caused the war, and the humiliations of many other sorts forced upon her and her leaders. It has created in Germany a form of morbid feelings and thinking allied to insanity. It is futile as well as wrong to condemn an insane person for his acts, especially if I have caused his insanity. A Chinese proverb says that a wise person uses other persons as his mirror. Germany

is the mirror held up to the Allies, in which, after a lapse of time, they are seeing the reflection of the evils within themselves They recoil with horror, but their horror, if it were understood, would be directed at their own disharmonies and shortcomings.

To jump to another aspect of the European situation, a number of keen observers, both in America and at Geneva, have stated, from reading the British White Paper and comparing it with Lord Runciman's activities and other straws in the news, that they believe that two weeks before Hitler's Nuremburg speech Runciman had told Sudeten German leaders that he was going to recommend to Chamberlain that Czechoslovakia should be dismembered, that this was transmitted to Hitler, who then knew that Chamberlain was tied to what Runciman would advise, so that Hitler knew that he could threaten war and compel Chamberlain to yield Without all the supporting evidence, which I do not have space here to add, that may seem like an unbelievable state of affairs. I can say assuredly, however, that the great majority of American opinion is that the British Tory Party believes that, if Hitler's prestige were severely shaken or he were defeated in a war, Germany would have a revolution and become Communist, and that Communism would hurt the British ruling class more than Nazism or Fascism. Therefore Chamberlain was ready to do anything to avoid war. That is, that the Tory party has put its class interests above its imperial interests, and has compelled France to follow suit I look for a long period of black reaction in England and France.

To put the matter in another form, it seems to me probable that the modern methods and weapons of war now threaten the safety of the ruling classes as much as that of the masses, both directly because of the indiscriminateness of modern weapons, and indirectly because of the much greater danger of revolution and destruction of civilization by war. The weapons will be kept as a means to terrify the masses (an English girl, who was in London at the time of the crisis, told me that the panic in the East End of London because of fear of air raids was exceedingly great, though of course no mention was allowed to be made in the papers). The masses, if terrified, can be made to adopt any sort of repression under the guise of protec-

tion against air raids. If the weapons are used, they will be turned not against another nation but against the working class. Capitalism is in its period of decline and desperate. As long as violence is threatened, the holders of power will do anything to keep their position of relative control, even though it means a general impoverishment in which they too share. Only by means of full-fledged *satyagraha* can they be altered, I believe.

Well, you can see that I do not have much hope for Western civilization. Your beliefs are going to be proved true much faster than anyone conceived likely. Your programme of *satyagraha*, *khadi*, Harijan reform, Hindu-Muslim unity, village industries, and revised education through handicraft, is not only absolutely sound, it is the hope of India and the hope of the world. For there is coming a great economic breakdown in the West which will make *khadi* as much needed there as it is in India.

Because of these beliefs I want to write this additional book on which I am now working, in an attempt to develop suggestions for methods of training in non-violence better than my pamphlet, answering criticisms and doubts of the programme, integrating the philosophy and the methods, and showing the importance of manual work. It cannot cover it all, but, as in the other instance, I am trying to put it all in Western terminology and concepts. In the disruption of former modes of sentiment and action, I believe there is very great need for a firm new philosophy of a better way of social living and the way shown how to bring it about Access to Western libraries is necessary for this writing I hope that such a book could be useful in India as well as in the West.

Perhaps I am mistaken, perhaps I am deceiving myself, but anyhow for that purpose I am staying on here, taking a risk of possible moral harm to myself, and a risk that war may break out and prevent my getting this written or published. If I am mistaken, I will pay for it in my next and succeeding incarnations.

I do intend to come again to India to see if I can help there I cannot disengage myself from India, nor do I want to Believing as I do that your movement is the hope of the entire world, I feel that as soon as I can get this book done I can thereafter help even my own country best by trying to help strengthen

the movement in India Just how soon I can get away I cannot tell. I want to see you and be with you again. Please forgive me that I do not start instantly. I am trying to use my abilities for you in the best way I can see So much trouble and suffering come from people being unable to think clearly on these problems I am trying to clarify thought.

I have a number of ideas as to how possibly I might be of aid after I get there, but they would all be subject to the needs as they actually are at that time. Even if I could not help much, I would like to be there

Harijan, 24-12-1938

V

WE ARE TREATED AS SUBHUMANS— WE ARE ASKED TO BE SUPERHUMAN

[The following is an abridgement of the article by Mr Hayem Greenburg from the *Jewish Frontier* referred to by Gandhiji in his article "The Jewish Question" — Chapter 77, page 205, in this book.]

In his article concerning the Jewish question, a statement for which certain elements in Jewry have long waited with impatience, the spiritual leader of *Young India* directs against us two important accusations. He blames us for not exhibiting the heroism of militant pacifism in those lands where Jews are persecuted, and especially in Germany. On the other hand he accuses us of following an aggressively nationalist — almost imperialist — policy in Palestine and of a desire to deprive the Arabs of their fatherland.

Gandhi's first accusation is quite natural and is in complete harmony with his entire world outlook. His temperament does not tolerate passivity and his ethical-religious convictions dictate to him the duty of heroic and active resistance according to the Indian principle of "Satyagraha"

The motivating idea of "Satyagraha" is not, as some claim, a practical strategy which Gandhi "made to order" to meet the concrete demands of the Indian situation. Long ago he advocated it as a universal ideal which could be applied by all the oppressed and injured everywhere and independent of the

specific historical situation. Personally I feel that the individual and group struggles according to the plan of "Satyagraha" — aside from its moral-religious implications — have proved to be practical and effective. The truth of the "Satyagraha" teaching — which in another form has been expressed by Jesus and other Jewish teachers many generations ago — is in my eyes as self-evident as a mathematical axiom. But I must admit to myself that in order to apply Gandhi's method of struggle it is necessary to accept it not only on a purely intellectual plane, it is also imperative that it be assimilated emotionally, that it should be believed in with all the force of one's being. Such faith the Jews of Germany do not possess. Faith in the principle of "Satyagraha" is a matter of special predisposition which, for numerous reasons, the German Jews have not developed The civilization in which German Jews have lived for so many generations, and to the creation of which they have so energetically and ably contributed, has not prepared them for the "pathos" of "Satyagraha" As a result they are now defenceless. The accepted defence methods of the European-American world cannot be applied by the German Jews They cannot resort to passive resistance because they lack the heroism, the faith and the specific imaginative powers which alone can stimulate such heroism. When Gandhi accuses German Jews of lacking that mentality which, in his estimation, is the only truly heroic mentality, I am ready to concur with him, but with one reservation which he also must accept — that this accusation should also be levelled against the millions of non-Jewish Germans who wear the yoke of the Hitler regime with impotent hatred and show no more affinity for "Satyagraha" methods than do the Jews; against the millions of Italians who for years have breathed the contaminated air of their own tyranny; against the tens of millions of Russians who have exhausted their strength in civil war and do not find their way to the Gandhi method of resisting the red despotism, against hundreds of millions of Chinese who by their military resistance aid the Japanese aggressors to ravage their country instead of following the path of non-co-operation.

It is true that one may demand, as Gandhi does, that Jews, and particularly the Jews of Germany, should be the "pioneers" of new forms of social struggle in the Western world and should

be the first to embrace the practice of "Satyagraha". Gandhi wishes that we should set an example to the non-Jewish Germans, that we should point the way to a spiritual crusade against their wicked Government. He may have a sound reason for believing that the incomparable suffering and degradation to which German Jews are subjected "compels" them to act more heroically and to be more "adventurous" spiritually than their neighbours. I do not question the idea implicit in Gandhi's demand that there is a mutual relationship between the intensity of suffering and the intensity of the moral reaction to suffering. But there is reason to assume that when suffering and insults transgress certain bounds it is quite natural that the reaction should be a feeling of futility and despair instead of that heroism which Gandhi suggests. This is especially true when the group concerned is historically and psychologically not prepared for such a catastrophe and therefore looks upon it as a sudden and unexpected occurrence. The prophet of *Young India* has in this instance exhibited an unusual lack of psychological understanding.

Gandhi should also have understood that it is far less simple to preach "Satyagraha" to German Jews than it is to Indian masses, even to the lowest caste of "untouchables". We all know the evils of English rule and administration in India. But one should be wary of drawing comparisons between the situation of the Indian masses today, or even twenty years ago, and the position of the German Jews today

* * *

A Jewish Gandhi in Germany, should one arise, could "function" for about five minutes — until the first Gestapo agent would lead him, not to a concentration camp, but directly to the guillotine.

If Gandhi demands heroism from the Indians, he demands of the German Jews a measure of super-heroism unexampled in history Gandhi's comparison between the situation of the Indians and that of the German Jews contains an element of unfairness which crept in against his will and against his intentions

But if Gandhi demands that we practise super-heroism in Germany, he requests that in Palestine we should renounce the

most elementary rights which every people may and should claim. When he asks why we do not "like the other peoples of the earth" make our home in the land where we were born and where we earn our livelihood, he indicates that he has not pondered the unusual drama of the paradoxical Jewish history. Jews have been dispersed for many generations, and it could not be an accident that after sojourning in so many lands and with so many peoples they have not become so rooted in those countries that these should cease being "stepmother lands".

* * *

But Gandhi refuses to recognize our right to a distinct territorial settlement, a right which is enjoyed, almost without exception, by all the peoples of the world. Were it not so, he would see the Palestine problem in an altogether different political and moral light. For when he says that "it would be a crime against humanity to reduce the proud Arabs so that Palestine can be restored to the Jews partly or wholly as their National Homeland," he forgets that, if national honour is at stake (this is the burden of his statement, and he knows full well that one may not repeat the discredited allegations of economic or cultural harm that Jews supposedly caused to Arabs), he should also have thought of Jewish honour. Either it is dishonourable to be a minority in a country or it is merely a question of fictitious prestige for which he can have no sympathy. If only pseudo-honour is involved, why should he be concerned lest the "proud Arabs" be deprived of the enjoyment of an inflated pride? But if real national honour is at stake, why should the Arabs enjoy it throughout the length and breadth of the Arabian peninsula, Iraq, Lebanon, Trans-Jordan, and Egypt (an area almost as large as the European continent), while the Jews should be deprived of this honour, even in an area which occupies about one per cent of the above-mentioned land; an area to which they have historical claims and the natural right they acquired during two generations of diligent work, initiative, heroism and suffering?

From a purely legalistic point of view, it may be possible to agree with him that "the mandates have no sanction but that of the last war." This does not mean, however, that the basic idea of the mandates, and the mandatory system as it has been practised during the past twenty years, was born from

the war. The idea underlying the mandate which, according to the constitution of the League of Nations, should be applied in territories where the population is not ready for self-government, or where local interests must be subordinated to more important considerations of an international character, is potentially of great humanitarian significance. It is a prelude to that "civil society" of which Franklin wrote in the eighteenth century; it is a way to a more rational and just collective international control of the world's wealth. I am not unaware of the shortcomings with which the League of Nations is weighed down nor of its sad fate during recent years which also brought misfortune to all humanity. But whoever observed closely the activities of the League in the administration of mandated territories—naturally excluding those areas mandated to Japan, a country which cynically mocked League control even when its representatives were still sitting at Geneva—must admit that the mandatory system is a step forward when compared with the uncontrolled colonial regimes of the past and the present The fact that a mandatory Government is responsible to the Permanent Mandates Commission, in which the majority of the members represent governments possessing neither mandates nor colonial possessions, is in itself an advance in the direction of internationalism and the humanization of the world

It is regrettable that Gandhi approached our problem without that fundamental earnestness and passionate search for truth which are so characteristic of his usual treatment of problems. He therefore missed the deeper implications of the Mandates system He therefore also failed to grasp the unequalled tragedy of Jewish existence. This is the reason why he can justify the phenomenon of five Arab States demanding in London the establishment of a sixth one on the eve of the founding of two other sovereign Arab governments in Syria and Lebanon, while at the same time sanctioning the denial of refuge to Jews in their old home.

This also explains his stand that Arabs must nowhere be reduced to the status of a minority while tens of millions of Russians, Poles, Czechs, Germans, Irish and Italians live in dozens of countries as ethnic minorities and while Jews live as a persecuted minority on the entire globe.

With all my respect for the Mahatma (I doubt if there is

another living man who evokes within me such a moral awareness of his loftiness) I cannot help feeling that in the present instance he has betrayed his inner nature. I cannot avoid the suspicion that, so far as the Palestine problem is concerned, Gandhi allowed himself to be influenced by the anti-Zionist propaganda being conducted among fanatic pan-Islamists. His understandable and praiseworthy desire for a united front with the Mohammedans apparently misguided and blinded him to significant realities and deprived him of that analytical clarity which is a part of his moral being. Years ago he was, for the same reason, misguided into supporting the agitation for the re-establishment of the Khalifate, an institution that is at such variance with his general views. Gandhi was wrong then; he is also mistaken in the present instance and the source of these mistakes seems to be the same.

Harijan, 27-5-1939

VI

GANDHIJI ABSOLVED

The following is the full text of the Working Committee's declaration of policy :

The Working Committee have been deeply moved by the tragic events that have taken place in Europe in startling succession and in particular by the misfortunes that have befallen the people of France These events have already had far-reaching consequences, and they are likely to be followed by other happenings which will lead to novel situations and complex problems.

Ever since the commencement of European war the Congress has followed a policy which was based on its principles and on the attitude of the British Government towards the demand that India should function as a free and independent country. This policy was confirmed in the Ramgarh resolution. The manner of the application of this policy will necessarily depend on the situation which changes from day to day Problems which were distant are now near at hand and may soon demand solution. The problem of the achievement of national freedom has now to be considered along with the allied one,

its maintenance and the defence of the country against possible external aggression and internal disorder.

The war in Europe, resulting from a desire for imperialist domination over other peoples and countries and a suicidal race in armaments, has led to human sorrow and misery on a scale hitherto unknown. It has demonstrated the inefficacy of organized violence, on however vast a scale, for the defence of national freedom and the liberties of peoples. It has shown beyond a doubt that warfare cannot lead to peace and freedom; and the choice before the world is uttermost degradation and destruction through warfare or the way of peace and non-violence on basis of freedom for all peoples. Mahatma Gandhi has presented to the peoples of the world, crying for relief from the crushing burden of war, a weapon in the shape of organized non-violence designed to take the place of war for the defence of a people's rights and freedom against armed aggression. He feels that at this critical phase in the history of man the Congress should enforce this ideal by itself declaring that it does not want that India should maintain armed forces to defend her freedom against external aggression or internal disorder.

While the Working Committee hold that the Congress must continue to adhere strictly to the principle of non-violence in their struggle for independence, the Committee cannot ignore the present imperfections and failings in this respect of the human elements they have to deal with, and the possible dangers in a period of transition and dynamic change, until the Congress has acquired non-violent control over the people in adequate measure and the people have imbibed sufficiently the lesson of organized non-violence. The Committee have deliberated over the problem that has thus arisen and have come to the conclusion that they are unable to go the full length with Gandhiji. But they recognize that he should be free to pursue his great ideal in his own way, and therefore absolve him from responsibility for the programme and activity which the Congress has to pursue under the conditions at present prevailing in India and the world in regard to external aggression and internal disorder

Many of the problems which the Working Committee have considered in this connection are not of the present, though they may be of the near future. The Committee wish to make it clear

that the methods and basic policy of non-violence in the national struggle for freedom continue with full force and are not affected in the least by the inability to extend it to the region of national defence.

The War Committees that are being formed are definitely aimed at increasing the war effort. In view of the Congress policy, they cannot be supported, and Congressmen cannot participate in them or contribute to war funds, nor can Congressmen associate themselves, under present political conditions, with Government-controlled Civic Guards

Congress committees should organize, wherever necessary, people in villages and other areas for self-defence and in order to maintain a sense of public security in their respective areas This should be done on a non-communal basis and in full co-operation with all other groups interested in this task

In view of the difficult times that loom ahead, it is essential that the Congress should function as an active and disciplined organization Provincial committees are enjoined to take necessary steps for this purpose. They should realize that it is of urgent and vital importance that the Congress should function in this way in these days of crisis, and should not be merely a roll of vast numbers of inactive members All members of the executive committees, in particular, are expected to take a continuous and active part in the Congress work, and those who are unwilling or unable to do so are failing in their duty to the country and are of no service to the organization

The critical situation that faces the world today requires vigilant attention and action whenever needed. For this purpose the Working Committee will meet at frequent intervals, and all members must keep in readiness to obey an urgent summons. The All India Congress Committee should be summoned to meet in the last week of July

Harijan, 29-6-1940

VII

WHAT LED TO THE DECISION

A Historical Retrospect: 14 Years

Gandhiji's decision to be responsible no longer to guide the deliberations and policy of the Congress may have come upon the members of the Working Committee and upon the country with a certain amount of suddenness. But it was the natural conclusion of a series of events since 1934 — or may I say since 1919? For his warning to the people first came to be uttered after the mob violence in Nadiad and Ahmedabad. He shocked his followers and amused his critics by proclaiming that in placing the remedy of civil disobedience in the hands of people who had not learnt the discipline to listen and to obey he had been guilty of a Himalayan blunder. Then came Chauri Chaura and the momentous Bardoli decision suspending *satyagraha* indefinitely. This gave the followers an even greater shock than that given by the "Himalayan blunder". But Gandhiji was adamant. He knew that the country would realize the wisdom of the decision in course of time, and that it would be able to prepare itself for another struggle. It took the country six or seven years — years punctuated by communal rioting and internecine strife—to develop non-violent strength to hurl defiance against the Government. We had learnt the virtue of non-violence, but it had with most of us a limited meaning and content. And yet it carried us through to a certain amount of success. Then came more organized repression on behalf of the Government, our limited non-violence proved a poor match for it, and with that came the third milestone on the march to non-violence — I mean the Patna decision of 1934 suspending civil disobedience and confining it only to himself.

Throughout this period of fourteen years he belonged to the Congress, he was a member of it, he attended meetings of the A. I. C. C. and of the Working Committee, and actively participated in the deliberations. But the Patna decision had set him thinking. Always anxious to lay the blame principally on himself, he had begun to discover that the arrest in the growth

of the Congress and the corruption in its ranks was due to the fact that the Congress "had degenerated into an organization dominated by one personality, and that in it there was no free play of reason" There were vital differences between him and the Congressmen, but his personality, he felt, had acted like an incubus on them and prevented their self-expression That was unconsciously encouraging hypocrisy and a subtle piece of violence Khadi and the spinning wheel were there on the Congress programme, "yet only a few Congressmen have a living faith in the India-wide potency of the wheel" And as for non-violence, as he said in his historic statement issued in September 1934, "after fourteen years of trial it still remains a policy with the majority of Congressmen, whereas it is a fundamental creed with me That Congressmen do not still regard non-violence as a creed is no fault of theirs It is undoubtedly my faulty presentation, and still more its faulty execution, that are responsible for this failure I have no consciousness of faulty presentation or execution, but it is the only possible inference from the fact that it has not yet become an integral part of the lives of Congressmen." We had paid lip-loyalty to non-violence, and even that non-violence was not of the strong but of the weak "If we were non-violent through and through, our non-violence would have been self-evident. Nor were we able to show to the terrorists that we had greater faith in our non-violence than they in their violence." The argument led irresistibly to the conclusion that he should leave the Congress organization as he was convinced that by being outside he would serve the Congress and the country better than by being in it. That was the fourth milestone — the step to withdraw from the Congress being solely dictated by the idea of making the experiment of non-violence in thought, word and deed more intense. "For this experiment," he said, "I need complete detachment and absolute freedom of action. I can only search Him through non-violence, and in no other way, and the freedom of my country, as of the world, is surely included in the search for Truth. I cannot suspend this search for anything in this world or another" The wisdom of the decision, which was so to say ratified by the Bombay Congress, was proved by a very simple test. The amendment suggesting the substitution of the words "truthful and non-violent" for "peaceful and legitimate" was

military, to that extent, in my opinion, we must admit our failure."

His soul had already begun to get impatient. How long would this experiment last, how long could he continue to give his guidance if it was ultimately to be futile? We discover this impatience of the soul in numerous utterances of those days. To some co-workers he said: "I am ashamed that our ministries had to call to their aid the police and the military. I am ashamed that they had to use the language that they did in reply to the Opposition speeches. I feel as if the Congress had lost and the British had won. Why does our non-violence fail on such occasions? Is it the non-violence of the weak? Even the *goondas* should not move us from our faith and make us say. 'We will send them to the gallows or shoot them down' They too are our countrymen If they will kill us, we will allow them to do so. You cannot pit against organized violence the non-violence of the weak, but the non-violence which the bravest alone can exercise We have, you will say, been sufficiently non-violent. We were non-violent during the civil disobedience campaign, we received *lathi* blows and worse My reply is this · We did, but not sufficiently. We could not get independence at the end of the Dandi March, as ours was not the unadulterated non-violence of the bravest"

The question of external aggression has cropped up just now, but the question of internal unrest was already there as alive as today. Had we the strength then to say to the *goondas*, "Kill us, we will not kill you"? We had not. But Gandhiji lived in the hope that we should have that strength soon "Supposing," he said to the Gandhi Seva Sangh people at Delang on the 25th of March, 1938, "the Viceroy were to invite the President of the Congress to meet him and to state the Congress terms, do you think he would have the strength to say, 'The Congress is capable of taking charge of the administration, the British may go'? Do you think we could tell him that we should be able to do without the police and the military, that we should be able to come to terms with the Princes, the Zamindars and the Mussalmans? I am afraid we could not honestly say we should easily be able to come to terms with these And yet, if we had real non-violence in us, we should be able to say and do these things."

But if the situation was bad in the middle of 1938, it is worse in the middle of 1940. Someone asked Gandhiji the other day: "You think we have not the non-violence of the brave Well, then, I ask you what would you do if independence were to be offered to you today? Would you say no?"

He said. "I would say no I am giving an absurd answer to an absurd question. The question is absurd, for independence is not going to be offered, because we are not ready. If we were ready, it would be there without our asking it."

But I am anticipating events. There was impatience in the soul, but he was patient with the people. He was arguing with the ministries, arguing with the people. "In spite of our having accepted the volunteer's pledge for 17 years we have not developed the irresistible strength that such acceptance of *ahimsa* means The reason is that we have not troubled, we have not laboured, to organize such a non-violent army If we cannot do so, if we cannot carry out the pledge, it would be well to reconsider our position. The tragedy is that the pledge is still in existence, but it exists on paper. If we had on a sufficient scale such a non-violent army as the pledge contemplates, we should not have had these riots, and if there had been, we would have quelled the riots or immolated ourselves in the attempt. We have heard of only one man who met his death I admire his self-immolation. But my breast would have swelled with joy if there had been several Guptas."

"If," he said in an article written in that period, "the Congress non-violence is merely confined to abstention from causing physical hurt to the British officials and their dependants, such non-violence can never bring us independence It is bound to be worsted in the final heat Indeed we shall find it to be worthless, if not positively harmful, long before the final heat is reached." And then this solemn warning: "If we feel that we shall not be able to displace the British power without a violent struggle, the Congress must say so to the nation and prepare accordingly We must do what is being done all the world over — *forbear when we can, hit when we must* If that is to be our creed or policy, we have lost precious seventeen years But it is never too late to learn and mend."

The long tours of the Frontier Province that followed were with the sole purpose of preparing the promising field for the non-violence of the brave.

And it was not that the ministries wholly failed.
What's done we partly may compute,
But oft not what's resisted.

If we were to go minutely into the history of the work of the Congress ministries in the eight provinces, we would find that they had some triumphs of non-violence to their credit But God fulfils Himself in many ways. If the ministries had continued, would they have been ultimately found wanting? May it not have been providential that the situation created by the war came as a godsend and compelled the ministries to resign? Who shall say? But again I am anticipating events

A Narrow Escape

The test had nearly come with the international crisis of September 1938 and the rape of Czechoslovakia. But the dictated peace at Munich saved us. The Working Committee sat and deliberated for some days, but the test in the shape of war did not come. Gandhiji placed the implications of non-violence before the Working Committee in a manner that could not be mistaken, and the Working Committee was so unanimously with him that he was in a position to report: "The Working Committee had almost come to the conclusion that it would deny itself the opportunity of striking a bargain with England, but would make its contribution to the world peace, to the defence of Czechoslovakia and to India's freedom by declaring to the world by its action that the way to peace with honour did not lie through mutual slaughter of the innocents, but that it lay only and truly through the practice of organized non-violence even unto death."

And in the course of a private conversation he used words which had a prophetic ring, at least so far as he himself was concerned:

"You may rest assured that whatever happens there will be no surrender by the Government For me, even if I stand alone, *there is no participation in the war even if the Government should surrender the whole control to* the *Congress*... Who would have thought aeroplanes to be a practical reality

fifty years ago? Who would have imagined in this country, thirty years ago, that thousands of innocent men, women and children would be ready smilingly to march to prison? The weapon of *ahimsa* does not need supermen or superwomen to wield it, beings of common clay can use and have used it before this with success. At any rate fifteen members of the Working Committee did express their readiness to put their *ahimsa* to the test That was more than I was prepared for"

But we were not weighed, and our honour remained intact At any rate we did not die before our death. What the Committee would have done if the crisis had actually come it is difficult to say, but as we have seen it filled Gandhiji with great hopes

The Crisis

Exactly a year after this the crisis was upon us In that first hour of trial, the Congress was weighed and not found wanting. It did not take the Working Committee long to issue its mandate to the Congress ministries to resign. Gandhiji heaved a sigh of relief The questions that frequently tormented him ceased to trouble his soul. If the Congress had accepted his advice, in the initial stage, of offering unconditional co-operation, or if later the British Government had acceded to the Congress demand for a declaration and the Congress had offered co-operation in response, the position for Gandhiji would have been the same For the co-operation he had contemplated was *moral* co-operation and no other. But who knows? It may be that even there there has been providential intervention. If the Working Committee had then taken the stand—the very honest stand indeed—that it has taken now, the difficulty both for the Committee and for Gandhiji would have been immense, the situation more delicate. But as I have said God intervened

The crisis in the shape of the collapse of one European nation after another in quick succession came none too soon. Gandhiji had waited in patience so long, trying to steer a difficult course through rocks and breakers. He had voluntarily accepted the position of the Generalissimo Was it wise? The speeches at the Ramgarh Congress were a mirror of the struggles of his soul. In private he had implored the Working Committee and the Maulana to relieve him of the position. It was simply with a view to being able to render greater service to the

Congress "I am putting upon myself an undue strain, but as it is not yet a strain on my conscience, I shall continue to serve you, if you will insist on my doing so. But it is a terrible strain, and I would implore you to free me. I should then pursue my experiment of *ahimsa* with absolute freedom." I am quoting from memory, but that was the substance of his entreaty to the Working Committee. And when they did not relieve him he poured out the depths of his soul in the two speeches at Ramgarh.

But the soul's impatience continued. Would the Working Committee face or quail before the coming storm? Perhaps they were taken by surprise, but not Gandhiji. The European situation had for him a lesson that was unmistakably clear. "It fills me with the utmost non-violence," he said. "I cannot think of a better thing to offer to Britain and the defeated nations than non-violence. It is impossible for me to enthuse over the deeds of Hitler or of those who fought or failed to fight him. There is nothing to choose between the victory of Hitler and the defeat of others. But I have no doubt in my mind that even a patched-up non-violent army would take the wind out of Hitler's sails. I need not have his aeroplanes, tanks, etc. He need not destroy our homes. Our non-violent army would welcome him, and it may be that he would not dare to come I know that this may be a day-dream. But I cannot belie the principle of a lifetime or wipe out my day-dreams of the past twenty years. If we have not the non-violent strength of the brave to fight anarchy and aggression, let us say so and reduce ourselves to a small minority hoping to develop non-violence of the strong in the days to come."

The Working Committee did some fierce thinking for days. Those were the days of heart-searching for them. The logic of Gandhiji's propositions was invincible, but could they go with those propositions to the people? Were they so thoroughly saturated themselves with the spirit of non-violence of the brave to be able to carry conviction to the people? Gandhiji even drafted a resolution for them. But fain as they would have it, sorely as they felt the wrench of having to do without his leadership and his advice, they could not be untrue to themselves "We feel we could not accept your position with our mind and heart and soul, and we feel we should not entangle you," they said to him in effect "And if that is so, why should

we accept an untenable position merely to retain your connection? It would be a fraud on ourselves and others."

But if they could not be untrue to themselves, neither could Gandhiji be untrue to himself He said to them: "I must be left free for my self-expression. I must be free to pursue my search, and I know you will believe me when I say that I go only to be of more effective service to the Congress, to you and the nation. Of course I shall be available when you want me But I can no longer identify myself with the direction of your policy and programme You will, therefore, try your best to do without me and have your meetings in future not in Wardha but elsewhere."

It was said of old by a Teacher to his disciples: "Ye are the salt of the earth: but if the salt have lost its savour, wherewith shall it be salted?" Had those who for us have been "the salt of the earth" lost their savour? One cannot say. But there were these words also uttered of old: "Ye are the light of the world A city that is set on an hill cannot be hid' And they may yet be those lights In the meanwhile Gandhiji waits and prays in silence "I should work single-handed in implicit faith that what I fail to make clear to my countrymen today shall be clear to them some day of itself or, if God wills it, through some apt word He may put in my mouth, or some apt work which He may prompt me to do." These words were uttered in 1934, but they might have been uttered only the other day.

New Delhi, 1-7-40 M. D

Harijan, 13-7-1940

VIII

A I. C. C RESOLUTION

The following resolution was passed by the A I C. C at Bombay on 16th September, 1940

The All India Congress Committee has given its careful attention to the events that have taken place since its last meeting held in Poona on July 27, 1940, and to the resolutions passed by the Working Committee at Wardha in August last The Committee approves of and endorses these resolutions

In order to end the deadlock in India and to promote the national cause, in co-operation with the British people, the Working Committee, even at the sacrifice of Mahatma Gandhi's co-operation, made a proposal to the British Government in their Delhi resolution of July 7th, which was subsequently approved by the A.I.C.C at Poona. This proposal was rejected by the British Government in a manner which left no doubt that they had no intention to recognize India's independence, and would, if they could, continue to hold this country indefinitely in bondage for British exploitation. This decision of the British Government shows that they will impose their will upon India, and their recent policy has further shown that they will not even tolerate free expression of public opinion in condemnation of their associating India in the war against Germany, against the will of a vast body of the people of India, and of exploiting her national resources and man power for this purpose.

The All India Congress Committee cannot submit to a policy which is a denial of India's natural right to freedom, which suppresses the free expression of public opinion, and which would lead to the degradation of her people and their continued enslavement By following this policy the British Government have created an intolerable situation, and are imposing upon the Congress a struggle for the preservation of the honour and the elementary rights of the people. The Congress is pledged under Gandhiji's leadership to non-violence for the vindication of India's freedom. At this grave crisis in the movement for national freedom, the All India Congress Committee, there-

fore, requests him to guide the Congress in the action that should be taken. The Delhi resolution, confirmed by the A. I. C. C. at Poona which prevented him from so doing, no longer applies. It has lapsed.

The A. I. C. C. sympathize with the British people as well as the peoples of all other countries involved in the War. Congressmen cannot withhold their admiration for the bravery and endurance shown by the British nation in the face of danger and peril. They can have no ill-will against them, and the spirit of *satyagraha* forbids the Congress from doing anything with a view to embarrass them. But this self-imposed restraint cannot be taken to the extent of self-extinction. The Congress must insist on the fullest freedom to pursue its policy, based on non-violence. The Congress has, however, no desire at the present moment to extend non-violent resistance, should this become necessary, beyond what it required for the preservation of the liberties of the people.

In view of certain misapprehensions that have arisen in regard to the Congress policy of non-violence the A. I. C. C. desire to state this afresh, and to make it clear that this policy continues, notwithstanding anything contained in previous resolutions which may have led to these misapprehensions. This Committee firmly believes in the policy and practice of non-violence not only in the struggle for Swaraj, but also, in so far as this may be possible of application, in free India. The Committee is convinced, and recent world events have demonstrated, that complete world disarmament is necessary and the establishment of a new and juster political and economic order, if the world is not to destroy itself and revert to barbarism. A free India will, therefore, throw all her weight in favour of world disarmament and should herself be prepared to give a lead in this to the world. Such lead will inevitably depend on external factors and internal conditions, but the State would do its utmost to give effect to this policy of disarmament. Effective disarmament and the establishment of world peace by the ending of national wars, depend ultimately on the removal of the causes of wars and national conflicts. These causes must be rooted out by the ending of the domination of one country over another and the exploitation of one people or group by other. To that end India will peacefully labour, and it is

with this objective in view that the people of India desire to attain the status of a free and independent nation. Such freedom will be the prelude to the close association with other countries within a comity of free nations for the peace and progress of the world

Harijan, 22-9-1940

IX

SEVEN DAYS IN BOMBAY

The Seven Days

An author of the history of the Russian Revolution wrote some years ago, in diary form, a history of the happenings of the ten days that preceded the birth of Soviet Russia. He gave that book the title *Ten Days That Shook the World* May it not be that the seven days in Bombay—12th to 18th—may come to be recorded in history as the Seven Days That Shook the World? And in a better sense perhaps? The days in Russia had all the colour and terror of a revolution The Seven Days here had a different character altogether. They were as peaceful as anything can be, and they mark a distinct stage not only in India's march towards peaceful freedom but also perhaps in the march of the world towards peace.

The whole setting was one of peace and not of war. For even if it was a gathering of men who were determined on war for the assertion of their right to self-existence, it was a gathering of people who would not hurt even a fly The Government had, therefore, no need for any elaborate police — not to say of military — preparations in anticipation of what might happen Those who met were the guests of a most peaceful body of people — the East India Cotton Association, who do not usually interest themselves in politics but who at this unique juncture in the history of India braved the risk of being regarded as having identified themselves with politics For they knew that the issue at stake concerned them no less than the rest of India, perhaps more.

The Working Committee was facing the hour of the greatest crisis in Congress history The days since June 18 when they

issued what is known as the Wardha Statement had been days of excruciating travail both for them and for Gandhiji For Gandhiji they had been days also of intense prayer. I may not produce here a diary of the seven days, but Pandit Jawaharlal in his speech at the A I C. C. described in one or two sentences what had happened. He was in bitter pain over the wild and irresponsible utterances of a handful of the members of the Committee, and was trying to plead with them to see the error of their ways. "You know," he said, "what Gandhiji has done? He has accepted full responsibility for the resolution. But do you know how it came into being? How great he spiritually is I do not know. But I know that politically he is without a peer. So far as matters of principle are concerned his iron will will not bend to any appeal for a compromise But he listened to us for hours, let every aspect and argument that we had to press work upon his mind. He produced one draft and then another and then a third until he saw that he had met the considerations advanced by every one of us, and was not content until the result was a resolution that was as far as possible the collective reflection of the mind of us all" (I am not quoting his words, I took no notes, but I am giving the impression left on my mind by his moving speech.)

And as one could see it, Gandhiji seemed to grow from moment to moment. He took two hours to produce the first draft which became the basis of the resolution as it ultimately emerged He did not know that, in spite of the intense mental labour that he had devoted to it, his draft reflected the minds of the members. When he addressed the house in Hindi, though he spoke in firm and measured tones there was trepidation and hesitation "The responsibility I am assuming," he said, "is perhaps the greatest I have ever assumed in my life. I do not know whether I shall succeed in satisfying you or even satisfying me. Nor have I the vaguest notion of the result. But for over 50 years I have trained myself never to be concerned about the result. What I should be concerned about is the means, and when I am sure of the purity of the means, faith is enough to lead me on All fear and trembling melt away before that faith, and once we have launched forth there is no looking back." In the English speech the firmness seemed to have increased, the soul in him cried out: "We have waited

long enough. Now comes the moment when the virtue of waiting has become vice." And yet he said: "There is impenetrable darkness before me regarding the future course of action. Would you trust your ship to a captain who is groping? If you would, well and good. Otherwise reject his generalship summarily." For days he had waited for light, but beheld obscurity A glimmer had appeared on the horizon with the confidence with which he advised the Working Committee. But still there was a pall of haze. The next day it had lifted, and he was able to declare almost in the exaltation of certitude. "The thing I have placed before you is not small. I myself had thought it was small But it was not so It has not come from my intellect. It has come from recesses of the heart where dwelleth the Innermost. It is He who has given it It was born at the end of infinite travail"

Verily once more the words of the Psalmist had come true: "Light is sown for the righteous and gladness for the upright in heart"

When Virtue Becomes Vice

I have in brief given the setting in which the event happened. I shall now proceed to give the outstanding portions of his three utterances They have appeared at length in the Press, but as they are unpruned and unauthorized, one is apt to miss the wood for the trees Mine will be a rapid summary with pauses at passages of outstanding importance.

An esteemed friend had sent him a telegram on the eve of the decision reminding him of the attitude of sympathy and of unconditional co-operation that he had adopted at the beginning of the war He had not forgotten it. But if he could not take the mind and the heart of the Congressmen with him, that attitude would have been useless. "If what I said did not find an echo in their hearts, they would have been of no use whatsoever to the Viceroy, to the great British Nation, and to India." But even after the first resolution on the war adopted at Wardha they had waited for exactly a year. He was not sorry that they had waited, the waiting had been all to the good, it had given him at any rate the strength to adopt the attitude he was taking Why did the very man who had day in and day out advised non-embarrassment now stand before them with the latest resolution? He said "There comes a time in man's life when

virtue itself becomes vice. Virtue which was virtue in its time, when torn from the purpose to which it was dedicated, becomes vice. The exercise of self-restraint was good enough, but when it threatened our very existence, the national spirit, whether it abided in Congressmen or non-Congressmen, when it threatened to kill the very spirit which enabled us to exercise self-restraint, then that virtue becomes vice." He proceeded: "I am speaking not only for the Congress, but for all who stand for national freedom — Muslims, Parsis, Hindus, Christians — even for those who are against the Congress, so long as they represent Indian aspirations, viz unadulterated independence. I should be untrue to all of them if I said now, 'No embarrassment to the British' My repetition of it would be just as useful for my salvation and for the guarding of my virtue as the repetition by a parrot of God's name which is but a mechanical vocal effort without intelligence, and which cannot take him near to God Therefore, if I exercised that self-suppression at this critical moment in the history of the nation, it would be suicidal, I should be untrue to myself. We cannot sit still when we see men like Dr. Ram Manohar Lohia and Jaiprakash Narain, than whom I know no braver or straighter men, being marched to jail not because they preached violence but because they carried out the behest of the Ramgarh resolution. It had become a point of honour with them It is not *satyagraha* to watch thus people being marched to jail in the exercise of their right of freedom of speech. If we looked on, the Congress would disappear and with it the national spirit. India, said the Maulana, at the time of Nankana tragedy, was one vast prison, and therefore, if we delivered her, we would easily be able to free not one but all the *gurudwaras*. If our liberty of speech is choked, the movement for the freedom of our country from bondage is choked Then, as I have said, the virtue of self-restraint is going to become vice. The virtue cannot be exercised, as the resolution says, to the extent of self-extinction — extinction of the national spirit, wherever it resides whether amongst Congressmen or non-Congressmen."

We Learnt It from You

Further exercise of self-suppression, he explained, would be an act unfriendly to Britain We have learnt, from Englishmen, to value the priceless boon of freedom of speech. In

expressing his sympathy for Britain and his admiration of the bravery of the British, Gandhiji brought this point out in a striking way "We have sympathy for all the suffering nations in this war. I wish well of Britain. The destruction of a single innocent child in England shocks me. When I heard of the impending damage to St Paul's I felt as much as I should have if the Kashi Viswanath and the Jumma Masjid were in a similar plight. I know the sentiment that surrounds St. Paul's. The news therefore shocked me But what can I do beyond saying that it has shocked me? In spite of the shock, the British people are playing and dancing and carrying on their business as usual, and we are told that an eleven year old lad writes to his father that he would far rather be destroyed in England with his parents than seek refuge in Canada That fearlessness is in their bones, and if we have learnt anything from them it is that. Even if London and the whole of the British Isles were to fall, they have the courage to carry on the fight from Canada, Australia or New Zealand. Therefore let us emulate their bravery, let us not be carried away by our sympathy to the extent of neglecting our clear duty"

That duty is to declare from the house-tops that we shall have none of this war, we shall do nothing by way of providing men, money or ammunition for the conduct of the war. To let the Viceroy tell the world that the whole of India is with Britain in this war, is to allow him to deceive the world. If the war was being fought for democracy, with what justice can that claim be advanced when democracy was being trampled to death in India? Dominion Status was being promised, but the Provincial Autonomy that was given to the provinces under their own Act was reduced to a mockery. In his impassioned Hindi utterance he asked: "Was Punjab consulted? Was Bengal consulted? Was Sind consulted? Was the Frontier Province with a population of over 90 per cent Mussalmans consulted before India was declared a belligerent? Were the people in these provinces asked whether they wished to join the war? How can one man, however good, be invested with so much power over nearly 40 crores of people? I am a friend of the Viceroy and hope to remain his friend until the end of my days, but the situation created by imposing him as a virtual dictator on India is intolerable. It is a negation of democracy."

The Demand

The Congress has during the past one year exhausted all the means in its power to bring about an understanding with Britain. It asked for a declaration of India's independence It declared that it would be content if British Government were prepared to accept a constitution framed by a Constituent Assembly. It offered a third alternative, and in doing so almost stultified itself, nearly went back, as it seemed to me, on the solemn professions of the past 20 years of its existence It was pledged to winning Swaraj by non-violence, it forgot that in promising Britain its active co-operation in the war effort in case Britain recognized India's independence, it was contradicting itself and offering to pay the price of its principle — for the sake of independence But happily even that offer was rejected Now comes the fourth offer in the field: "Simply declare that India is free to carry on, non-violently and openly, anti-war propaganda, that India is free to preach non-co-operation with Government in their war effort, and we will have no civil disobedience."

That is the lowest possible test of their honesty Civil liberties are the very foundation of democracy, and if you deny them you deny democracy Only the other day a British judge defined the essence of democracy Delivering judgment in an action brought against *The Daily Worker*, Mr Justice Stable observed. "The expression of views, no matter how unpopular, how fantastic or how wrong-headed they may appear to the majority, is a right, and a right which I, among others, are paid to see preserved I subscribe wholeheartedly to the words that fell from the lips of Mr D N Pritt, K C (defending counsel), that it is those views which are held by only a few, those views that are unpopular, those views which do run counter to the views of the great majority of mankind, particularly in times of national emotional crises such as war, it is those views which this court should be particularly zealous to protect." It would seem that what is going on in India is the most flagrant negation of this elementary principle of democracy

This is how Gandhiji put it in his direct simple language: "In order completely to clarify our position, I propose to approach the Viceroy, if he will be good enough to see me — and I have no doubt that he will — and to place my difficulties before

him I will approach him in your name I will tell him that this is the position to which we have been reduced. We do not want to embarrass you and deflect you from your purpose in regard to war effort. You go your way, and we go ours, undeterred, the common ground being non-violence. If our advice prevails, there will be no war effort on the part of our people. If, on the other hand, without your using any but moral pressure you find that they respond, then we cannot help it. If you get assistance from the Princes, from the zamindars, from anybody high or low, you can have it, but let our voice also be heard If you do so, it will be eminently honourable, it will certainly be a feather in your cap It will be honourable of you — although you are engaged in a life and death struggle — that you have recognized this liberty. It will be honourable of you that you do so although you have limitless powers to choke our voice and give us the fullest possible freedom, consistently with the observance of non-violence, to tell the people of India not to join the war effort."

That is the position that would put the Congress and the British Government at ease with one another As Gandhiji put it: "The British can then say to the world, 'Judge us by our conduct. Here in India we are playing the game.'"

What of Freedom?

Then, it may be asked, what about our demand for independence? Independence is contained in the right to declare and preach non-co-operation with the war effort "That liberty," he said, towards the close of his speech, "is the foundation of your Swaraj. If that foundation-stone is in danger, you have to exert the whole of your might to defend that single stone" He explained this thought at greater length in his closing speech:

"I am sorry that your mind has failed to grasp a very simple but a very important point that I made yesterday. I described the right of free speech as the foundation-stone of Swaraj. Let me explain this. I have yet to come across a complete definition of Swaraj I do not think I have been able to give it myself. I do not think even Jawaharlal has ever given it. It is possible that someone from amongst you may be able to give it, as it has often happened that others have defined for me things I have been labouring for If anyone can do so,

I shall gladly make him my *guru*. But this resolution contains the seed of Swaraj. If you hold fast to it, all that the movers of the various amendments desire is automatically attained. If we can win this liberty of free speech — either by fighting for it or by negotiation — we have secured everything — free speech for a communist like Dr. Ashraf, and free speech for a Forward Bloc-wallah like Sardar Sardul Singh. If I give it a religious colour, I can call it full religious liberty, the liberty cultural and religious that the Muslims are asking for. You say you want independence. But you forget that it is beyond their power to give it to us. Nor can we be independent by simply declaring that we are independent. We can be independent only when, after the British have left, we can remain unafraid and rule ourselves unafraid of any attack from any foreign power — Japan, Germany, Russia or Afghanistan. It is absurd to launch civil disobedience today for independence. How are we to fight for independence with those whose own independence is in grave peril? Even if independence can be given by one nation to another, it is not possible for the English. Those who are themselves in peril cannot save others. But if they fight unto death for their freedom, if they are at all reasonable, they must recognize our right of free speech. This they can certainly do. It is our duty to fight for that right. If they resist it and we have to fight and they are embarrassed, they will have invited embarrassment themselves.

"This liberty is a concrete issue which needs no defining. It is the foundation of freedom, especially when it has to be taken non-violently. To surrender it is to surrender the only means for attaining freedom.

"When Rajaji told me that his own offer was easier for the British to accept than mine, he was expressing half the truth. It may be easy indeed for them to recognize our independence, but I can understand, during the war, their reluctance to grant us central responsible government. For once they grant it, they have to carry on through us. One day it may be Rajaji, another day it may be Jawaharlal, and then it may be a Damodar Menon. This co-operation that we offer them would be to them a commodity of doubtful value, for they do not trust us, and if I was in the Viceroy's position, I should understand his misgivings. It is risky for them to carry on war through those whom they

do not trust But where is the risk in letting everyone declare that he is free to refuse all co-operation in war and preach that non-co-operation to everyone he comes across, unless they want to enforce co-operation at the point of the bayonet ?"

The Only Limiting Condition

But the right sought to be asserted is subject to the condition that we observe non-violence. "The condition is necessary, because a Government that is based on violence cannot tolerate violence. That is the unavoidable logic of violence. But even those who are violently inclined need not despair, if they will listen to me today. For if we win Swaraj through non-violence, even the violent, if there are any, will have not only the liberty to preach but to do violence There would be no army. But that will not prevent a Sikh or a Khaksar from possessing a sword or swords. If the rest are all believers in non-violence, what violence can these inflict ? The restraint of speech and action of the majority will automatically exercise similar restraint on those who are contrarily inclined At any rate, if I had my way as the president of a non-violent Indian republic, I should not hesitate to give those who are violently inclined, the liberty of violent speech. On the other hand, let me tell them that under a violent State they cannot expect that liberty

"Let me tell those who want mass civil disobedience, strikes, no-rent campaigns, that they forget that all those things are there in our old resolution of 1920. In fact I have been striving all these years to prepare the country for all these things We should have been able to do all of them and to bring about a new social order by now, if only we had observed the conditions. And if we did not observe the conditions and were not ready, none but we were to blame.

"But you are free to organize mass civil disobedience if you like, though you will then be guilty of indiscipline. But, as I said to Subhas Babu who put me the same question, I shall gladly congratulate you if you succeed, but you cannot have my blessings. Of course, the honourable course for you would be to leave the Congress.

"But you know that you cannot organize mass disobedience at the present stage without violence For similar reasons I am asking students not to leave colleges to join *satyagraha*, unless

they will leave them for good Similarly about strikes. Dr Suresh Bannerji, who was once a co-worker but who has for some years strayed away from me but seems now to be coming back, said to me some time ago that only I could conduct a labour strike to a successful conclusion. I can do all these things, if I have your full co-operation and complete discipline."

The Effect

As regards the immediate effect Gandhiji said "When we come to our own, the Government will be in charge not only of the Congressmen but of non-Congressmen There will be adult suffrage, and the Government will be formed by the elected representatives of all the adult voters — Hindus, Muslims, Parsis, Christians, Jews, and those whom we foolishly have regarded as untouchables. What a mixed parliament of these representatives will do I do not know. But I hope that the Congress vote will always be in favour of non-violence If it is in a minority, it will record its vote, if it is in a majority, it will administer the State non-violently. For the Congress as Congress can do nothing else Supposing the Government accepts our demand tomorrow and says. 'Do what you like in future, but now please do help us with men and money' I am afraid even then we shall have to say to them 'We are sorry we cannot give the co-operation. We wish you well, and may God help you. But committed as we are to the policy of non-violence, nothing that you can offer can persuade us to give you violent assistance.'"

In the interval between now and Gandhiji's return from a visit to the Viceroy, what is to be done? Gandhiji said "I am going to see the Viceroy, but I may fail in my mission I have never approached a mission in despair. I have approached it with the consciousness that I may be faced with a blind wall. But I have often penetrated blind walls. But if I fail, we shall be free to assert our right to carry on anti-war propaganda and take the consequences. You are of course free to carry on the propaganda from now, but it will not be fair, it will not be restraint. I would, therefore, ask you to be patient until my return from the Viceroy."

Since this was written, the Working Committee passed the following resolution which leaves no manner of doubt about the Congressmen's duty during the interval:

"In view of the resolution just passed by the A I C. C, the Working Committee calls upon all Congress organizations to stop all civil disobedience, individual or other, pending definite instructions of Gandhiji. He regards this suspension as indispensable for his pending interview with H E. the Viceroy, and as a test of the discipline of registered and unregistered Congressmen and all Congress-minded men and women, and also as a short course of obedience to law before recourse to civil disobedience, should it become necessary."

The Hindu-Muslim Question

In his closing speech Gandhiji referred to the Hindu-Muslim question, as it had been raised during the debate He said "If a conflict is there in store for us, who can prevent it? We have to be prepared even for anarchy and chaos, but we must have the faith that non-violence cannot result in chaos But if some how disorders take place, they will be a test of our non-violence. Non-violence is a force that gains in intensity with the increase in the violence that it has to deal with I hope that you will get that power of non-violence before I die. But there is a message I should like to reach the ears of every Mussalman. India cannot win independence, if eight or more crores of Mussalmans are opposed to it But I cannot believe that all of them are so opposed, until it is proved to me by the vote of every adult Mussalman. Let them declare that they want to have their political salvation apart from that of the Hindus. India is a poor country full of Hindus and Mussalmans and others staying in every corner of it. To divide it into two is worse than anarchy. It is vivisection which cannot be tolerated — not because I am a Hindu, for I am speaking from this platform as representative of Hindus, Muslims, Parsis and all else. But I will say to them, 'Vivisect me before you vivisect India You shall not do what even the Moghuls, who ruled over India for over two centuries, did not do' What I have said about the Muslims applies equally to Sikhs. If 30 lakhs of Sikhs will obstruct Indian independence, we shall deal with them non-violently. Non-violent Swaraj cannot be won except by non-violence. There are other obstacles, too, imposed by the existence of an alien power. But we must strain every nerve to achieve communal peace Islam means peace That peace cannot be confined to the Muslims It must mean peace for the whole world."

Implications of the Resolution

The representative of the American Associated Press saw Gandhiji in Bombay within twentyfour hours of the termination of the A I C C said "Hundreds of our papers have printed the A. I. C C resolution and a summary of your speeches wired by me, and have commented on it Four questions emerge out of these comments, and I seek your answers to the questions." Gandhiji was immersed up to the ears in work, and was actually attending a meeting of the Working Committee when Mr Stimson arrived. He interrupted the meeting and dictated his answers to the four questions

The first question was "How do you desire not to embarrass Britain with your demand to preach anti-war propaganda in a non-violent way?"

Gandhiji said "Because non-violence is the Congress creed which involves Congress hostility to all war Hence it is a vital necessity for the Congress to dissociate itself from identifying itself with any war Hence my desire not to embarrass Britain was necessarily limited and conditioned by the counter necessity of preserving the Congress existence, and therefore civil disobedience is definitely restricted to freedom of speech and action, provided they are absolutely non-violent Therefore I have claimed in my speech that, if the full import of Congress action is understood, it must in the end help Britain and the world"

"Why," wondered Mr Stimson

"Because in the midst of the conflagration all round there is one powerful body pinning its faith to uttermost non-violence If it succeeds, then the groaning world can heave a sigh of relief and find a way out of these monstrous armaments."

The second question was "How do you visualize the future of India in the event of a Nazi victory?"

A. All I can say is—I am not dismayed by the prospect, if my country remains true to the cult of non-violence But that does not mean that I should be in any way pleased with the prospect of a Nazi victory What terrifies me is that as things are going on at present defeat of Nazism will be bought at a terrific price, viz, superior Nazism, call it by any name you like

Q In view of what has happened in Malabar, is there any hope of mass civil disobedience being carried on non-violently?

A Not at present, and therefore, as you must have noticed, in my speech I made an emphatic declaration that so far as I was concerned there was no prospect of my embarking on mass civil disobedience But if you ask me whether it is possible to conduct mass civil disobedience without its resulting in violence, I would emphatically say 'yes'. But my country is not at present ready for mass action, and in a way I am thankful that the unfortunate events in Malabar have come as a warning to the country and a pointer for me also.

Mr. Stimson's last question was Does your policy mean disapproval of America's help to Britain in the shape of planes and munitions?

A Not in the slightest For the simple reason that America does not believe with the Indian National Congress in non-violent action. I wish it did Then America's contribution to peace and help to Britain would be infinitely more substantial than any number of planes and any amount of material that America can supply to Britain And if the weekly correspondence I receive from America and visits from Americans who come to see me is any index to American opinion, I expect America to take a leaf out of the Congress book and outrun the Congress in the race for the establishment of peace on earth through universal disarmament

Am I Inconsistent?

The correspondent of *The News Chronicle*, who interviewed Gandhiji in Bombay on the 18th, saw him when he had already had a heavy day on the top of a night of insufficient sleep, and when he had heavy engagements yet to fulfil But as he presented a conundrum on behalf of the British people, Gandhiji gladly found time for him and replied to his question which was as follows "While expressing your sympathy with the British people in their plight, don't you think there is an inconsistency between your earlier decisions and your latest decision?"

"I thought," said Gandhiji replying to the question, "that I had clearly and sufficiently explained my position in my speech, in anticipation of the charge of inconsistency. If there is any inconsistency, it is due in this particular instance to changed circumstances. My sympathy is not only the same as it had been expressed in Simla on the declaration of war, it has

become deeper because what was imaginary had become vividly real In Simla almost a year ago I had expressed my grief over what might befall Britain Today the dreaded thing has happened and is still going on. By nature I am so framed that every calamity moves me irrespective of the people whom it may overtake But my sympathy, even though it is deeper today than a year ago, has undoubtedly changed in form I was unprepared for the recent Government declarations, and I claim that it is the genuineness of my sympathy which has made me single out the one fact which Britain can easily recognize and yield without any hitch in her prosecution of war I readily grant that there might be some reason for not dividing the responsibility for the conduct of the war with those who are the determined opponents of British imperialism and all it implies, and therefore I felt that, if the Congress continued to abide by its policy of non-embarrassment which is inherent in its non-violence, the Congress should for the moment abate agitation by way of direct action for indepepndence But freedom of speech and corresponding action is the breath of democratic life. Freedom of propagating non-violence as substitute for war is the most relevant when indecent savagery is being perpetrated by the warring nations of Europe The Congress will forfeit all its right to be considered a non-violent organization, if out of false sympathy or what would be worse—fear of consequences—it ceased to agitate against the inhumanity that is being perpetrated in Europe and which, if not checked by somebody or some organization, may overtake the whole world. I hope this statement of Congress policy as I interpret it, as its sole guide, will not only satisfy the British public opinion, but will make it range itself on the side of the Congress, so as to enable the Viceroy to recognize the justice of the Congress claim, which is a claim not for itself but which is a claim for freedom of speech no matter by whom exercised so long as it does not promote violence in any shape or form"

M D.

Bombay, 18-9-40

Harijan, 22-9-1940

X

MORE ABOUT THE RESOLUTION

For the Masses

In my hurried notes last week on the A. I C C. resolution there were certain points that I omitted and would like to take up this week What has pained Gandhiji most about the Delhi resolution and the Poona resolution was not only that we would contradict ourselves by paying non-violence as the price of independence, when we had all along been declaring that we would win independence by non-violence, worse than that contradiction was the fact that the resolutions, if acted upon, would have been a sad betrayal of the masses whom for twenty years we had made non-violence-minded and whom we would now make war-minded It is for the sake of them, he explained in his Hindi speech, that he welcomed the Bombay resolution "For," said he, " the Congress strength is derived not from the members on the Congress registers but from the millions who have never entered the Congress but who feel that the Congress represents them It is that character that makes the Congress speak for the Hindus, Muslims, Parsis, Christians, Jews and all Throughout its history of over fifty years it has had not only Hindu presidents, but Muslim and Parsi presidents, who made no distinction between different communities, and who proved that those who rendered the highest service to the nation and who desired its freedom could take charge of it. The Congress had been described by some of its critics as a fascist organization But if they admit that the Congress has no other weapon but non-violence, they disprove their charge For Fascism, Nazism and Imperialism depend for their existence on violence The Congress could not hold together without non-violence. Non-violence and *satyagraha*, its active principle, constitute the sovereign remedy which could be applied by all the minorities in the country, and which was for even the smallest minority a sure safeguard against the tyranny of the majority however big it might be Non-violence was thus bound up with the very existence of the Congress, and to give it up would be to betray it and the masses for whom it stood"

A visitor recently asked Gandhiji if his re-entry meant that the Congress had become again non-violent at heart. "So far as I can say from things in the Punjab," said the visitor, "I think that, though there is a good deal of self-restraint, there is little non-violence."

Gandhiji replied. "I agree The resolution is the result of several days' full-fledged debate among the members of the Working Committee I may confess that many of the members have no independent faith in non-violence but they are trying to imbibe it But you must know that the Congress is not merely the Working Committee, nor the A. I C. C, nor the members on the Congress registers, but the dumb millions. They are all peace-loving, and we have to represent them truly. These millions, before the Congress identified itself with them in 1919, had taken no part in any violent or non-violent or even a so-called constitutional fight But they rose like one man on the 6th of April 1919. They took up peaceful rebellion as their *mantra*, and without any organization, without any country-wide tour—for I had not then toured in the interior—they instinctively took up the call, and the Congress organization became a peaceful rebel organization. The Bombay resolution had these masses in mind."

No Threat

It is surprising that a paper should have accused Gandhiji of an intention to go to the Viceroy with the threat of civil disobedience. The very same paper published copious extracts from Gandhiji's speech from which the following are enough to prove that there was no such intention.

"There is one thing which is not mentioned in the resolution. In giving me the authority that the Working Committee has done, there is a reservation. I do not want to hurl civil disobedience or anything in the face of the Government without making my meaning clear, the meaning I attach to the sum total of Government actions—actions beginning with the declaration of the Viceroy, then the statement of the Secretary of State for India, and then the series of actions and the policy that the Government have pursued since.

The sum total of all these things has left an indelible impression on my mind that there is something wrong, some injustice, being perpetrated against the whole nation, and that

the voice of freedom is about to be stifled. It is there in the resolution, not in the exact language which I am using now, but you will see the meaning clear as day-light. In order completely to clarify our position, I propose to approach the Viceroy with a request that he will be good enough to see me, and I have no doubt that he will I will place my difficulties before him; I will place the Congress difficulties before him. I will approach him in your name I will tell him that this is the position to which we have been reduced · We do not want to embarrass you and deflect you from your purpose in regard to war effort We go our way, and you go yours, undeterred, the common ground being non-violence Left free to ourselves, there will be no war effort on the part of our people. If, on the other hand, without your using any but moral pressure you find that they respond, then we cannot help it. If you get assistance from the Princes, from the zamindars, from anybody high or low, you can have it; but let our voice also be heard. If you do so, it will be eminently honourable, it will certainly be a feather in your cap. It will be honourable of you, although you are engaged in a life and death struggle, that you have given us this liberty It will be honourable of you that you take this great step although you have limitless powers to choke our voice, and give us the fullest possible freedom, consistently with the observance of non-violence, to tell the people of India not to join the war."

The Demand

While some have regarded the Congress demand as too difficult to be accepted, some have regarded it as too trifling. A talk that Gandhiji had with an *ashram* youngster may be summarized for the benefit of both. "If," asked the youngster, "the Congress demand is complete, it means that we do not need Swaraj, and that we shall be content with the liberty of speech and the Press."

"Our objective is complete independence, as you know. But do you know the means to attain it?"

"Carrying out the constructive programme."

"That is one of the principal branches of the tree But what is the root?"

"Truth and non-violence."

"Well, then, we want the right to preach truth and non-violence"

"But will newspaper articles and speeches be enough for the propagation of truth and non-violence ?"

"No, we have to do much more But the right to preach truth and non-violence is threatened The law seems to say that we may not tell people that they have every right not to co-operate in the war effort, and that it is their duty as non-violent people not to co-operate thus"

"But you have yourself said that we can get Swaraj if we fulfil the constructive programme Why then this trivial issue ?"

"It is not a trivial issue, it is a concrete and all-important issue If we surrender it, a time will come when we may have to surrender all, when we may have to forget even the name of truth and non-violence To preach these is our birthright, and to forfeit it is to forfeit our existence."

"But what I am troubled about is the emphasis on mere speech and writing."

"It is the liberty we claim How and when to use it and whether to use it or no is our concern. If we cannot stand a strong breeze, we shut the windows and even doors. But how should we feel, if someone were to shut us up ?"

"I see But may it not be imaginary ? If you ask a little child not to eat earth, it will eat it It is the inhibition that is galling Do you want to remove the inhibition ?"

"The analogy is wrong, for the little child has no right to eat earth, whereas no one has a right to prevent us from enjoying free air. But let me take another example for you, as you are a child Prahlad was ordered by his father not to take the name of Rama (God). He might have argued with himself, 'I lose nothing by not repeating the name of Rama, for Rama is in my heart' But if he had resorted to this argument, he would have deceived himself He did not do so, and he defied his father to do his worst, he was not going to give up repeating *Ramanama*. And because he braved the most fearful hardships and dared even death for that sacred right, we have today a living faith in *Ramanama*. If he had yielded, that faith would have vanished from the earth. Even so, if we give up our right to preach truth and non-violence, we give it up for ever."

"But abstract truth and non-violence no one prevents us from preaching."

"No principle exists in the abstract. Without its concrete application it has no meaning And when I want to preach non-violence I want to preach it as an effective substitute for war, and thereby to be able to wean Britain and other warring countries from violence and barbarism"

Why for All ?

"But," someone argued, "why do you claim that right for all ? In your speech you said that you would speak as a conscientious objector, but that others might use any reasoning they liked — e.g., the financial or the imperialistic argument"

"I have no business to claim the right only for myself and for those who are conscientious objectors For some other grounds may be as important as, if not more than conscientious grounds , and if I may not be smothered, how can I suffer those others to be smothered ? If, moreover, the liberty was restricted to conscientious objectors, we should compel a number of our people to be hypocrites, for they would take shelter under the conscientious argument. All that is necessary is the acceptance by all of the condition of non-violence That cannot be relaxed"

A 'Columbus'

Twice during the past few days Gandhiji likened himself to Columbus, — first after the last Wardha meeting of the Working Committee, and then in his speech at the close of A. I. C. C. proceedings. On this second occasion he went into some detail in giving the comparison · "I do not ask you to appoint me your captain as a favour. I would entreat you to do a number of other things, but I do not entreat you to make me your captain If you with eyes open and your minds clear feel that you should make me your captain, you may do so, not otherwise. For, unless I have your minds and hearts with me, we cannot reach the promised land I am in the same predicament as Columbus when he started on his voyage to discover India. Perhaps I am in a worse plight in that I have no chart and no compass which Columbus had. I therefore ask for your implicit obedience and discipline as the ship's officers and crew. We have to be an army

in action tomorrow, and unless you give unflinching discipline, we shall be nowhere. A ship's captain never brooks indiscipline, and may even relieve the ship of the insubordinate crew"

It would be interesting for a moment to see the points of similarity and dissimilarity in the comparison. Columbus before he started on his voyage made himself admiral and assumed full powers as Gandhiji has done His journey was perilous, and though he had a chart and compass, the journey was over uncharted seas But the comparison ends there The bulk of his 88 crews were criminals and vagabonds who had been given the choice between imprisonment and this perilous voyage. They were so indisciplined and wrong-headed that they did threaten to mutiny and even to push Columbus overboard some dark night. To control these men Columbus had to have recourse to deception, threats, promises! The very nature of our struggle makes these conditions impossible, and Gandhiji has promised in advance to make room for a better captain in case he fails, or even without, if people do not want him.

M. D.

Sevagram, 23-9-40
Harijan, 29-9-1940

XI

CREED v. POLICY OF NON-VIOLENCE

[More than a month ago Dr. Katju sent to Shri Kishorlal Mashruwala what may be called a short thesis on non-violence The latter was to decide what to make of it and, if he liked, show it to me. Shri Mashruwala gave it to me some time ago. But I could get time to read it only during the journey. I read it carefully. It was too long for publication in *Harijan* ; and yet I felt that it should be placed before the readers of *Harijan* in some form. The following is the result. I have spent considerable time in reducing the original without leaving out the essential argument I wholly agree with Dr. Katju that non-violence cannot make further headway without the Congress making it a creed He suggests that there should be a plan to show how it can be worked under given circumstances. He suggested a book or a series for the guidance of the votaries. Much literature has sprung up on the subject. Richard Gregg has spent years of labour in research He has written text-books for the guidance of the votaries of the West. His books are very readable. Let Dr. Katju set apart time for producing a book which would be a guide for us in India at this critical hour.

On the train to Wardha, 5-4-42 M. K. G.]

The efficiency of non-violence with non-co-operation as a potent instrument of defence against external aggression has been very much discussed during the last three years Gandhiji has emphasized that what can win us our freedom from British control must also prove equally useful to protect that freedom from violent attack. But the Working Committee were not prepared to go that length, and they said so in the well-known Wardha resolution last year They thought that the country was not prepared to go that length In this, I think, the Working Committee were quite right

The shattering events of the last two years in the present war have influenced men in different ways From many, with the growing realization of India's utter defencelessness, there is an insistent demand for rapid 'militarization' and industria-

lization. On others, however, the reaction has been in a contrary direction, and they now realize vividly, as never before, the utter futility of resort to violence for purposes of defence The collapse of strong, well-armed armies in Poland, Norway, France, Yugoslavia and Greece, and last of all the carnage in Russia, have shown to them — and I am one of those persons — that armed defence leads nowhere. If civilization and human liberties are to be saved, we must seek and ensure peace by means of non-violence It is obvious that non-violence can be of no use to nations imperialistically inclined to conquer and exploit others. That, I take it, is not the ambition of any Indian. So the question is only of the defence of a free India.

It must be conceded that, though the Congress has been working on non-violent lines for the last twenty years, that has only been as a matter of policy Leaving Gandhiji aside our leaders have not presented to the people the excellent doctrine of non-violence with all its possibilities and implications. Few, very few, have believed in it as a creed, and anyone who advocates its adoption as a mere matter of policy is likely to break down at the critical juncture I think that on those of us who believe in the matchless efficacy of non-violence and non-co-operation as an instrument of defence and protection against external aggression now rests the duty of infusing into, and inspiring, our countrymen with that faith.

Previous preparation is essential Just as you cannot turn an ordinary citizen into a trained and disciplined soldier overnight or manufacture guns and aeroplanes in a week, similarly you require time and patience to train a whole people in the art of non-violence and non-co-operation.

Violence has stolen a long march during thousands of years. There are so many misconceptions to be removed Non-violence is unintelligible to so many To some it is a counsel of perfection, fit for *rishis* only To others it is stupid; there is seemingly such a childlike faith in violence Non-violence ceases to be stupid when it is considered as non-co-operation. The present-day aggressor does not conquer in order to exterminate and settle down. He conquers or seeks to conquer in order to exploit, and it is at that stage that non-co-operation comes in

We have to explain, to expound, to remove doubts, and to endeavour to make even the lame, the halt and the blind into

heroes All this requires time and devoted effort And this can only be done with any reasonable prospect of success by those who themselves have a firm faith in the efficacy of the method of non-violence and whose personal lives are regulated by that faith. You cannot possibly divorce this doctrine from the purest and highest morals. And no distinction should be drawn between personal and public affairs. What is ethically bad in an individual is equally bad for a community and a nation. We must believe that it is feasible to conduct public affairs on that basis, and we must ourselves act accordingly to the best of our power and ability; and we must endeavour, by precept and example, to create a similar belief in our countrymen at large Without this belief, I am convinced, non-violence cannot make any headway at all Therefore those who advocate non-violence must live a life of non-violence, not resisting evil with evil but sterilizing it with non-co-operation The non-co-operation has to be above reproach.

I am by no means pessimistic of success, if only those of us who profess faith in non-violence are true to our creed. All philosophy seems to inculcate non-violence and chastity in thought and conduct and love of truth The masses will follow, if properly educated in the doctrine. The doctrine of *ahimsa* has ever been there, and people have, all through the history of the race, regulated their private life by it. It is Gandhiji's contribution to world welfare that he has taught us to apply non-violence and non-co-operation in the field of politics and public affairs also. This is a signal experiment and should succeed particularly in India if we proceed on right lines

It is from this point of view that I suggest that, while we must endeavour by all truthful and non-violent methods to persuade our countrymen to cling to the path of non-violence to aim at conducting the government of the country by non-violent methods and to win our independence and retain and protect it by non-violence and non-co-operation, there should be no such thing as an effort to capture power in the vulgar sense of the word. The idea of 'capture' more often than not connotes use of Tammany Hall methods, political jugglery and cunning, and improper and corrupt practices There is no place for these in our schemes of things. We can only succeed, if we have the people genuinely behind us. Political power, not resting on

the willing and genuine consent of a vast majority of the people, can only function, in the last resort, by the use of force and violence; and that use we forbid to ourselves. So our whole programme of action must be directed to persuade the people to our ways of thinking by methods of public education, study circles, public meetings, private discussions and the Press, and above all by our exemplary lives.

Insistence on mere non-violence without concrete action is apt to be misleading You have all sorts of conundrums put before you to show that non-violence is merely another name for cowardice and timidity. This false notion is partly due to the fact that equal insistence is not laid upon non-co-operation Non-violence and non-co-operation are two wheels of the chariot And taken together they require the greatest courage and sacrifice in their votaries. It is the weapon, as Gandhiji is never tired of saying, of the bravest, and what is more, it is the bravery of spirit that is the requisite thing Thus it permits everyone, man, woman and child, the strong as well as the weak in body, to avail of it, provided one is stout-hearted and firm of faith and devoid of fear. It is non-co-operation which gives power and strength to non-violence, readiness to die, and a determined refusal to obey and co-operate with evil

Just consider our present situation. We blame Gandhiji for lack of complete success in our political struggle for freedom, and we cavil at the method he advises But have we really followed his advice ? Is not British rule in its entirety based on our own co-operation ? Do we not serve our masters and often even take pride and glory in such service ? The army, the police and the civil services are manned by us, and our brightest young men labour strenuously to put on such shackles and consider them adornments We are ourselves — I refer particularly to the English-educated middle classes — the greatest sinners, and yet we blame Gandhiji And has the method really failed ? In spite of our tragic shortcomings, in spite of our weakness of faith, in spite of our half-hearted and apologetic pursuit of the method as a mere matter of policy, our success has been great indeed. The political awakening among the masses is marvellous.

The administration of a big country like India cannot be carried on by any foreigner, the Englishman or whoever he may

be, without Indian co-operation. And if that co-operation is withheld, foreign domination is not possible This is the lesson we have to burn into our minds, the minds of the masses as well as of the classes

It is in this sense wholly true to say that non-co-operation is a powerful weapon for the defence of the country against an aggressor. As I have already said, extermination is a thing of the past Aggressors aim at exploitation by making the subject people work for them and for their profit. And work is co-operation Non-co-operation will make exploitation impossible and thus aggression itself unprofitable The process of non-co-operation — we are aware of it — will put our non-violence through terrific tests. But if we can stand assaults, torture and shootings without any retaliation or even an attempt at retaliation from our side and persist in non-co-operation, the struggle must end in success Examples of such heroic sufferings are not wanting Guru-ka-Bagh *morchas* of the Sikhs, the Dharasana episodes, and the behaviour of processionists at innumerable places are shining examples of the practice of non-violence on a large scale. And the struggle of Bardoli peasants for relief against excessive land taxes is a valuable lesson in non-co-operation

Much is said, in order to terrify people, of the results of the use of violence by the invaders and aggressors. But the thing is obviously overdone. I believe in the first place that human nature, though often bad and vile enough, will ultimately recoil from persistent cruelty and torture and slaughter of non-resisting non-co-operators. Sufferings cheerfully borne will melt the heart of even a savage.

And after all does war — without non-co-operation — really sustain freedom ? In the first place, national policies based on force and violence as instruments of defence lead to a race for armaments, and that race by itself indubitably leads to war. Moreover there is no finality in armaments The machine masters and ultimately overwhelms mankind , and then what happens to the vanquished ? Once the army is overthrown and defeated, people are forced to surrender.

What we have really to show is that people can act courageously and suffer hardship and even death without the temporary stimulus, and indeed intoxication, of war. A well-or-

ganized mass movement based on non -violence and non-co-operation for the noble purpose of winning or retaining national independence must raise people, not only young well-built soldiers but even the old and the infirm and men and women alike, to greater heights than a bloody war ever can. It is the hearts of the leaders that doubt and quail, but the heart of the common people is sound enough.

Non-co-operation with the aggressor and the foreigner is plain enough. That will obviously involve the non-payment of taxes, a refusal to enter his service or do his bidding, a refusal to work in his mills and factories, and non-resort to his law courts But it may become necessary even to non-co-operate with those of our countrymen, our own kith and kin, who flout the national will and co-operate with the aggressor. The thing to remember is that our non-co-operation has always to be non-violent and will, therefore, never do personal injury to the opponent. So there should be no interference with the supply of water to the thirsty or food to the hungry, medical assistance to the sick or burial to the dead Within these broad limits a community is entitled to protect itself from its own backsliding brethren And where public interests so demand, even filial and all family considerations must yield National freedom is above everything Human history teaches us that no nation has ever been able through violence to retain freedom and defend its independence against superior violence Given the correct training, this is possible only through non-violent non-co-operation

Non-violence is not a mere negative concept It is essentially a constructive contribution to world politics The present structure of society puts a definite premium on violence Vast aggregates of private wealth in the hands of individuals who also constitute the ruling classes in the community naturally favour violence as a means of defence. It is now a commonplace that often so-called national interests are in substance nothing but the interests of private capitalists and investors in foreign lands. And even in local disturbances it is the moneyed man, the man of property, who shouts the loudest for the use of force for his protection The adoption of non-violence as a national policy will inevitably involve a radical alteration in the social and economic spheres.

It is obvious that national wealth must be more widely and equitably diffused and social inequalities must disappear. Every citizen must have equal opportunities to grow. Much in socialism and communism will be found useful and admirable in a non-violent society, the basic difference being that a believer in non-violence holds that changes can be brought about by peaceful persuasion and non-violent methods, whereas those who profess the other doctrines do not share this faith and think that violent expropriation is the only course

A believer in non-violence does not wish to abolish capitalism by violence. He wishes the community to avail itself of the results of individual enterprise, and he takes all sting out of capitalism when he converts a capitalist into a trustee for the benefit of the nation. If a capitalist were really to consider himself a trustee, he would never endeavour to swell his profits by sweating labour If his wealth were ultimately to be applied for the benefit of the community, there would be no incentive left to enrich oneself by improper and Shylockean methods. He would, even in the process of earning his profits, genuinely endeavour to benefit his fellowmen as widely as possible

The real considerations in a society founded upon non-violence may well be two-fold Firstly, we have no outside countries to exploit, no foreign markets to capture, with the aid of our armies; and secondly, there must be profitable employment for every fit person I do not think that in a free India the adjustment of industries will be beyond the bounds of human ingenuity The needs of the hand-spinning industry will have to be particularly borne in mind, capable as it is of giving employment to millions of people who would otherwise be wholly without work In planning an economic programme suited to our needs we can draw with great profit on the great social experiment in Russia. I personally believe that working on co-operative lines is the true way to national salvation and to non-violence also.

For the propagation of non-violence on sound lines it is necessary that a picture — at least in broad outline, but distinct and well-defined, not vague and shadowy — of a well-planned non-violent society should be drawn and placed before the public. Such a scheme would serve many purposes. It would educate and make people think, and the plan itself would benefit

by public discussion and criticism. As it is, people are left in doubt and do not know where non-violence will land them and India.

I realize that, like the experiment in Russia, the way to perfecting a plan of a non-violent society must be through trial and error But basic principles must be settled and (as far as possible) precisely stated, and outlines well drawn. Shri Mashruwala has recently made attempts in that direction His articles published in newspapers and magazines were both thoughtful and thought-provoking.

I dare say that adequate material for drawing up a preliminary outline as a basis of discussion for planned non-violence can be found in the writings of Gandhiji spread over a period of 40 years Valuable hints can also be gained from Tolstoy and other eminent writers. But at present the material is all scattered, the picture much too hazy. Like the famous *Communist Manifesto*, we should have a manifesto of non-violence and non-co-operation

The political and economical structure of non-violent society must be broad-based on the conscious will of the people as a whole If force as a dominating sanction is eliminated, then cheerful and willing obedience to the national laws is the prerequisite to any durable constitution. People will only offer non-violent resistance to an aggressor, if they are proud of their country and prize their national freedom. But if national freedom merely means for a vast majority of the people a slavish, care-worn existence with ever-present fear of want and hunger, then it will not be surprising if there is not too keen a readiness to offer non-violent resistance and to die for an order of things which has done so little for the individual

In a non-violent State there should not be any communal questions or any minority problems. When the highest rules of morality and ethics become the mainsprings of political policy and administrative action and non-violence actually becomes the foundation of national life, then there will not be the bickerings and rivalries and struggle for power with which we are nowadays so familiar

The aim, it is said, of all good government is to cease to govern, for the State to fade away and for society to become classless. These aims can only be realized by genuine non-

violence. Government will be truly the government by consent. Problems of poverty and social amelioration confront all communities and require similar treatment. It is the fear that majorities will resort to the use of force and violence which poisons the air. The moment non-violence becomes the instrument of national policy the majority will have no sanction at its back other than the willing consent of the minority, and if it wishes to rule (if rule it can be called at all), it can only do so by winning the confidence and disarming all suspicion of the minorities. Indeed in a fully non-violent society there are no minorities, for all are one people

The conclusion, in my view, is that in our present environment non-violence can only succeed as a creed and nothing else Just as Soviet Russia made a deliberate attempt to alter the whole structure of society and to modify all the current and accepted notions of human nature by conscious State effort, similarly those who believe in non-violence and non-co-operation as the future hope of humanity must acquire power in the body politic in non-violent ways, and then endeavour to make non-violence the only essential instrument of our national policy. I personally think that the ideal of non-violence is not unrealizable. On the contrary I think that it can be reached, and that humanity, after the experience of the horrors of two world wars in the course of a quarter of a century, will willingly strive after it But to attain that goal we shall have to educate public opinion in its favour, more particularly by radically planning the whole field of education on the basis of non-violence We must begin with the child, and mould the mind of the coming generation accordingly.

So far non-violence (*ahimsa*) has been practised to regulate men's personal lives, and therefore one life, well-lived according to the principles of *ahimsa*, has influenced many others But the sphere of influence has been limited to the region of private and personal affairs. Unfortunately, during thousands of years, mankind has been taught that private life and national life are things apart, governed by totally different principles. Gandhiji teaches us that there is no such distinction. That lesson can only be learnt by the people as a whole, if those in charge of national affairs first learn it themselves and then endeavour to impart it to others I fear we have not done our

duty by the people. We have doubted and hesitated ourselves. How then can we blame the people? Let Congressmen first be firm believers themselves, give a little further time to the people in general, and then put them to the test.

K. N. KATJU

Harijan, 26-4-1942

INDEX